AIRPLANE DESIGN

PERFORMANCE

BY

EDWARD P. WARNER, A.B., M.S.

Fellow of the Royal Aeronautical Society; Fellow of the Institute of the Aeronautical Sciences; Member of the Society of Automotive Engineers; lately Professor of Aeronautical Engineering at the Massachusetts Institute of Technology; lately Vice-Chairman of the Federal Aviation Commission; lately Editor of Aviation

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PREFACE

Nine years ago this summer I published a book on the Aerodynamics of Airplane Design. Nine years is a long time in technology, and nowhere more so than in aeronautical science. The volume that bore the imprint of 1927 has fallen badly out of step with the art, and its revision and modernization have become progressively more important. This is the revision.

Or, at least, it was started as a revision. As the work went forward it became apparent that there was so much that needed to be added, and so little that could remain unchanged, that revision gave way to complete rewriting and that in turn to subdivision. A treatment of the aerodynamics of design adequate for the serious student can no longer be confined within a single pair of covers and not grow too bulky to handle. The subject has been split, and performance alone, and the basic aerodynamic laws and phenomena and collected data which control performance, are treated here. Stability and control will follow in another volume.

In writing in 1927 it was possible to look back at short range on the work of the pioneers who designed and built aircraft before there was any true science of aeronautical engineering. Those days seem very remote in history now; and by a large proportion of the present practitioners of the arts of aircraft design the existence of a time when research did not dominate the designer and guide his every step can hardly be comprehended, it lies so far outside their personal experience.

Aside from such particular, and sometimes revolutionary, innovations in design as the system of cowling for air-cooled engines developed by the National Advisory Committee for Aeronautics, the most important features of the decade in applied aerodynamics have been the provision of wind tunnels allowing tests to be made substantially under full-scale conditions, and an enhanced recognition by engineers of the practical usefulness of basic aerodynamic theory. In 1927, except for an occasional and gradually increasing use of induced-drag formulas, the typical

designer thought of himself as having very little use for the offerings of the mathematicians, and he had very little disposition even to inquire into their application to his own work.

He is much more disposed in that direction now. The higher branches of mathematical physics have taken a place, and gained a universality of respect, such as they hold in no other branch of engineering science, unless it be possibly in the application and transmission of alternating currents. But however favorable the designer's inclination may now be toward the mathematician, his competence to cooperate directly in the mathematician's work is still in most cases very limited. Mathematical aerodynamics remains a pursuit returning results only to the specially trained and specially gifted few. Among students of aircraft design not one out of fifty will ever make, or will have the special qualities required for making, a personal contribution to the extension of basic theory. Not many more than that one out of fifty will ever experience any need to have followed the theory's derivations from source to final conclusion, and indeed only a small minority of students have the training and the gift in general mathematics that would qualify them even to tread those lofty paths in the footprints of a preceptor.

For the few so favored and distinguished, preceptors are at hand. I have made no pretension to adding to their number. Few as the successful students of the theory may be, all who engage in aircraft design must use the theory's results. For them, a physical explanation of the phenomena, a mechanical analogy, a demonstration that the indications of the rule and the formula furnished by the mathematicians are essentially in accord with common sense, seem far more important than an attempt to hammer through the processes of mathematical analysis.

In introducing the book that appeared in 1927 I observed: "I have written with the object of exacting from my readers only a very modest knowledge of mathematics of collegiate grade. Proofs and derivations have been ruthlessly omitted wherever they would have involved undue mathematical complication or where they were not necessary to an understanding of the results and their application. Bare formulas without explanation, to be applied by rote, have been avoided where possible, but even a crude picture of the physical phenomena which can be tied up to the formula, and from the adoption of which by assumption

the formula could be reproduced if forgotten, has been accepted in lieu of complete analytic treatment: The charge that rigor is lacking must then leave me unmoved. For rigor *qua* rigor I have not gone out of my way to seek."

Those sentences still define the essential purpose that I have kept in mind, and the spirit in which I have sought to pursue it. The results of the theory and its applications are infinitely more important now than they were nine years ago. I have given them correspondingly more space; I have enlarged in some detail upon such matters as the nature of the boundary layer and the effect of conditions there. But I have still written primarily for those who are more concerned with being able to use the results of the theory and to relate them to practical experience and observation than with having a full comprehension of the processes whereby they actually came into being.

As material useful in design accumulates with accelerating rapidity, digestion and correlation become more and more necessary. It becomes progressively more hopeless for a student of the art to maintain a personal familiarity with the whole store of primary source. It has been my purpose throughout so to choose and to arrange the data and the citations that they would fall into a coherent pattern and as far as possible illuminate each other. Coherence and mutual illumination have called for the development of new groupings in most cases, and of new forms of presentation in many. Wherever new groupings have been made or new forms and methods introduced, clarity has been the primary objective.

When I first wrote on airplane design I wrote as a teacher, and thought of the book primarily as a text. Since then I have come as an editor to know at first hand students and would-be students far outside the academic halls, and I have encountered evidence that a book written primarily as a text, and which to be sure I have had the satisfaction of seeing quite widely adopted for that purpose, has been quite as extensively used by enthusiasts bent on self-education. The questions that many of them have raised in correspondence have been in mind in rewriting, and I express my appreciation to them now as I expressed my appreciation nine years ago to the students with whom I had been working before that time. I hope that they and their successors, both inside the schools and outside, will find that their interest has

provided the means of making this book more useful than its predecessor.

Acknowledgments are due to a number of friends in the American and European industries who have allowed the use here of material that they had collected for their own purposes, and most especially to the personnel of the Wright Aeronautical Corporation and the Pratt & Whitney Aircraft Corporation, from whom came the illustrative charts of power-plant characteristics used in Chapter XIII. They are due to the teachers of various aeronautical engineering courses who have made suggestions and pointed out defects in the presentation first adopted. Acknowledgments are due particularly to Dr. George W. Lewis of the National Advisory Committee for Aeronautics, and to Miss Margaret Muller, in charge of the Committee's Technical Information Division, whose assistance in discovering references and in procuring material for illustrations has been unfailing. Acknowledgment is due to my wife, Joan Warner, for constant help in the preparation of material and for suggestions without number.

EDWARD P. WARNER.

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BIBLIOGRAPHICAL NOTE

The data of aeronautical engineering spring from many different sources, but, at least for those readers who give a natural preference to material presented in English, two of them are primary. The American National Advisory Committee for Aeronautics and the British Aeronautical Research Committee are responsible for the presentation of most, and on the whole the most important, of the aerodynamic data that become newly available. The great majority of what they publish is the product of their own laboratories, but not all of it by any means originates there; for both the American and the British official bodies present from time to time the results of the more important work, or work that might otherwise escape public attention, of other governmental bodies and of completely unofficial ones, or even of individuals.

To avoid a tedious reiteration in almost every footnote of the names of one or the other or both of the official committees, reference to their work is made throughout the present volume with identification of the individual organization concerned only by the form of the reference. Thus, the National Advisory Committee for Aeronautics, more generally referred to as N.A.C.A., produces three main series of publications: its *Technical Reports; Technical Notes;* and *Technical Memoranda.* They are referred to in footnotes throughout this volume simply as *Rept.*, as *Tech. Note*, and as *Tech. Memo.*, with the appropriate number following, and wherever any of those abbreviations appears it is to an N.A.C.A. publication that reference is made. Similarly, the British Aeronautical Research Committee publishes its results in the form of a long series of Reports and Memoranda, more commonly known as *R. & M.*'s, and so referred to, with their appropriate numbers for identification, in the footnotes.

The N.A.C.A. publications are divided into three categories, on the general line that the *Technical Reports* include the most important work, or that considered particularly worthy of permanent preservation for reference; the *Technical Notes* record less significant studies or, in many cases, preliminary

phases of investigations which will later be more exhaustively covered in the *Report* form; while the *Technical Memoranda* are in almost every case translations of important papers that have appeared in languages other than English. The *Technical Reports* are bound into annual volumes; the *Notes* and the *Memoranda* are available only in pamphlet form. The British *R. & M.*'s are also bound up annually, and the particular annual volume which preserves any individual *Report* or *R. & M.* can be determined in every case for the American publications and in most cases for British ones from the table here appended. The tabulation in the British cases is not rigorously applicable in every instance, for the annual compilation of the British Reports is not made on a strictly serial basis, and sometimes an *R. & M.* appears interpolated among numbers much higher or much lower than its own, in an annual volume other than that in which it would naturally be expected to find a place. The tabulation is extended to include an indication of numbers of the *Technical Notes* and *Technical Memoranda* published in each year, to show how rapidly they have been appearing and to allow a check on the date of issue of any particular item in the series.

Year	*Rept. Nos.*	*Tech. Note Nos.*	*Tech. Memo. Nos.*
1915	1– 7		
1916	8– 12		
1917	13– 23		
1918	24– 50		
1919	51– 82		
1920	83–110	1– 21	
1921	111–132	22– 56	
1922	133–158	57–114	
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1926	233–256	228–247	332–381
1927	257–282	248–266	382–431
1928	283–308	267–298	432–481
1929	309–336	299–319	482–532
1930	337–364	320–349	533–584
1931	365–400	350–393	585–639
1932	401–440	394–431	640–686
1933	441–474	432–472	687–723
1934	475–507	473–504	724–754
1935	508–541	505–542	755–776

Date of British Annual Rept.	*R. & M. Nos.*	*Date of British Annual Rept.*	*R. & M. Nos*
1909–10	1– 26	1921–22	741–818
1910–11	27– 42	1922–23	819–858
1911–12	43– 66	1923–24	859–903
1912–13	67– 94	1924–25	904–970
1913–14	95–145	1925–26	971–1028
1914–15	146–186	1926–27	1029–1085
1915–16	187–242	1927–28	1086–1149
1916–17	243–329	1928–29	1150–1236
1917–18	330–471	1929–30	1237–1328
1918–19	472–630	1930–31	1329–1397
1919–20	631–675	1931–32	1398–1449
1920–21	676–740	1932–33	1450–1535
		1933–34	1536–1589

STANDARD NOMENCLATURE AND SYMBOLS

The nomenclature followed throughout the volume is that of the National Advisory Committee for Aeronautics, as laid down in *Rept.* 474, except where specific indication to the contrary is given. The symbols used in formulas and charts, and in mathematical work generally, adhere to standard practice so far as there is any standard, but in many cases there is none. The tabulation on this page and the next indicates practices adopted as standard for the purposes of the present book. Some of them have the sanction of widespread use, but others are adopted as reasonable and convenient by the present writer and have no sanction except his preference.

ρ Air density (in mass units not in weight; as, in slugs per cu. ft. rather than in lb. per cu. ft.)

μ Viscosity of a fluid.

Kinematic viscosity of a fluid $\left(=\frac{\mu}{\rho}\right)$

q $\frac{1}{2}\rho V^2$ (total kinetic head)

E Modulus of elasticity, either of a solid or of a fluid.

g Acceleration due to gravity.

N Reynolds' number.

W Weight

S Area (with a subscript to indicate the particular area meant. When there is no subscript, it is the area of the wings of an airplane or the cross-sectional area of a streamline form or other nonlifting object that is referred to.)

b Span

c Chord

R Aspect ratio

G Gap of a biplane

D Propeller diameter

V Speed

Velocity of sound (also used, in stability calculations, for component of velocity parallel to the wing span; but not so used in this volume.)

h Altitude

H Ceiling, or limiting altitude

P Power

T Thrust

Q Torque
n R.p.m. or r.p.sec. of a propeller
η Efficiency of a propeller
L Lift
D Drag
Z Force normal to some axis fixed in the body, as to the chord of a wing
X Force parallel to some axis fixed in the body
w Vertical component of velocity
α Angle of attack
β Blade angle (for propellers)
ϵ Angle of downwash
C_L Lift coefficient
C_D Drag coefficient (Symbols can be made in accordance with this same principle for the coefficients of normal force, longitudinal force or any other force component)

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CHAPTER I

INTRODUCTION AND NOMENCLATURE

"Aerodynamics," says the official guide to American aeronautical nomenclature, "is the branch of dynamics which treats of the motion of air and other gaseous fluids and of the forces on solids in motion relative to such fluid." It treats, in other words, of all the relations between an aircraft and the medium in which it moves, and it is obviously a science which lies at the very foundation of the whole aeronautical art and which is of vital importance alike to the designer and to the operator of aircraft. An airplane is a vehicle supported by the dynamic reaction of the air against inclined surfaces, and the study of that reaction and of the laws which govern it is a branch of aerodynamics. In moving steadily through the air, certain resistances are encountered, and they, too, come under the head of aerodynamic phenomena, as does the action of the air on the blades of the propeller which furnishes the thrust necessary to overcome the resistance. Finally, the direction of the aircraft in accord with the pilot's will necessarily depends in the case of an airplane, and to some extent in the case of an airship as well, on a modification of the dynamic reaction of the air against the various parts of the machine, so that the study of control and stability, too, fits within the scope of the broad definition with which this paragraph began.

Classification and subdivision are necessary preliminaries to the consideration of a subject so extensive, and the general science of aerodynamics may conveniently be divided into two parts: one relating primarily to phenomena existing under steady conditions, corresponding to steady flight of an airplane, and the

other involving departures from that steady motion. The distinction between the two is not and cannot be absolutely rigorous, but it serves as a means of making a convenient if rough separation. Problems of the first class may be called those of performance, since practically all of them work back finally to a study of the speed and climb of the airplane and of their variations with changes of form and surrounding conditions, while the second group may be similarly gathered under the general heading of stability and control.

Nomenclature.—The possession of a common language is a prerequisite to any discussion. The terminology of aeronautics has become highly specialized since 1910, and a scientific nomenclature has been superposed on the purely practical vocabulary that the men who fly and maintain airplanes have developed for their own purposes. The unfortunate result is that several different words are often used with the same meaning, and that the same word often means different things in different localities. An examination of the nomenclature is therefore a step to be taken very early in any study of aeronautical engineering.

Although this volume nominally deals only with performance, it is difficult to go very far in that subject, and especially in its practical application, without needing a knowledge of the general constitution of an airplane, the names of its parts, and the terms used in describing its principal dimensions and characteristics. The subject of nomenclature may therefore be treated as a whole and at once. In such a treatment there need be no attempt to give lengthy and exact definitions. Those have been furnished in the reports of governmental and other organizations. Both in the United States and in Great Britain great attention has been given to nomenclatural questions, and the aeronautical dictionaries that have been issued are the results of lengthy conferences among all the interested parties.[1]

The generic name of *aircraft* is applied to every vehicle which is supported by the air. When that support is static, the result of buoyancy, as in the case of the balloon and airship, the aircraft

[1] For the latest American practice, the last of a number of revisions and now accepted as standard by all government departments dealing with aeronautical matters, see "Nomenclature for Aeronautics," *Rept.* 474. Standard British nomenclature has been covered in a report of the British Engineering Standards Association.

is referred to as *lighter than air*. When the support is dynamic and the result of relative motion between the air and some part of the structure, the aircraft is described as *heavier than air*. It is with heavier-than-air craft only that the present volume is concerned.

Types of Aircraft.—Heavier-than-air craft may in turn be divided into several classes. There is the *helicopter*, which depends for its lift on the direct vertical thrust of air propellers, and the *ornithopter*, which has wings flapping in imitation of those of a bird. No such vehicles as these, however, have attained any measure of success up to the present time. The helicopter has shown occasional signs of promise, but as yet the field belongs primarily to the *airplane*, an aircraft fitted with fixed wings and driven by some auxiliary mechanical means, and to the *autogiro*, an aircraft sustained by a free-running rotor which revolves in a horizontal plane above the fuselage and which is kept rotating only by the action of the air due to the passage of the machine through the air. The rotor is not connected to the engine, and therein lies the fundamental point of difference between the autogiro and the helicopter. The autogiro, like the helicopter, can descend vertically, but it normally requires a short run along the ground prior to ascent (though in 1936 it is just in process of being emancipated from that necessity by a technique known as the "jump-off"). Akin to the autogiro in that they depend on a constant motion of the lifting surfaces with reference to the body of the aircraft are the *gyroplane*, in which a free-running rotor has blades that feather and change their angles to the plane of rotation as they revolve, and the *cyclogiro*, a paddle-wheel device with its lifting surfaces driven to rotate around an axis extending out laterally on either side of the body. Both are highly experimental, the latter so much so that no cyclogiro has yet flown. The only other member of the family of heavier-than-air craft which need be mentioned at all is the *glider*, an airplane lacking an engine and having the mechanical drive replaced by a propelling component of the force of gravity.

The present treatment will be confined almost exclusively to the design of the airplane. Gliders and other specialized types will be introduced from time to time for purposes of comparison, but there will be no attempt to cover those forms at all fully. The autogiro is at present a semiproprietary device

covered by patents held by the inventor Juan de la Cierva, and the basic theory has been developed almost exclusively, and is to a large extent held in confidence, by Señor de la Cierva and his associates.

The term *airplane* has been adopted for official use in the United States and has gained universal acceptance here. In Great Britain, however, the older word *aeroplane* is still used with exactly the same meaning.

The classification of airplanes can proceed on several different lines of division. They may, for example, be divided into *landplanes*, *seaplanes*, and *amphibians*. Those terms, almost self-explanatory, relate to airplanes arising from and returning to the land, to the water, and to either at will.

Second, a division may be made to depend on the number and location of the supporting surfaces. A *monoplane* is an airplane which has all of its supporting surfaces located approximately in the same plane. Monoplanes are further grouped into *low wing*, *high wing*, and *mid wing*, describing the level on the body or *fuselage* at which the wing is mounted, and *parasol*, where the wing is clear above the body with an open space between. A *biplane* is one with two sets of superposed surfaces, while a *triplane* has three such sets. For further superposition such terms as quadruplane and octoplane are sometimes coined, but as a rule any arrangement more complex than that of the triplane is simply called a *multiplane*. A *tandem* machine is one in which the main supporting surfaces are arranged one behind another, a grouping first used on a large scale by the late Professor Langley.

Final division may be made to depend on the location of the propellers which drive the airplane. A machine with the propellers at the front is a *tractor*, one with the propellers in the rear a *pusher*. The terms are useful in description of a new design, but some difficulty arises and more elaborate explanations become necessary when there are several propellers located in various positions. In certain airplanes fitted with three engines there have been one tractor and two pusher propellers, while on the four-engine types two pairs of propellers in tandem have often been employed, giving two propellers in front of the wings and two behind. While speaking of propellers and their location, incidentally, mention should be made of the fact that

British writers have abandoned the term *propeller* in favor of *airscrew*, on the ground that the older word connotes the idea of something which pushes, while an airplane is generally pulled through the air. *Airscrew* is sometimes, though not often, heard in America.

Airplane Parts.—From the classification of the types of airplanes, a natural step is to the listing of the principal parts of the structure and to the way in which the characteristics of those parts are specified. The structure of an airplane may in general be divided into four main groups, *wings, body, landing gear,* and *tail surfaces.*

No one would hesitate in pointing out the wings of an airplane, and yet curiously enough the definition of the term *wing* has occasioned very great difficulty and given rise to much difference of opinion. The fact that a wing is ordinarily a one-sided structure has made it seem desirable to keep the same type of definition for an airplane. A bird admittedly has two wings; yet a monoplane which follows the bird in general arrangement of surfaces is often spoken of as having one. The officially approved definitions have changed from time to time, but the one now accepted endows the monoplane with only one, the biplane with two, a single wing being considered as extending clear from tip to tip. When only a part of that structure is referred to, the term *wing* is still employed, but with a distinction established by the addition of another adjective, the upper wing being broken up into the right upper wing and left upper wing. A popular nomenclature under which two things are added together to produce one bearing the same name as each of the two is admittedly unscientific, but hard to escape in this instance.

The *wing cell,* or *cellule* as it is sometimes called, is the complete assembly of wings and bracing members. In a machine with two or more superposed wings the cell is divided into *bays*, each bay including all the structure between two sets of points of attachment of interplane bracing members. The analogy with the use of the same term in building construction is obvious. A biplane or a triplane may then be described as single bay, two bay, and so on, depending on the number of bays in the cell on each side of the center line.

In a wing combination, especially in a biplane or triplane, there are several elements of the bracing system which receive distinct

names. The *interplane struts*, as the name implies, are those members which run vertically or nearly vertically between the upper and lower wings. The term *strut* is used despite the fact that designs are sometimes so made that a number of these members are in tension under normal loading conditions.

To complete the arrangement of trussing, the struts are ordinarily supplemented by certain wires. In the simplest and most straightforward type of biplane truss every bay has the form of a six-sided solid, and structural stability and rigidity generally prescribe a separate system of braces in each of the six faces. A pair of diagonal wires is therefore used in each face, so that there are altogether three sets of wires, each including the diagonals in two opposite sides of the bay. The appearance from the front is shown in Fig. 1. The wires which run upward and outward are called the *lift wires*, or *flying wires*, since they carry the load in normal flight, while the opposite

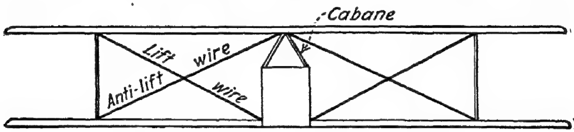


FIG. 1.—Biplane truss, front view.

diagonals are the *antilift* or *landing wires*. The lift wires are often double, as a safeguard against complete collapse of the structure in the event of the failure of a single wire. The plan form of a wing is illustrated in Fig. 2, and the *drag* and *antidrag wires* are there shown, the former being those which run back-

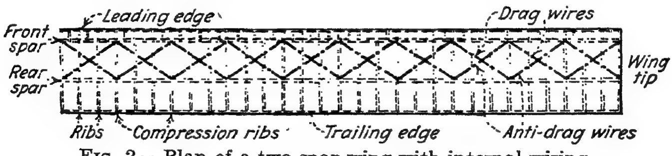


FIG. 2.—Plan of a two-spar wing with internal wiring.

ward and outward away from the center line of the airplane. Most of this internal gear is now eliminated, in many cases, by the use of a sheet-metal skin on the wing in place of a fabric covering. Metal-covered, the wing acts as a unit, and the loads that it bears are distributed through the shell without need for internal trussing members.

Finally the side view (Fig. 3) shows the wires which primarily save the trusses from twisting or vibrating, and which are referred to in America as *stagger wires* and in British parlance as *incidence wires*.

In recent practice there has been a general tendency to depart from the simple six-sided box form of bracing for a biplane cell, modifying the directions of the wires to make them serve two or more purposes. The rear lift wire, for example, is often led forward at its lower end to somewhere near the inner end of the lower front wing spar, so that it may have a component of tension resisting the *drag*, or rearwardly directed air force, on the upper wing. In some cases the front and rear lift wires are actually crossed, the attachment of the inner and lower end of the former being behind that of the latter. (The wires always take their names of "front" or "rear" from the location of their attachments to the upper wing.) The lift wires then help to perform the normal function of the stagger wires in resisting torsion of the wing cell. Frequently, too, the stagger wires themselves are replaced by a rigid diagonal capable of sustaining either tension or compression, so that the truss in profile has the collection of struts and wires shown in Fig. 3 replaced by a single N-shaped assembly of three steel tubes.

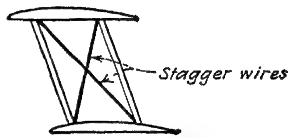


FIG. 3.—Wire-braced biplane truss, profile.

Figure 2, which shows the interplane bracing arrangement in the plane of the wing, displays also certain structural parts, the

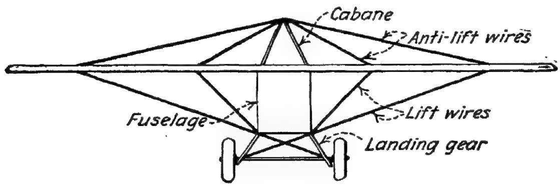


FIG. 4.—Wire-braced monoplane, truss arrangement.

most important of which are named in the illustration. The detailed discussion of their nature and functions, however, will be postponed. The only other structural element of the wing truss which needs special mention appears in Fig. 1. A *cabane* is a small pyramid of struts rising from the body to support the

upper wing, or in some cases directed downward to carry the lower one. The same name is applied to an assembly of similar form which runs up to support the antilift wires of a wire-braced monoplane, as shown in Fig. 4. Monoplanes of that form, however, although once very popular, are now relatively rare. They have given way to the *strut-braced,* or *semicantilever,* monoplane illustrated in Fig. 5, and to the pure *cantilever* type with all the structure contained within the relatively thick tapered wings and no external bracing at all, as in Fig. 6.

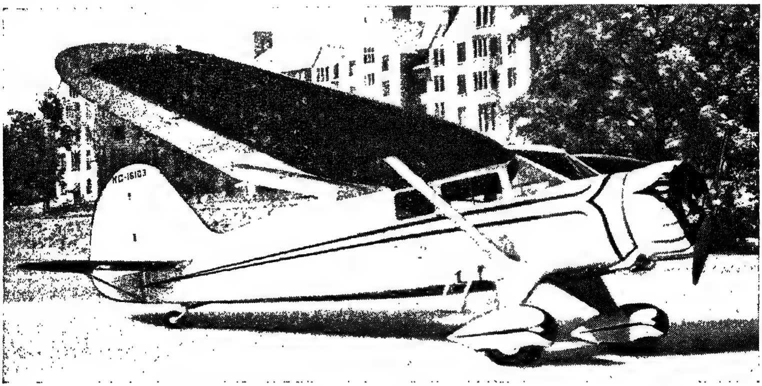


FIG. 5.—Strut-braced high-wing monoplane.

With reference to the nomenclature of the other parts of the structure, the body group can be quickly disposed of. The commonest type of body goes by the name of *fuselage,* a term borrowed directly from the French. In order to come properly within that class, the body must serve as a connecting member between the wings and tail surfaces and also provide space for the housing of part or all of the crew or for some part of the airplane power plant. A structure so slender that it serves merely as a support for the tail should rightly be described as a *tail boom.* When the body is designed merely to accommodate the crew or an engine and does not extend to the tail, it is called a *nacelle* instead of a fuselage. Nacelles are most commonly employed on multiengined airplanes, to contain or support the wing engines.

The portion of the structure which embodies the wheels or skids and their supports is called the *landing gear* in America and the *undercarriage* in Great Britain. The commonest form of landing gear consists of two main wheels and a *tail wheel* or *tail skid*, the latter placed near the rear of the fuselage. Other skids are sometimes used to protect the structure from injury in a bad landing, notably near the tips of the lower wings, but those are not figured as parts of the landing gear, as they are designed to come in contact with the ground only under abnormal conditions.

Landing gears are sometimes made *retractable* or so designed that they can be drawn up within the wings or fuselage to reduce

FIG. 6.—Cantilever monoplane, low-wing.

the air resistance and increase the speed. Under the control of the pilot, and using either manual or electric power, they are folded up after attaining flight and lowered again just before landing.

The landing gear almost always contains some sort of *shock absorber*. In the past that was commonly of rubber cord or a little pile of rubber disks, but it is now general practice to use a *shock strut* or *oleo strut*, an oil-filled cylinder in which a piston moves to dissipate the shock of landing by forcing oil through a small orifice calculated to offer a substantially constant resistance to the motion of the piston, and so to the relative vertical motion of the axle and wheels relative to the fuselage and other rigidly connected parts of the airplane, immediately after the landing impact occurs.

When an airplane bases its operation on the water instead of on the land, the landing gear is replaced by a *float* or *hull*, the

former term being used for a closed structure having no function except to furnish buoyancy, while the latter applies to a more elaborate flotation member accommodating the crew and combining the functions of a landing gear with those of a fuselage or nacelle. The general group of *seaplanes* may be correspondingly divided into *float seaplanes* and *boat seaplanes* or (more commonly) *flying boats*.

In the case of an amphibion, a wheel landing gear and a float or hull as well must be fitted, with means for raising and lowering the wheels to get them out of the way when operating from the water. There is, however, an intermediate class of devices known as *flotation gears* which are used on landplanes required to fly over the water, especially those in the naval service. The object of a flotation gear is to make it possible to remain afloat after a forced landing, but not to permit the machine to take off from the water again.

The fourth main subdivision of the structure was referred to as the tail-surface group, but in dealing with its nomenclature all control and stabilizing members may be included, whether or not they are a part of the tail.

Taking the control surfaces first, they are three in number, designed to produce rotation of the airplane around its three principal axes. The *elevator* (sometimes there are two or more) is a surface which is normally horizontal or nearly so, and which can be rotated at the will of the pilot about a horizontal axis perpendicular to the direction of flight. The elevator is ordinarily placed at the rear of the machine as a part of the tail, but it may be put at the extreme front, well forward of the wings, without losing its identity or changing its name. Airplanes with the elevator in that position were once common but are seldom seen now. They are classed together as of *canard* type (or *Ente*, from the corresponding German expression), from the fancied resemblance to the appearance of a duck when in flight.

The *rudder* fulfils a function corresponding to that of the elevator and is mounted in much the same way, but rotating about a vertical axis to steer to right or left. It too may be placed at the front instead of the rear, but such a location is even rarer for the rudder than for the elevator.

Finally, the third control is furnished as a rule by *ailerons*, auxiliary surfaces attached to the rear edges of the wings near

the tips. They are fixed so that they rotate about a transverse axis parallel to that on which the elevator turns, and the object is, by turning the ailerons on the two sides oppositely, to increase the lift on one side and decrease that on the other so as to cause the airplane to roll. In rare instances the ailerons are mounted between and independently of the wings of a biplane and are then called *interplane ailerons*. Another possibility is the use of *tip ailerons*, extended beyond the tip of the wing instead of hinged to its trailing edge. The arrangement of the wing structure so that the wings themselves can be warped is still another alternative, once much used but long since abandoned because of the loss of structural rigidity. Still others, showing promise of growing favor, are a variety of schemes for controlling the flow of air around or through the wing near the tip. The *slot-lip aileron*, a partially shuttered opening through the wing developed for lateral control purposes by the National Advisory Committee, seems at the moment of writing to be giving the most encouraging results.

All these control surfaces are operated by the pilot with a *stick* or *wheel* and a *rudder bar* or *pedals*. The stick control is most commonly employed on all machines up to about 4,000 lb. gross weight, the wheel above that point. The elevators are operated by a fore-and-aft motion of the stick or of the control column which the wheel surmounts, the ailerons by transverse motion of the stick or rotation of the wheel. The rudder bar or pedals (almost always the latter in recent design practice) supply directional control.

A control surface of a sort, used as a rule only at the moment of landing, is the *flap*. Some airplanes have a part of the rear-most section of the wing made up as a separate element, like an aileron, and so connected up that it can be pulled down at a very large angle to its normal position to improve the performance in landing. The movable portion, usually covering only the part of the wing between the ailerons, is the flap. The theory and practice of its functioning are to be described in a later chapter.[1]

The stabilizing surfaces are various in number and arrangement on different machines, but a few types nearly always appear. The first and the most important is the *stabilizer* in

[1] Chap. X, *infra*.

American nomenclature, or *tailplane* in the British, and is a normally fixed surface, normally nearly horizontal and forming part of the tail. The elevator is as a rule hinged directly to the rear of the stabilizer. In a few instances the stabilizer has been eliminated, the entire horizontal tail surface being made movable and serving as an elevator. The stabilizer is in many cases movable through a few degrees of angle with a special slow-motion control, the object being to permit the pilot to adjust the balance of the airplane and reduce the load on the elevator control in steady flight. Such adjustability is much less common now than a few years ago, the same end being attained much more simply by attaching little separately adjustable flaps, or *tabs*, along the rear edges of the control surfaces themselves. Tabs may be used on the rudder and ailerons, as well as on the elevators.

All other stabilizing surfaces are generically described as *fins*. When the term is used without qualifying adjective it refers to a part of the tail that bears the same relation to the rudder that the stabilizer does to the elevator, being a fixed vertical surface to which the rudder is hinged. The complete omission of the fin is more common than that of the stabilizer.

Though all airplanes in regular service or regularly upon the market have a full complement of control surfaces arranged in the normal fashion, *tailless* machines have been the subject of much experiment, beginning in 1908 but particularly since 1929. On those craft, as a rule, all control comes from ailerons, which can at the will of the pilot be operated either oppositely on the two wings in the usual manner or in the same direction on the two, and from movable fins above the wing tips.

Another possible aberration from normal practice that has been the subject of much study since 1934 is the omission of a movable rudder and the limitation of the vertical tail surfaces to a single rigid fin. The object is to simplify the operations of control by reducing the independent controlling members from three to two, and to eliminate stalling hazards.

Nomenclature of Airfoil Characteristics.—The nomenclature of the most important parts of the structure having been thus summarized, attention may be given to the naming and method of measurement of dimensions and characteristics. Most of the more important figures appertain to the wings and to the wing trusses, and a start may be made with those which relate to the

single wing. Typical wing sections are shown in Figs. 7, 8, and 9. Such a form is technically known as an *airfoil section*, or *aerofoil section*, the distinction corresponding to that between airplane and aeroplane. Fundamentally, a wing considered from the aeròdynamic point of view, quite apart from any question of structure, is properly called an *airfoil*. The models

Fig. 7.—Airfoil section with concave lower surface.

made of metal or wood for testing in aeronautical laboratories, for example, are ordinarily spoken of as model airfoils rather than as model wings.

The first and most important dimension of an airfoil section is the *chord*, as that is the datum line from which all other characteristics are measured. It is a quantity of which the definition becomes a little difficult in some instances, particularly

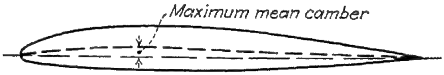


Fig. 8.—Double convex airfoil section.

in airfoils of types recently developed and adopted. In the case of the section shown in Fig. 7, it is obvious that the chord should be taken as the line connecting the *leading* and *trailing edges* of the wing, these being the points farthest forward and farthest back, respectively. In Figs. 8 and 9, however, the choice is less obvious. Where, as in the first of those illustrations, the lower surface of the contour is convex throughout, the chord is

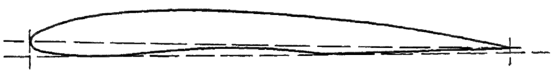


Fig. 9.—Airfoil section with doubly curved lower surface.

arbitrarily taken as the line connecting the two points which are farthest apart in the section, as the leading and trailing edges are generally arcs of circles instead of coming to sharp points. The case in Fig. 9 is more difficult, as either one of the two lines shown might be used. No real standard has been reached, but as a convenient working rule it may be said that the method

which applies to the double convex section should be followed wherever there is the slightest room for doubt. There is in fact an increasing tendency to use it in *all* cases. The chord is the length of projection of the airfoil section on the chord line just defined; that is, the length of the chord is the distance between two perpendiculars drawn tangent to the section at its foremost and rearmost points.

The chord is the longitudinal dimension of an airfoil section, and the vertical dimension is the *camber*. The camber of the upper or lower surface at any point is the height of the corresponding point on the contour above the chord and is measured perpendicular to the chord. The *maximum mean camber*, or maximum camber of the median line of the section (shown in Fig. 8), is much used by students of aerodynamics as a basis of comparison of airfoil properties. It is commonly specified either as a fraction or as a percentage of the chord.

The specification of the cambers of the upper and lower surfaces at a series of points along the chord of course fully defines the form of the airfoil section. The aerodynamic performance of a section is, however, dependent not only on the form but also on the attitude at which the surface is presented to the air. That is defined by the *angle of attack*, or in British usage *the angle of incidence*,[1] which is the angle between the chord of the section and the direction of the relative wind. Since the performance depends so directly on the angle of attack, the importance of accuracy in specifically defining the chord becomes more apparent than before, the chord being used as a uniform datum from which to lay off angles. For a fast airplane flying at its maximum speed, a change of a single degree in the angle of attack may change the lift as much as 30 per cent. The relative wind is of course the resultant of the motion of the airplane relative to the ground and the motion of the air relative to the ground. For an airplane flying through still air the direction of the relative wind is directly opposite to that of the flight path, and its velocity is equal to the speed of the airplane relative to the ground. It should be emphasized that the angle of attack is always taken with respect to the relative wind and so depends

[1] The term *angle of incidence* is more often used in American practice to define a permanent design characteristic, the angle which the chord of the wing of an airplane makes with the axis of the fuselage.

primarily on the direction of the airplane's flight path, not on the attitude of the machine with respect to the earth. An airplane may be diving very steeply, possibly at an angle of as much as 45 deg. below the horizontal, and yet at that instant have an angle of attack very near to 0 deg. or, on the other hand, it may climb at an angle of 15 or 20 deg. while the angle of attack is only 6 or 8 deg. The angle between the axis of the airplane and the ground is called the *angle of inclination* or *angle of pitch* and is quite different from the angle of attack or of incidence. Only when the airplane is flying a horizontal path is the angle of pitch equal to the angle of attack.

The angle of attack is generally denoted by α.

Dimensions of Wings and Wing Combinations.—When, instead of only the section of the airfoil, the wing of a monoplane is considered as a whole, other dimensions are found, and other factors which require specification. The first is of course the *span*, which is the distance from tip to tip across the wings. The term *span* should never be used for a measurement running only to the center line. In those cases in which the wing tip is not square or in which part of an aileron projects beyond the tip of the rigid portions of the wing, the span should still be taken between the extreme tips. Practically defined, the span of an airplane is the width of the narrowest door through which the airplane could pass, moving along a line perpendicular to the plane of the door.

The ratio of the span to the chord of an airfoil is called the *aspect ratio*, and is a very important factor in airplane performance. It is, of course, always much larger than unity, as the airplane wing works most efficiently when shaped to present its longest dimension perpendicular to the direction of motion. When the wing is tapered so that the chord is not constant, the determination of aspect ratio by the definition just given becomes more difficult, and it is then more convenient to use the formula always employed in any case by many writers, $R = b^2/S$, where R is the aspect ratio, S the wing area, and b the span. The two methods obviously give identical results for square-tipped wings of constant chord.

As a general rule, the tips of the wings are not square. They may simply be beveled off so that either the leading or trailing edge is materially shorter than the other, an arrangement called

rake, or they may be rounded off in a substantially symmetrical fashion to form a semielliptical tip. The latter shape, though perhaps less pleasing to the eye than the raked form with small fillets at the leading and trailing edges, has been gaining favor rapidly in recent years by virtue of being superior to all its rivals in aerodynamic efficiency.

Wings have been built with a great variety of plan forms, and a variety of considerations may dictate the choice of a particular one. Cantilever wings, for example, are generally tapered both in plan and in thickness, chord and thickness both being steadily reduced from the root toward the tip in the obvious interest of structural efficiency and to keep down the wing weight. A shape occasionally employed to improve the balance of an airplane, to give it more lateral stability, or to increase the pilot's field of view, has the leading and trailing edges bent backwards so that the tips of the wings are behind the inner ends. This arrangement is called *sweepback*, and the amount is defined by the angle between a line drawn midway between the leading and trailing edges of the wing and a line perpendicular to the plane of symmetry of the airplane. The angle of sweepback is seldom greater than 6 deg. in modern designs, although a few types designed for specialized stability characteristics, particularly among the tailless machines, have run it as high as 30 deg.

For wing area as for aspect ratio, no problem of measurement would arise if the plain rectangular form were the standard rule. Here again complications occur in special cases, and especially in determining the allowance to be made for the part of the wing screened or totally occluded by the fuselage or nacelles. The National Advisory Committee for Aeronautics has agreed that of several solutions, no one of which is wholly satisfactory or theoretically sound, the simplest is the least objectionable. It stands therefore as the official decision that "for the purpose of calculating area a wing shall be considered to extend without interruption through fuselage and nacelles" and that "that part of the area which lies within the fuselage or nacelles shall be bounded for purposes of measurement by two lateral lines that connect the intersections of the leading and trailing edges with the fuselage or nacelle, ignoring fairings and fillets." This becomes clearer in the light of Fig. 10, where a particularly

troublesome case is shown with the area as given by the definition crosshatched.

Similar problems arise in determining control surface areas, and are similarly solved. The stabilizer is given credit for the area of fuselage included within a continuation of the normal stabilizer contour. The rudder and fin, on the other hand, receive no credit for the portion of the fuselage that immediately underlies them. Only the area of rudder and fin proper, project-

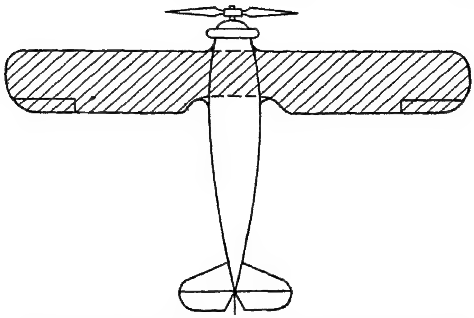


FIG. 10.—Convention for measuring wing area.

ing above or below or behind the fuselage, is measured in that case. In all these instances, as with the wings, fairings and fillets are ignored in measuring.

Considerations of lateral stability, which lead to the use of sweepback on some occasions, furnish a motive also for a setting

FIG. 11.—Wing with dihedral angle (front view).

of the wings at a *dihedral angle*, or in other words for the raising of the wing tips, as shown in a front view of the wings in Fig. 11. The use of at least a small amount of dihedral is almost universal.

The amount of dihedral is measured by the angle between the plane of a wing and the plane parallel to the wing chord and perpendicular to the plane of symmetry, as indicated in Fig. 11. This practice is not in accord with the measurement of the dihedral angle between two planes in texts on solid geometry, but it has marked advantages of simplicity and convenience.

When the amount by which the wings are raised is doubled, it is much more rational to think of the dihedral as changed from 2 to 4 deg., rather than from 176 to 172 deg., as the conventions of solid geometry would require.

Finally there is a group of definitions that relate only to biplanes and multiplanes. The *gap*, the first of these terms, is the distance between the chords of the upper and lower wings or, in the case of triplanes, between the upper and middle or middle and lower wings. So long as the wing chords are parallel, there is little opportunity for confusion in measuring the distance between them, but when they make an angle with each other more careful definition becomes necessary. The gap is then taken as the distance between the planes of the chords of the

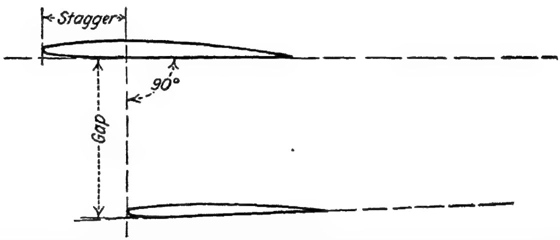


FIG. 12.—Staggered biplane wing arrangement and dimensioning.

upper and lower wings, measured along a line perpendicular to the upper chord from the leading edge of the lower wing.

Another term used only in connection with machines having two or more superposed wings is *stagger*. That is the amount by which one wing is set ahead of the other and is measured by the distance from the leading edge of the upper wing to the foot of a perpendicular dropped from the lower leading edge to the upper wing chord. The stagger is positive when the leading edge of the upper wing is ahead of the foot of the perpendicular or when the upper is in advance of the lower. The method of measuring gap and stagger is illustrated in Fig. 12.

There is one other angle for which a definition should be given, more especially as it is the source of some linguistic confusion. The term *decalage* in American practice refers to the angle between the chords of superposed wings and is usually taken as positive when the upper wing is set at the larger angle. Exactly

the same word, however, means *stagger* in French, and it has also been used on occasion to denote the angle at which the stabilizer is set with respect to the wings. In any event, it is seldom encountered in English at the present time.

The upper and lower wings of a biplane are not always alike in size or form. They sometimes differ in chord, and very frequently indeed in span. When the latter is the case, *overhang* is said to exist. Overhang is the amount by which the tip of one wing projects beyond the tip of the other. It is therefore equal to half the difference of spans, and is called positive when, as is usually the case, the upper wing has greater span than the lower.

These are the most important and most general terms, the ones which are constantly recurring in any discussion of aeronautical science applied to the airplane. Other words, equally specialized in meaning but used only in connection with a particular part of the subject, may be neglected until their actual use in the text becomes necessary.

CHAPTER II

SOME GENERAL PRINCIPLES

Fluid Resistance.—When any solid object, of whatever form it may be, is moved through a fluid, there is a necessary disturbance of the particles of fluid which lie within or immediately adjacent to the space through which the solid passes. In disturbing those particles and moving them to new positions, while they or others flow back to fill the space after the obstruction has passed, energy must be expanded. That expenditure of mechanical energy is expressible in terms of a force which must be applied to propel the solid body through the fluid. The required force is commonly described as the *fluid resistance*, or, in the particular case of a body moving through air, the *air resistance.*

It is mathematically demonstrable that a solid free from sharp edges and discontinuities of form, moving through a "perfect fluid" (defined as one which is incompressible and which has no viscosity or power of opposing shearing motions between the fluid particles and so of creating frictional resistance), would encounter no resisting force.[1]

In actual practice, however, no perfect fluids exist, and a resistance is always present, its magnitude depending on the nature of the fluid and on the size and the form of the solid body.

It is easy to demonstrate, again, in the special case of the perfect fluid, or in the case of a fluid considered as made up of an infinite number of solid elastic particles each following its own course without interference with the others, that all forces and pressures must be proportional to the square of the speed of motion, other things being equal, and that the total forces acting

[1] To reproduce or to discuss at length the mathematical proofs of such conclusions as this would merely confuse the issue in a text intended for the use of the aircraft designer. A full discussion of the motion of a perfect fluid around a solid obstacle and of the pressures and forces resulting from such motion will be found in any good book on hydrodynamics (see, for example, "Hydrodynamics," by Horace Lamb, Cambridge, 1916, or the article "Hydromechanics," in the Encyclopaedia Britannica).

must vary as the extent of the area on which they act. In the particular case of a jet of fluid striking a solid obstruction at right angles to the jet's own course, it is plain from the most elementary theoretical mechanics that the force exerted by the fluid upon the obstruction must be equal to the reactive force of the obstruction upon the fluid, and that the latter is equal to the product of the mass of fluid being acted upon at any one time by the acceleration imparted to the mass. It is impossible to determine the two factors separately, but it is clear without mathematical proof that the aggregate of all the effects for all the particles of fluid will be equal to the product of the total mass of fluid reaching the obstruction in a unit time by the total vectorial change in velocity undergone by each particle. It is, in other words, equal to the total change of momentum in the fluid in unit time, taking into account changes in direction as well as in absolute velocity. The mass of fluid arriving each second is $\rho S'V$ where ρ is the density, S' the cross-sectional area of the jet at a distance from the obstruction, before it has shown any effect of the approaching impact, and V the speed of the jet.

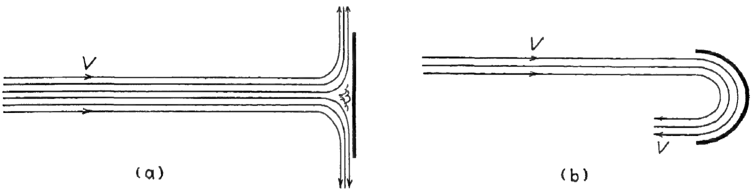


FIG. 13.—Reaction of fluid streams against obstructions (idealized).

If the obstruction is shaped as in Fig. 13*a*, so as to let the fluid pass off just at right angles to its original direction of motion, the change of velocity is equal to V and the total pressure on the obstruction should be $\rho S'V^2$. If on the other hand the shape is that of the bucket of a Pelton wheel, as indicated in Fig. 13*b*, so that the jet is turned back on itself, the velocity is changed from V to $-V$, a decrease of $2V$, and the pressure is twice as large as in the other instance.

When a solid object is immersed in a great mass of fluid, relatively to which it moves, the demonstration is not quite so simple but the momentum principle still remains valid. It is still true that the total force upon the solid is substantially

equal to the total change of momentum taking place in unit time, and that the direction of the resultant force is directly opposite to the direction of the resultant momentum. In speaking of "momentum of the fluid" we are of course concerned exclusively with relative motion, and all velocities are measured relative to the solid upon which the aerodynamic force is to be studied. It makes not the least difference whether the solid is being drawn through still air at velocity V or whether it is being held stationary in a steady wind of the same velocity.

The general validity of the area-and-speed-squared law of resistance variation is then plain, and it becomes possible to write for a given fluid and a given form of solid the resistance equation

$$R = R_C S V^2 \tag{1}$$

where R is the total force, R_C a resistance coefficient to be experimentally determined, unless it should happily prove possible to compute it for some particular cases, S an area selected as a convenient index of the size of the moving object (in the case of an object such as a sphere or the hull of an airship the area selected is that projected on a plane perpendicular to the line of normal motion, but the area of a wing is always taken as the actual extent of its surface projected on the plane of the chords, even though the plane of that surface is normally nearly parallel to the wind direction), and V the relative speed between the fluid and the body.

A further study of the theory, with more regard to the actual nature of real and therefore imperfect fluids, shows however that Eq. (1) cannot be rigorously justified, and a little experiment suffices to prove that in some cases it definitely does not express the truth. For certain objects the effect of doubling the speed of motion may be only to triple the resistance; for others or under other conditions it may be to augment it to five times its original value, instead of increasing it fourfold as the formula indicates that it should. The law of correspondence between air resistance and rate of change of momentum, stated above, remains essentially valid, but under some conditions the whole type of flow of the fluid about the solid may change as the speed changes, and the change of momentum is then no longer proportional to V^2.

A slightly more elaborate analysis, employing the theory of dimensions,[1] leads to a somewhat more involved and a strictly correct equation of the form

$$R = \rho S V^2 f_1 \left(\frac{VL\rho}{\mu}\right) f_2 \left(\frac{V^2\rho}{E}\right) \tag{2}$$

In this equation ρ is the density of the fluid in which the motion takes place, L is a convenient linear dimension of the moving solid, μ the viscosity coefficient, or coefficient of internal friction,[2] of the fluid, and E its modulus of elasticity; the other symbols having the same meaning as before. $VL\rho/\mu$, often written VL/ν where ν is equal to μ/ρ and is called the kinematic viscosity, is known as the Reynolds' number. It takes its name from Osborne Reynolds, British physicist of the mid-nineteenth century, pioneer worker on fluid resistance and dimensional theory.

The function $V^2\rho/E$ is a measure of the compressibility of the fluid in relation to the velocity of motion. It is often written V/v, where v is the speed of transmission of sound through the fluid (1,130 ft. per sec. in air at standard sea-level atmospheric conditions), for the rate of propagation of a sound wave through a gas or a liquid is proportional to $\sqrt{E/\rho}$. The velocity-ratio form has the advantage of having a certain physical significance, for experiment and theory agree that compressibility of the fluid begins to have an important influence on its motion, and so on the forces set up, as V/v approaches unity. When the ratio is less than 0.5 the compressibility effect is almost insignificant. In practice it is of negligible importance for most of the current problems of flight, and can be disregarded in studying the forces acting on airplane parts. It becomes serious only in its effects on the flow around propeller blades, of which the tips

[1] This proof also is omitted, belonging rather in a textbook on aeronautical research methods than in one on the application of aerodynamics to design. It is given in a very simple form in "Aeronautics in Theory and Experiment," by W. L. Cowley and H. Levy, Chap. IV, New York, 1918, and in a somewhat more conventional one in "Aeronautics," by E. B. Wilson, New York, 1919. A much more complete discussion of dimensional problems, including this particular one, from the point of view of the mathematical physicist is contained in "Dimensional Analysis," by P. W. Bridgman, New Haven, 1922.

[2] Defined and discussed in Chap. VI.

may be moving through the air at from 700 to 1,300 ft. per sec., and upon the air forces on the rigid structure of racing planes or of military machines that have to include high-speed diving among their tactical exercises. We shall return to it at a later point.[1]

Some function of $VL\rho/\mu$ thus replaces the constant resistance coefficient, if compressibility be neglected. The common practice in making a series of aerodynamic studies has been to determine the value of the function, or in other words of the resistance coefficient, at a number of different values of Reynolds' number and then to plot a curve of its varying magnitude and extrapolate to the full-scale flight range. As a general rule, fortunately, the variation is confined within comparatively narrow limits and can be adequately taken care of by applying a *scale correction* to the coefficients determined from a test made on a model of some particular scale and at some particular speed. The scale correction is of course determined by just such a series of trials at varying Reynolds' numbers as were just described.

In any case it is the common practice, and for our present purposes it will suffice, to treat the aerodynamic coefficients as constants, of course obtaining their values under conditions as nearly as possible like those that are of immediate practical interest. All aerodynamic forces can then be written, and to a first approximation and with suitable reservations from time to time they can be treated, as varying in proportion to a projected area or to the square of a linear dimension of the object tested and to the square of the speed of relative motion between that object and the fluid. We may return our equation to the original simple form of (1).

The fact that in most cases aerodynamic coefficients are to a first order of approximation constants gives *wind-tunnel testing* its practical utility. The wind tunnel, of which something over a hundred now exist in various parts of the world, is a tube or channel through which air is drawn at high speed—though usually far inferior to the speed of flight of an aircraft. A miniature model of the object of which the aerodynamic characteristics are to be studied is placed in the air stream, mounted on a balance so that all the forces acting on it can be measured. By knowing the forces, the air speed, and the scale

[1] P. 111, *infra*.

of the model or its projected area, the values of the corresponding coefficients can be determined by substitution in (1).

The range of extrapolation of the results, by scale correction or otherwise, is of course terrific. The typical wind tunnel operated by a university or by an aircraft manufacturer for his own testing is likely to allow the use of a model wing or fuselage of about one-twelfth full scale and to give a maximum wind speed about half that of flight. The Reynolds' number during the test is then commonly about one twenty-fifth the full-scale value. For a rigid airship model many tunnels are incapable of reaching a Reynolds' number exceeding one one-hundredth that of normal flight conditions. There are, however, three or four wind tunnels in the world, all of them operated by governments, that are large enough and capable of producing a high enough air speed to permit of testing many aircraft parts and even complete airplanes in full scale and at full flight speeds. The American National Advisory Committee for Aeronautics has such a tunnel,[1] and the largest of all those that now exist, in its laboratories. Figures 14 and 15 give some idea of its dimensions. The committee includes in its equipment another tunnel in which, though it is of small size, full-scale data are secured and any question of scale corrections to the results eliminated by the highly ingenious device of enclosing the whole apparatus in a huge steel tank in which the air pressure can be raised to several times the normal atmospheric level.[2] Dimensional theory has just shown us that, compressibility aside, full-scale conditions are attained if $VL\rho/\mu$ is kept constant. The density of a gas is of course proportional to its pressure, other things being equal, while viscosity is virtually independent of pressure. If a one-tenth scale model is to be tested, then, and if the conditions of test permit of attaining only one-half the full flight speed, a full-scale value of Reynolds' number can none the less be reached and the test results be made rigorously applicable to full-scale prediction by running the test

[1] "The N. A. C. A. Full-scale Wind Tunnel," by Smith J. DeFrance, *Rept.* 459.

[2] "The Variable-density Wind Tunnel of the National Advisory Committee for Aeronautics," by Max M. Munk and Elton W. Miller, *Rept.* 227. "The N. A. C. A. Variable-density Wind Tunnel" (describing a new and modified design), by Eastman N. Jacobs and Ira H. Abbott, *Rept.* 416. "Report of the Scale Effect Panel on the Variable Density Wind Tunnel," *R. and M.* 1149.

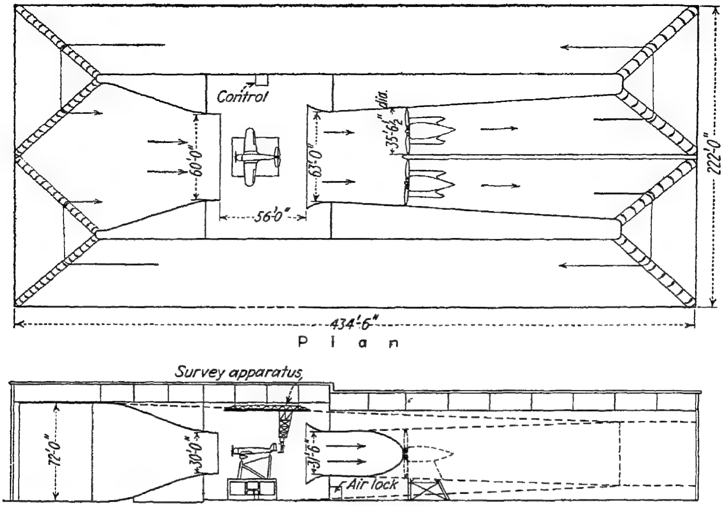


FIG. 14.—National Advisory Committee for Aeronautics full-scale wind tunnel.

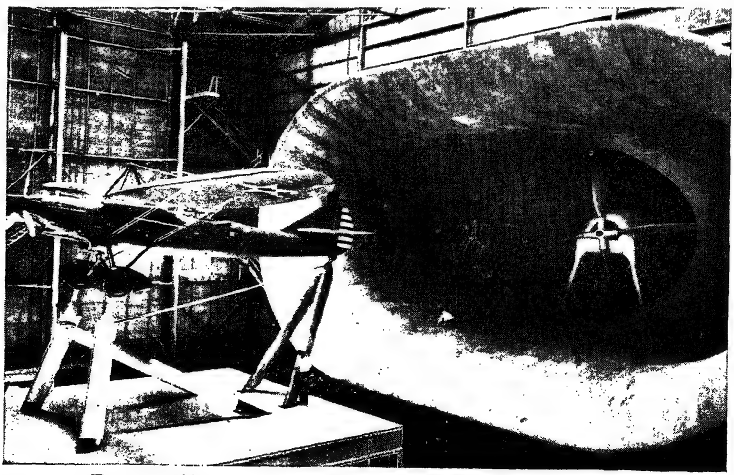


FIG. 15.—N. A. C. A. full-scale tunnel, interior arrangement.

under a pressure of 20 atmospheres, thus keeping $VL\rho$ the same in the tunnel as in flight. The compressibility effect is of course quite different from that in free air and full scale, but as already noted that makes little difference except where speeds of 400 m.p.h. or more have to be anticipated.

All this refinement becomes increasingly important, and engineers become increasingly exigent in their demands for it, for two reasons. First, because as design becomes increasingly an exact science, and as designers are required to subject themselves to ever closer specifications and advance guarantees for their product's performance, errors of 5 or 10 per cent in aerodynamic data, that might once have seemed too small to notice, come to assume nightmare proportions. Second, because the accumulation of experience has shattered such comfortable illusions, once commonly held, as that the variation of aerodynamic coefficients with Reynolds' number is in practically every case a smooth curve, that in such variations as exist there is an asymptotic approach to a constant level at high Reynolds' numbers, that sudden and violent changes of behavior in any case do not occur there, and that the data obtained at a Reynolds' number of one-tenth full scale or thereabouts can therefore be taken as valid for any higher figure. Those things unfortunately are not true.

In place of illusion has come understanding that although a gradual variation and a smooth one may be typical they are far from being universal. Both the study of data taken in the tunnel and the development of an understanding of the ways in which the fundamental type of flow may change with changing Reynolds' number[1] have taught us to view without surprise the most irregular of scale-effect curves, with the trend of the values of the coefficients plotted undergoing three or four reversals in successive Reynolds' number ranges and showing substantial changes even at Reynolds' numbers of 1,000,000 or more. There is still enough stability so that the small tunnel operated at moderate speed is by no means without value as an instrument of research, especially when its results are interpreted in the light of our present understanding of theory, but scale effect becomes increasingly a plague, and the more completely it can be eliminated by using test data taken at the Reynolds' number at which

[1] Discussed at length in Chap. VI, *infra*.

the aircraft is finally to be operated the better off the designing engineer will find himself.

Aerodynamic Force Components and Coefficients.—In some cases it is required only that the magnitude of a resultant force be determined, and a single resistance equation then suffices. It is quite obvious, for example, that the force on a sphere or on a flat plate set at right angles to its direction of motion through the air will act in direct opposition to that motion, being expressible by a vector parallel to the line of motion but running in the opposite direction. The direction of the force thus being known, the measurement of its magnitude would fully determine its characteristics. If the flat plate be set at 10 deg. to the wind

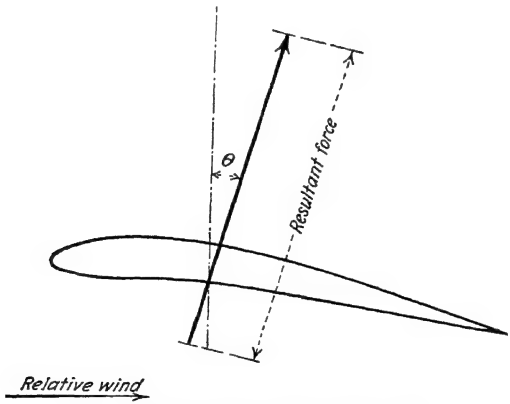


Fig. 16.—Resultant force on airfoil.

direction instead of at a right angle, however, or if the sphere be replaced by an object of wholly irregular form with no axis of symmetry, the direction of the vector of resultant force is no more certain in advance than the magnitude, both alike being the subject of experimental determination.

The force might be fully defined by specifying its magnitude and the angle between its line of action and some conveniently specified axis, such as the perpendicular to the relative wind direction, expressing the results either numerically or by plotting a vector to scale. The vector method is illustrated for an airfoil section in Fig. 16. For the purpose of practically all calculations made in airplane design, however, it is more convenient to

deal with components of force along specified axes than to introduce the concept of angle.

There are of course various sets of reference axes which might be used in breaking the resultant force up into components. For most purposes, especially in the calculation of performance, it is most convenient to divide the resultant force into *lift* and *drag*, defined, respectively, as the components perpendicular to and parallel to the direction of the relative wind; the use of those axes is almost universal. In straight horizontal flight of an airplane the lift and drag therefore act vertically and horizontally,

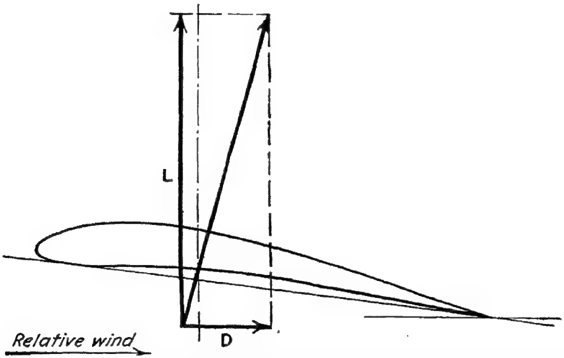


FIG. 17.—Composition of air reaction on airfoil, axes fixed relative to the wi...

respectively, the first being in direct opposition to the weight and the second having to be balanced by the propeller thrust.

For certain purposes, however, especially in connection with the stress analysis of the wing structure, it is more convenient to refer the component forces to axes fixed in the structure than to a set dependent on the relative wind direction. Such axes may of course be chosen at will, but in the particular case of the airfoil they are customarily taken parallel and perpendicular to its chord, the components in the two directions being called the *longitudinal force* and *normal force*, respectively, and denoted by the symbols X and Z. The decomposition of a given resultant force with respect to the two alternative sets of axes is shown in Figs. 17 and 18. As drawn in Fig. 18, X happens to be positive (directed backward), but with good airfoils it frequently proves to have a negative value. The resultant force on the

in other words, is often directed forward of the perpendicular to the chord. Obviously it can never be directed forward of the vertical, and D can never be negative, as the airfoil would then be impelled to move straight upwind without external driving force and the law of energy conservation would be outraged.

If there were no scale correction to be reckoned with, the direction of the vector of resultant force would be governed only by the form of the object on which the forces acted and by the attitude in which it was presented to the moving air, and the laws of variation for the individual components of force

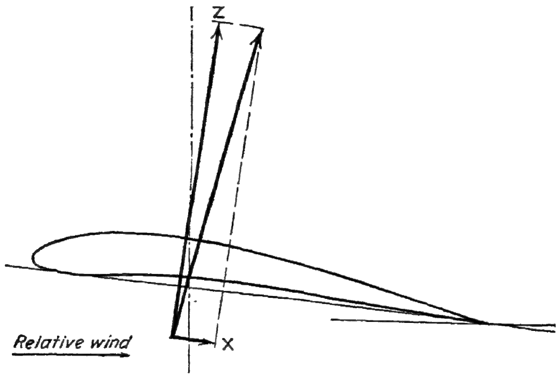


FIG. 18.—Composition of air reaction on airfoil, axes fixed relative to the airfoil.

are therefore the same as those for the resultant. The fundamental equations for lift and drag can then be written, subject to scale corrections,

$$L = L_C S V^2 \tag{3}$$

and

$$D = D_C S V^2 \tag{4}$$

L_C and D_C, the *lift coefficient* and *drag coefficient,* are the expression of the primary aerodynamic characteristics of an airfoil. When the factor of scale correction is introduced it is necessary to deal with it separately for the two components of force, as they are likely to be quite differently affected. The change of flow around the airfoil which takes place with a change in Reynolds' number is likely to modify not only the magnitude of the

total resultant force but also its point of application and its line of action, and so to change L_C and D_C in different ratios.

There is a wide diversity of practice in the selection of symbols for the aerodynamic force coefficients, the lift coefficient being variously denoted in reports from American and British laboratories by L_C, C_L, k_L, and k_y, while for the drag coefficient the corresponding forms D_C, C_D, k_D, and k_x are employed. The forms C_L and C_D are reserved for use with a special type of coefficient, as will be explained later.

Manifestly the numerical values of the coefficients of force will depend on the units employed for the other quantities entering into the equation. That will be true even though the units are consistent throughout, which they frequently are not, as the form in which the equations have been written does not make the coefficient dimensionless. The coefficient, being equal to a force divided by the product of an area and a square of a velocity, itself has the dimensions of mass divided by volume or, as already indicated in (2), of a density.[1]

The four sets of units most commonly employed are, in tabular form,

Number	Force	Area	Velocity
1	Pounds	Square feet	Feet per second
2	Pounds	Square feet	Miles per hour
3	Kilograms	Square meters	Meters per second
4	Kilograms	Square meters	Kilometers per hour

Two more sets of units are introduced by a slight change in the form of the equations of force. It will have been noted that the density of the air or other fluid appears as a factor in the general Eq. (2), the density of the air of course being variable with changes of temperature, barometric pressure, and other conditions. Equations (3) and (4) can then only be true, and the coefficients contained therein of constant value even at a constant Reynolds' number, for a specified standard air density. To take account of variations the forms

$$L = L_C \rho S V^2 \tag{5}$$
$$D = D_C \rho S V^2 \tag{6}$$

[1] Readers unfamiliar with the theory of dimensions will find an explanation of this process in such works as those of Cowley and Levy, of Wilson, and of Bridgman, already cited.

may be used, and they have the additional advantage of being dimensionally correct. As the coefficients were previously found to have the dimensions of a density, the introduction of a density as a factor leaves them as pure numbers, their magnitudes therefore independent of the system of units used so long as that system is consistent. In other words, in (5) the lift coefficient for a given airfoil at a given attitude would have the same value whether the quantities involved were expressed in the foot-pound-second or the meter-kilogram-second system or any other consistent arrangement, but it would not be legitimate to introduce two different units for the same thing, as, for example, by giving area in square feet and velocity in miles per hour. If area is expressed in square feet, the distance covered per unit time must also be in feet.[1]

Certain laboratories use a form like that just given, except that ρ is replaced by $\rho/2$. The reason for the change is that the product $(\rho V^2/2)$ has a physical significance, being the expression for the pressure equivalent of the kinetic energy of a moving fluid as measured with a pitot tube, a physical instrument frequently used in the determination of air speed. When the equation is written in this last form, therefore, the coefficient represents the ratio of the mean force per unit of area on the body under test to the pressure recorded by a pitot tube exposed to the same relative wind. It is only for that form of coefficient that the use of C_L and C_D is common practice, and only in that connection will they be used here.

Writing these last two forms to give the coefficients directly;

$$k_L = \frac{L}{\rho S V^2} \tag{7}$$

$$C_L = \frac{L}{\frac{\rho}{2} S V^2} \tag{8}$$

There is available a total of six sets of units and types of coefficient, all of which find some use in practice. Numbers 1 to 4, inclusive, are sometimes described as *engineering coefficients*,

[1] It is further to be noted that density must be given in mass and not in force units, the figure in terms of force units being divided by the acceleration due to gravity. Thus, for example, the standard density of air at sea level and 60° F. is stated not as 0.07608 lb. per cu. ft. but as that divided by 32.17 ft. per sec. or 0.00237 slug per cu. ft.

while Nos. 5 and 6 are similarly called *absolute coefficients* in reference to their non-dimensionality.

Numbers 1 and 4 are now encountered only very rarely. Number 2 was long the standard of the U. S. Army and Navy and of most of the privately controlled aerodynamic laboratories in America, while No. 3 was long used in most of the reports from the Eiffel wind tunnel and other French laboratories. Of the absolute forms, No. 5 is substantially universal in British practice, and No. 6 has been adopted as a standard by the National Advisory Committee for Aeronautics in the United States, to an increasing degree by laboratories in France and Italy, and by all German institutions, although the Germans in tabulating the coefficients commonly multiply them by 100 to avoid decimal-point complications. It was in Germany, during the World War, that the $\rho V^2/2$ system of coefficients originated.

There is a steady drift of favor away from the engineering units and toward the absolute type among practical designers. The inclusion of density as a factor in the force equation seemed to most users of wind-tunnel data a needless and a trouble-making refinement until a few years ago. Now, however, the designer has to deal so largely with the performances of aircraft at high altitudes and with the choice of a best altitude of operation that he is wise to welcome a type of equation that keeps air density omnipresent as a factor and so facilitates allowance for its changes. The system of absolute coefficients employed by the N. A. C. A., and designated herein as No. 6, will therefore be the general standard of presentation throughout this volume except where there is some very special reason for turning to one of the five alternatives.

Coefficient Conversion Factors.—Since there is so little agreement on this subject, and since the results of aerodynamic work taken from different sources obviously cannot be compared until they are all in terms of the same units, frequent transformation from one system into another is necessary. The conversion factor may of course be directly calculated whenever the relations between the fundamental units involved are known,[1] but conversions among the six forms here described can be made directly from the factors listed in Table I. To convert from the

[1] The method is explained in "Aeronautics," by E. B. Wilson, *cit. supra*, Chap. I.

type of coefficient listed in the vertical line at the left to that chosen from the horizontal line along the top, it is necessary to multiply by the figure found at the intersecting point in the body of the table. Thus, to convert the results from a British report into a form directly comparable with work published by the U. S. Army Air Corps Engineering Division prior to about 1933, all the values of coefficients must be multiplied by 0.0051.

TABLE I

	1	2	3	4	5	6
1	1.000	2.151	52.7	4.05	421	843
2	0.464	1.000	24.5	1.88	196	392
3	0.0190	0.0409	1.000	0.0769	8.02	16.0
4	0.247	0.532	13.0	1.000	104	208
5	0.00237	0.00510	0.125	0.00958	1.000	2.00
6	0.001185	0.00255	0.0622	0.00479	0.500	1.000

CHAPTER III

GENERAL CONSIDERATIONS ON AIRFOILS

It has already been pointed out that the coefficients of force on an airfoil vary with changes in its attitude with respect to the wind as measured by the angle of attack, or angle between the airfoil chord and the relative wind direction (commonly denoted by the symbol α). To take the simplest possible case, it is clearly apparent that a flat plate set either parallel or at right angles to the wind direction will have no lift, as the plate in either of those positions is symmetrical with reference to the wind and there can be no unsymmetrical component of force. When the angle is set at anything between 0 and 90 deg., however, the symmetry being destroyed, the existence of a lift component is to be expected. The magnitude of that component and of its corresponding coefficient of course depends on the exact value of the angle.

Air Flow about Airfoils.—It is equally clear, since the variation of lift with angle is unlikely to be discontinuous, that the lift will be small when the angle of attack is very near either 0 or 90 deg., increasing as those extremes are left behind and reaching a maximum for some intermediate angle. In the early days of the study of fluid dynamics speculation ran rife on the manner in which the force would change with changing attitude of flight, but it is no longer necessary to speculate. The advent of the wind tunnel and of measurements of the forces on models under controlled conditions has brought positive knowledge and has shown that at small angles of attack, those with which the student of the practical problems of flight is most concerned, the lift of a flat plate is almost exactly proportional to the angle. The same experiments have further demonstrated that the lift on a flat plate of the plan form conventional in airplane wings (with an aspect ratio of 5 or 6) reaches its maximum value at an angle of about 20 deg. The curve of lift coefficient against angle of attack for a flat plate of aspect ratio 6, as found from a wind-

tunnel test, is given in Fig. 19. The coefficient in this case is in terms of lb. per sq. ft. and m. p. h.,[1] and the maximum lift coefficient of 0.0021 at an angle of attack of 17 deg. therefore indicates a lift of 21 lb. per sq. ft. when moving at a speed of 100 m. p. h. through air of standard sea-level density. The corresponding maximum on the absolute system to be used elsewhere in this volume is 0.82.

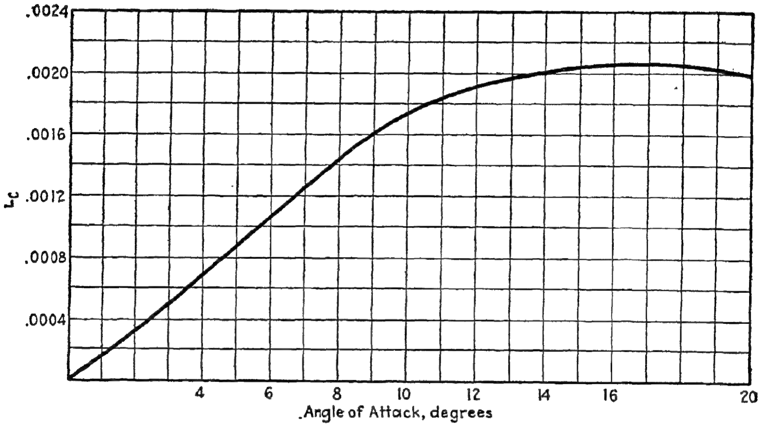


Fig. 19.—Lift coefficient of flat plate.

The form of the flat plate obviously tends to force a sudden change of the direction of motion of the particles of air passing below the plate, and an even more violent disturbance of those passing closely above the leading edge. The motion in the neighborhood of the leading edge is badly disturbed as a result, espe-

Fig. 20.—Deflection of leading edge of airfoil to meet relative wind.

cially at large angles. The shock to the particles of air meeting or passing close to the plate can evidently be reduced by modifying the form in the neighborhood of the leading edge as indicated in Fig. 20, and this is the first step in the evolution of an airfoil.

It is clearly desirable, in order to secure a high lift, that the lines of flow over both the upper and lower surfaces should be

[1] Form 2 in the previous chapter (p. 31).

continuously concave downward, the center of curvature of each portion of every streamline which passes close to the airfoil lying below it. A lift must be the result of an increase of pressure on the lower surface, or a reduction below atmospheric pressure on the upper surface, or a combination of the two effects; and if there is a pressure gradient from an excess or deficiency of pressure in the immediate neighborhood of the surface to atmospheric pressure at a great distance away the streamlines must be curved, as an unbalanced force will act on each particle of air to produce

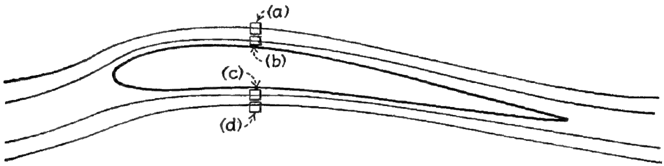


FIG. 21.—Relation between flow of air and pressure gradient.

a central acceleration. In Fig. 21, for example, with the streamlines curved as shown, there must be an unbalanced force acting in a downward direction on each of the small volumes of air diagrammatically indicated, the force required to produce a central acceleration with respect to the center of curvature of each streamline somewhere below the airfoil. To produce such an unbalanced force on each particle of air, there must be a pressure gradient, the pressure at a being greater than that at b and that at c exceeding that at d. It follows that the pressure adjacent to the airfoil is below atmospheric on the upper surface and above the datum on the lower. The existence of a lift may then be regarded as the consequence of the downward curvature of the streamlines. Given that relationship, obviously the largest lift will be attained with the airfoil which bends the streamlines on the smallest average radius. If the contour of the airfoil is too sharply curved, however, the air will fail to follow the surface. There will no longer be a smooth flow, and no deductions can then be drawn directly from the contour's curvature.

Another corollary of the existence of lift on an airfoil about which there is a non-turbulent flow is the existence of a difference in speed of motion between the various streamlines and along

any given line of flow. Bernoulli's theorem[1] makes mathematical expression of the fact that in a streamline flow the total head, or total of potential and kinetic energy contained by a particle of fluid, remains constant along the length of a streamline, or

$$\frac{\rho V^2}{2} + p = \text{constant} \tag{9}$$

This form obviously neglects the influence of compressibility, treating density as though it were a constant. To include the missing factor complicates the mathematical form somewhat, but makes no change in the fundamental reasoning.

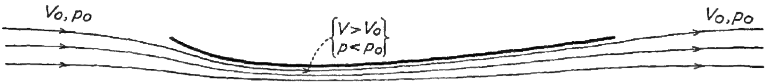


FIG. 22.—Venturi tube, with indication of velocity rise and pressure drop at throat.

It follows from Bernoulli that in a streamline flow the velocity is high where the pressure is low, and vice versa, the most straightforward application of that fact being the familiar venturi tube (Fig. 22) used for measuring the speed of fluid flow. Obviously if the diameter of the throat of the tube shown in Fig. 22 has half the diameter of the entrance end of the conical approach the ratio of the areas at the two points will be 1 to 4, and in order to get the same amount of fluid past all sections of the tube in a given time the speed at the throat will have to be four times as great as at the entrance, where it equals the speed of the undisturbed stream. There results a difference of pressure proportional, for any particular tube, to the square of speed of the fluid, and when the pressure difference is measured by a suitable gauge at the throat the initial speed can be calculated from (9). Exactly the same method of taking pressure drop can be and commonly is used to measure the velocity of the air through the throat of a wind tunnel, which is nothing more than a giant venturi tube.

[1] For a discussion and proof of the theorem, see any text on fluid mechanics, such as those already referred to on p. 20.

In exactly the same way, it develops from Bernoulli that the reduction of pressure above an airfoil must be accompanied by an increase in air velocity, while the air moving along the lower surface must travel at a speed lower than that of the undisturbed air stream at a considerable distance.[1]

The velocity being lower below the airfoil than in the undisturbed stream and the pressure being higher there, there is a resistance to the flow of air from in front of the wing on to the lower surface, and in seeking the easiest path the particles of air tend to flow upward, above rather than below the leading edge. The streamlines therefore begin to be diverted from their

FIG. 23.—Streamlines around curved plate

normal course some little distance ahead of the leading edge, the airfoil exerting a sort of "advance influence" on the fluid and giving its motion an upward trend.[2] A perceptible influence may exist as much as two chord lengths ahead of the leading edge. The lines of flow around an elementary curved-plate airfoil at a small angle of attack are therefore as diagrammatically represented in Fig. 23. The fact that the streamlines are, as in that figure, crowded more closely together above the airfoil than below is the natural consequence of the higher velocity of flow along the upper surface.

The curved plate into which the original flat plate was developed as the first step in the evolution of an airfoil will not do for a final form, being unsatisfactory on two counts. In the first place, an actual airplane wing must have some thickness within which the spars and other structural members that furnish its strength can be stored. Second, aerodynamic considerations also make it desirable that the curvature of the upper

[1] The actual magnitude of these velocity differences will be discussed in Chap. V.

[2] An interesting study of the amount of this "upwash" is contained in "Air Flow Investigation for Location of an Angle of Attack Head on a JN4H Airplane," by R. G. Freeman, *Tech. Note* 222.

surface be much sharper than that of the lower, for when the wing is presented to the air at a small angle of attack the flow of air is much less disturbed by being forced upward by the direct obstruction of an upper surface dipping sharply downward to the leading edge than it is by a correspondingly sharp rise behind and below the leading edge. If the lower surface is deeply concave, the streamlines fail to accommodate themselves promptly to its contour, and there is left under the foremost

FIG. 24.—Effect of excessive concavity of lower surface near leading edge of an airfoil.

part of the section a region of "dead water," in which the motion of the air is eddying and disturbed, with results very unfavorable to efficient performance. These effects and the difference between the action on the upper and lower surfaces are diagrammatically illustrated in Fig. 24, representing the flow around a plate of deep curvature at a small angle.

If to the requirement of curvature near the leading edge is added that of finite thickness, and so of different form for the

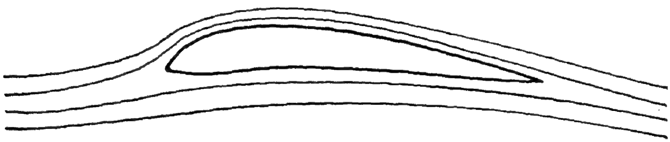


FIG. 25.—Stream-lines around airfoil.

upper and lower surfaces, there results a typical airfoil section of the type shown in Fig. 25. The infinite variety of sections that have been designed and tested are produced by varying the thickness and the amount and distribution of the curvature, but the general principle is the same for all.

All this works out nicely at small angles of attack. The flow of air accommodates itself readily to the airfoil section, and the flow photograph presented in Fig. 26 gives evidence that the actual conditions make a reasonably close approach to the ideal

ones. At larger angles, unfortunately, there is another story to be told.

Separation.—Its telling brings in a phenomenon already hinted at—the phenomenon of abrupt departure of the streamlines from the contour that is supposed to be guiding them.

FIG. 26.—Photograph of flow around airfoil at small angle of attack. (*British Aeronautical Research Committee.*)

Called *separation*, such a break from the geometrical contour develops whenever the conditions that would be imposed if it were to be followed become physically untenable. It is impossible for a streamline to attain an infinite velocity. It is impossible for a fluid to drop to an absolute negative pressure, since a fluid is incapable of sustaining tension. It is impossible for it, since each particle of the fluid possesses momentum, to make an

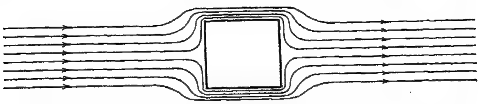


FIG. 27.—Flow around square strut (idealized).

absolutely abrupt change of course. Less obviously, but no less truly, it is impossible for it to stand a high positive pressure gradient or very rapid decrease of velocity along a continuing streamline.[1]

To begin at the beginning, it requires no study of hydrodynamics to agree that the flow condition shown in Fig. 27 is a

[1] This statement, which may be by no means clear, becomes the subject of detailed treatment in Chap. VI.

fiction. It simply could not happen. Even with the corners rounded off somewhat, the impossibility remains obvious. When the rounding off has proceeded to the point of replacing the square section by a circular one, as in Fig. 28, one might feel a little more uncertainty, but even then the flow looks improbable and direct experiment shows it to be unreal. Figure 29 illus-

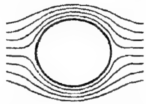


FIG. 28.—Flow around circular strut (idealized).

trates the actual motion round a cylinder and displays not only the fact of separation, at a point not far from the diametral plane perpendicular to the wind direction, but also the extreme turbulence and variability that typically characterize the flow after it has lost its direct physical guidance by the contour of the object immersed in the storm. When a streamline separates from a rigid surface, generally speaking, it ceases to be a stream-

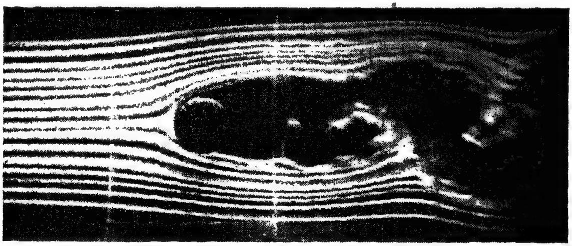


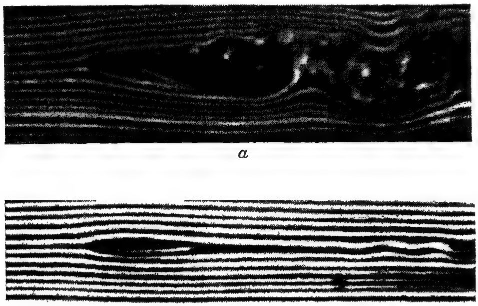
FIG. 29.—Actual flow around a circular strut.

line, and the particles of fluid that once composed it embark on almost unpredictable wanderings in a confused and shifting field.

The particular application to an airfoil at a large angle of attack is shown photographically in Fig. 30(*a*)[1], which should be compared directly with the study of the same airfoil at a smaller angle in Fig. 30(*b*).[1] At the high angle the flow still makes a comparatively easy contact with the lower surface at its leading edge,

[1] Taken from studies in the visible-flow tunnel of the N. A. C. A. Such photographs are made by introducing filaments of chemical smoke into the air stream, the smoke emerging continuously from a row of pointed nozzles placed a short distance upstream from the airfoil or other object.

and the curvature of the surface forces the air gradually around and down to the steep angle at which it leaves the trailing edge. On the upper surface, with its still sharper curvature, there is no mechanical forcing of the air to follow the surface after it has once begun to do so, and the mere influence of the pressure differential[1] is not strong enough to lead the fluid to so large a deviation from its original course as would be required. Separation ensues. In the particular case of the airfoil separation is



a

Fig. 30.—Flow around an airfoil of symmetrical section, at two different angles of attack.

called *burbling*, and the angle of attack at which it first appears is the *burble point*.

The burble point is seldom very clearly defined, as the change from smooth to turbulent flow usually develops gradually over a range of angle of 4 deg. or more, with the point of separation working steadily forward from the trailing edge until the turbulent region covers virtually the entire upper surface. Occasionally, however, the flow breaks down with spectacular suddenness at a critical angle.

The breakdown is accompanied by a rapid decrease of lift, as might be expected from the explanation that has been given of the lift as the product of the curvature of smooth streamlines. Inventors of freak airfoil sections, with sudden discontinuities of form near the leading edge, sometimes adopt the concept that it should be possible to increase the lift by deliberately forcing the air to break away from the upper surface and rush off in

[1] Explained on p. 37, *supra*.

some other direction, so leaving a "partial vacuum" in the deserted space above the airfoil. The idea is a pleasantly naive one, without sound support in theory and amply disproved in practice. Where oddities in airfoil form give abnormally high maximum lift coefficients[1] it is because they prevent burbling, not because they go out of their way to introduce it.

Distribution of Lift.—Since it is practicable to make the curvature of the airfoil contour sharper on the upper than on the lower surface, and since lifting action increases, as already explained, with increasing curvature of the streamlines, it would be expected that the reduction of pressure on the upper surface

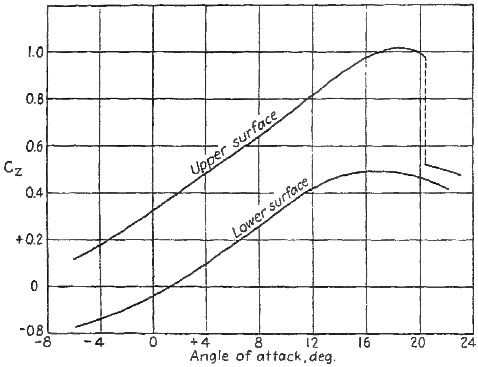


FIG. 31.—Distribution of lift between upper and lower surfaces of an airfoil.

would make a larger contribution to the resultant force on the airfoil than would the increase on the lower side, at least so long as burbling is avoided. It is found by experiment that even in the case of a flat plate the upper surface contributes more to the lift than does the lower at moderate angles, the proportion of total force contributed by the upper side decreasing as the angle of attack increases until with the plate set at right angles to the wind much the larger part of the force is due to the direct "impact effect" of the particles of air against the front of the plate, while a relatively small part of the resistance comes from the failure of the streamlines to close in instantaneously after passing around the edges of the plate and from the resultant creation of a region of lower pressure in the rear.

[1] As, for example, the slotted wing, described in Chap. X.

To illustrate numerically the dominating importance of the upper surface of an airfoil, the proportion of the total resultant force coming from that surface has been tabulated in Table II for a flat plate and for a typical airfoil section, each at a number of angles of attack. At the last of the angles listed for the airfoil burbling has already begun, and the upper-surface lift is falling off rapidly.

In Fig. 31 curves representing the lift drawn from the upper and lower surfaces, respectively, of a good airfoil are plotted against the angle of attack.

TABLE II

Flat plate[1]		Airfoil[2] N. A. C. A. 69	
Angle of attack, degrees	Ratio of force on upper surface to total reaction	Angle of attack,	Ratio of lift on upper surface to total lift
5	0.80	0	1.00
10	0.78	5	0.74
20	0.73	10	0.68
40	0.56	15	0.70
90	0.33	20	0.62

[1] "La résistance de l' air et l' aviation," by G. Eiffel, p. 80, Paris, 1911.
[2] "Pressure Distribution over Thick Airfoils," by F. H. Norton and D. L. Bacon, *Rept.* 150.

The lift of a flat plate must obviously be zero at 0 deg., from considerations of symmetry. There are no such limitations on airfoils in general, since the flow along the upper surface may easily be such as to give a lift when the chord is parallel to the relative wind.

Centers of Pressure and Moments.—In treating of the forces on an airfoil, mention has so far been made only of the magnitude of the resultant force and of its direction, as determined by and determining the relation between the components of lift and drag. For a complete definition of the force, a knowledge of its line of action or of the location of some point on the vector representing it is, of course, necessary. For purposes of convenient specification, such a point is commonly taken at the intersection of the vector of resultant force with the chord of the airfoil and is called the center of pressure. Figure 32 shows the resultant in a typical case, now fully located, and with its components and the

center of pressure of the section indicated. The moment of force around any point can evidently be determined when once the center of pressure is known. In particular, the moment around the leading edge will be equal to the product of the normal component of force, Z, by the distance from the leading edge to the center of pressure. For angles of attack within the range ordinarily used in flight this is very nearly equal to that same distance times the lift, Z and L differing from each other but very little within that range. Just as lift and drag are commonly

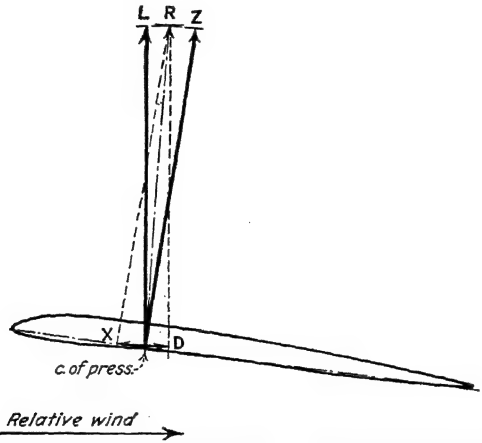


FIG. 32.—Location of center of pressure, and composition of air reaction.

expressed by coefficients eliminating the factors of speed and scale of the object tested, so also the moment and center of pressure are reducible to coefficient form. As the center of pressure can be defined by a simple linear measurement along the chord, the center-of-pressure coefficient is merely the ratio of the distance of the center behind the leading edge to the length of the chord. The moment, being the product of an aerodynamic force and a distance, varies for a given airfoil as the product of the area, the square of the speed, the air density, and the chord length, or

$$M = C_M \frac{\rho}{2} cSV^2 \tag{10}$$

where c is the chord length and C_M an experimentally determined moment coefficient. The moment about the leading edge is also equal to

$$M_{LE} = -Z\bar{x} \tag{11}$$

Z being the component of force normal to the chord and $\bar{x}$ the distance from the leading edge to the center of pressure. The negative sign is inserted to comply with the usual convention which measures center-of-pressure distances as positive back from the leading edge and which makes a moment positive when its direction is such as to tend to depress the trailing edge of the wing. By writing the force and moment in terms of their coefficients, this becomes

$$M_{LE} = C_M\frac{\rho}{2}cSV^2 = -C_Z\frac{\rho}{2}SV^2\bar{x} \tag{12}$$

whence it follows that

$$\frac{\bar{x}}{c} \qquad -\frac{C_M}{C_Z}$$

$\bar{x}/c$ is, of course, the center-of-pressure coefficient, and, normal force and lift being nearly equal at ordinary airfoil angles, the center-of-pressure coefficient is approximately equal to the coefficient of leading-edge moment divided by that of lift.

For most purposes, the moment coefficient proves a more useful tool for the designer than the center of pressure.

Taking the flat plate again as the simplest possible approach to the airfoil, it is evident that when the angle of attack is 90 deg. the center of pressure will be in the middle of the plate, which is then perfectly symmetrical about an axis passing through its middle point and parallel to the wind direction. The center-of-pressure coefficient is then 0.5. It is equally clear that at 0 deg. the center-of-pressure position is indeterminate, as the coefficients both of moment and of normal force are zero, and their ratio might have any value. Physically expressed, the vector of resultant force is coincident with the chord of the plate, and the two are therefore in contact at every point instead of at one alone.

When a plate is set at a small positive angle the disturbance of the lines of flow is most marked near the leading edge, as those streamlines which would normally impinge on the plate near its

trailing edge are forced downward while still far below the plate by the interference of the particles of air above them, as shown in Fig. 33, where the dotted lines prolong the original paths of the particles of air. The pressure being most intense where the cur-

FIG. 33.—Flow of air around flat plate (idealized).

vature of the streamlines is sharpest, it is obvious that the center of pressure will be somewhere well ahead of the middle point of the chord. As this concentration of force becomes more and more marked as the angle of attack approaches zero, it would be expected that the center of pressure will go steadily forward with decreasing angle. Wind-tunnel measurements verify the anticipation. The actual travel of the center of pressure on a flat plate of aspect ratio 6 is as shown in Fig. 34, where the center-of-pressure coefficient is plotted as ordinate against angles of attack as abscissas, that being the conventional device.

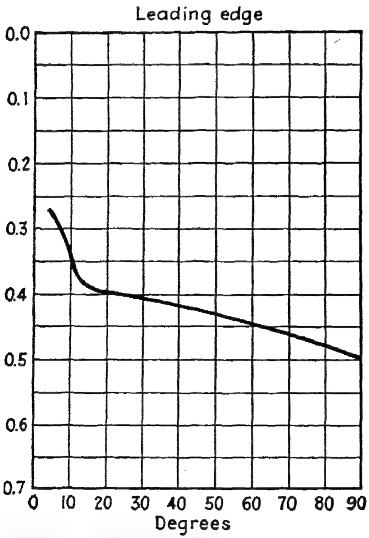


FIG. 34.—Center-of-pressure travel on a flat plate.

An airfoil, like a flat plate, has a center-of-pressure coefficient of substantially 0.5 at an angle of attack of 90 deg., but at the attitudes actually used in flight the distribution of pressure is quite different for the two forms. At an angle near that at which the lift coefficient reaches its maximum value (say 14 deg.), the air flows smoothly on to both the upper and lower surfaces of the airfoil without being forced to turn upward appreciably when very close to the surface, as the normal lines of flow of the air, allowing for the upward deviation which manifests itself well

ahead of the leading edge, would become tangent to the contour of the upper surface of the airfoil very near the leading edge at so large an angle of attack. The pressure is then most intense where the curvature of the surface is sharpest, in the forward part of the section, and under this condition the center of pressure of the airfoil, like that of the flat plate, is well forward toward the leading edge. The approximate distribution of pressure intensities over the upper and lower surfaces is shown vectorially in Fig. 35.

At a small angle, near that of zero lift, conditions are quite different. The leading edge of the section now dips downward so sharply with respect to the normal lines of flow that the stream-

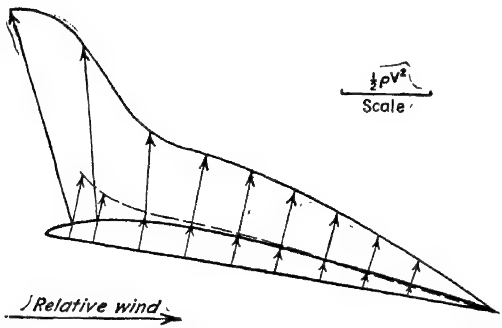


FIG. 35.—Pressure-distribution diagram at a high angle of attack.

lines are forced to turn upward abruptly in that region, and there is locally an excess of pressure on the upper surface and a reduction on the lower, instead of the reverse. After the streamlines accommodate themselves to the contour the pressures are of course in the expected direction, but the points of maximum pressure difference, both above and below the section, are considerably farther back at the small angle than at the large, and they continue to move steadily to the rear as the angle is still further reduced. Instead of moving forward as the angle approaches that of zero lift, as on the flat plate, the center of pressure therefore retreats farther from the leading edge, although, as already indicated, at very large angles (above 20 deg.) the airfoil and the flat plate act substantially alike. There is therefore a reversal of center-of-pressure motion on the airfoil, with the coefficients

decreasing in value up to an angle near that of maximum lift, and thereafter increasing with further increases of angle.

The backward movement of the center of pressure continues indefinitely as the angle is further reduced toward its zero lift

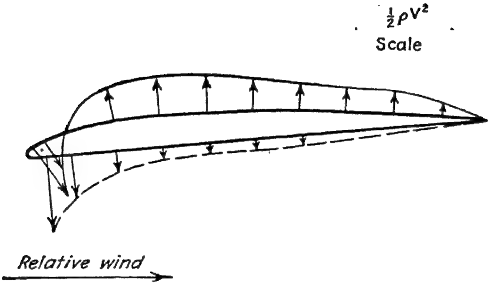


FIG. 36.—Pressure-distribution diagram at a low angle of attack.

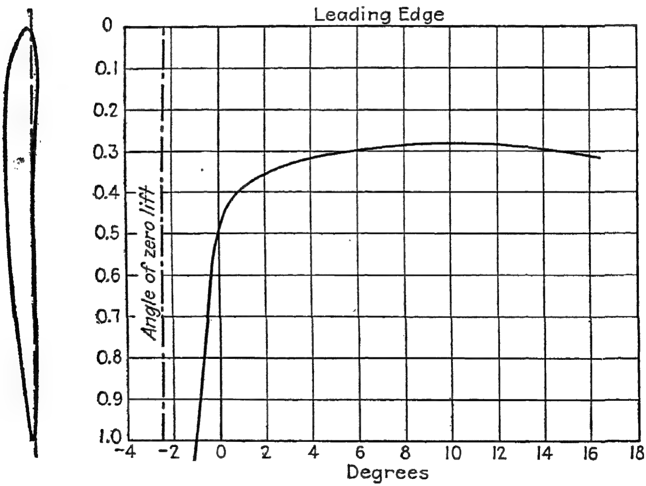


FIG. 37.—Center-of-pressure travel on a typical airfoil.

value, and when the angle of zero lift, or to speak with more rigorous accuracy that of zero normal force, is finally reached, a typical distribution of pressure is as indicated in Fig. 36. The upward and downward components of force exactly balance each other in magnitude, and the resultant normal force is therefore

zero, but since one of the two opposed sets of forces is centered near the leading edge and the other far back on the chord their resultant is a pure couple. The moment coefficient has a finite negative value (indicating a diving moment), and the coefficient of center of pressure, the ratio of moment to normal force, is therefore infinite. The center of pressure runs back off the wing at small angles, and its coefficient approaches infinity as the angle approaches that of zero normal force. The actual motion for a representative airfoil is plotted in Fig. 37, corresponding to the curve given in Fig. 34 for a flat plate.

"Stable" and "Unstable" Airfoils.—The fundamental difference in the nature of the center-of-pressure motion for the flat plate and the curved one, or airfoil form, can easily be illustrated by experiment. If an ordinary index card is slightly creased along a line through its center and parallel to its shorter side, giving it a dihedral angle for lateral stability, and then is lightly ballasted along one of the long sides with letter clips or some such objects so that the center of gravity is approximately one-third of the way back on the chord, it will perform very nicely as a model glider. Gently launched from an elevation with the ballasted edge leading, it will descend steadily to earth along a path inclined at 10 or 15 deg. to the horizontal and if slightly disturbed will quickly recover its equilibrium. If now the same card is given a section nearer that of an airfoil by bending it into a curve along the chord and is again launched in the same way, it will be found to flutter irregularly to earth, assuming no steady motion but falling to and fro in response to each slight upsetting influence.

The explanation is simple. Let it be supposed that the card is so ballasted in each instance that its center of gravity coincides with the position of the center of pressure for an angle of attack of 6 deg., and that in each case after the flight has been started at that angle some vagrant air current increases the angle momentarily from 6 to 8 deg. In the case of the flat plate the center of pressure then moves backward as the angle increases, and the air load acting at the center of pressure and the weight acting at the center of gravity combine to produce a couple tending to depress the leading edge of the card and so to decrease the angle back toward its original value and restore equilibrium. After the camber has been given to the card, the center-of-pres-

sure movement is reversed, and the increase in angle causes the c. p. to move forward. The resultant couple in that case tends to aggravate instead of rectifying the departure from the normal attitude, and the card would need to be equipped with a horizontal tail surface to restore its stability.

Pressure-distribution Plotting.—Although it is sufficient for most of the purposes of the airplane designer to know the total magnitudes and lines of action of the resultant forces on an airfoil, it is sometimes necessary, or at least very useful, to know in detail how the elementary forces are distributed over the surface or how the intensity of pressure varies from point to point.

Such information is essential for stress analysis. It is helpful, too, as a basis for attempted improvement of airfoil performance. The analysis of pressure-distribution diagrams may even lead directly to the modification of airfoil sections or plan forms,[1] with resultant amelioration of the aerodynamic qualities. In particular, pressure-distribution diagrams taken at various points along the span serve to show how nearly the theoretical ideal of lift distribution[2] along the span has been approached, and to suggest what changes in plan form might serve to approximate it more closely.

Pressure distribution can be determined with a considerable degree of accuracy from calculations based on pure theory[3] and from the computed curvature and velocity of the streamlines at various points around the contour, but it is more generally secured in the aerodynamic laboratory.

The expedient used in Fig. 35 for showing pressure variation serves to give a simple picture of the conditions under which the airfoil works, but for careful study and analysis it is better to plot a curve of pressure from atmospheric pressure as the datum and with positive pressure intensities, or pressure increases, at

[1] "Modification of Wing Section Shape to Assure a Predetermined Change in Pressure Distribution," by A. Betz, *Luftfahrtforschung*, Dec. 5, 1934, translated as *Tech. Memo.* 767.

[2] See p. 89, *infra*.

[3] "Theory of Wing Sections of Arbitrary Shape," by Theodore Theodorsen, *Rept.* 411; "General Potential Theory of Arbitrary Wing Sections," by Theodore Theodorsen and I. E. Garrick, *Rept.* 452; "Determination of the Theoretical Pressure Distribution for Twenty Airfoils," by I. E. Garrick, *Rept.* 465; "The Theoretical Pressure Distribution around Joukowski Airfoils," by W. G. A. Perring, *R. and M.* 1106.

various points along the chord plotted above that datum line, pressure reductions below, regardless of whether they act on the upper or lower surface. Distances along the chord are plotted as abscissas. Figure 38[1] presents such plots for the pressure distribution at six different angles of attack, including one beyond the burble, on an airfoil section much used in design.[2] The curves should of course be closed ones, for it is not to be expected that there will be any discontinuity in pressure in passing over the leading edge, especially when, as in this case,

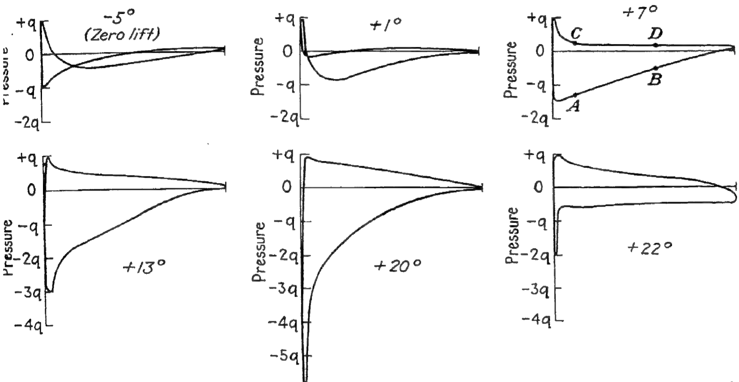


FIG. 38.—Variation of pressure distribution with angle of attack. ($q = 0.5\rho V^2$)

the leading edge is not perfectly sharp. The change of pressure in the immediate neighborhood of the leading edge, however, is so abrupt as to baffle experimental analysis, and what appears as a discontinuity, extending in one case from $+0.5\rho V^2$ to $-2.8\rho V^2$, actually represents the continuous change of pressure between a point 1 per cent of the chord behind the leading edge on the upper surface and one three times as far back on the lower surface. The actual distance between the two points of measurement on the model tested was only ¼ in.

It is noticeable that in all cases the pressure rises to a maximum of $+0.5\rho V^2$ at some point of the surface. That, as Bernoulli's

[1] Taken from "Airfoil Pressure Distribution in the Variable-density Wind Tunnel," by Eastman N. Jacobs, John Stack, and Robert M. Pinkerton, *Rept.* 353.

[2] The Clark Y, one of the most popular of airfoils in design practise (see p. 162).

theorem will be recalled to have shown, is the pressure which results from bringing the air completely to rest, being correspondent to a complete conversion of kinetic into pressure energy. In aerodynamic literature it is commonly denoted, and it will be denoted here, by the symbol q. On every object, therefore, whether the most perfect of streamline forms or a normal flat plate, the pressure reaches the same maximum at the *stagnation point*, or point of division between the lines of flow passing to one side and those going to the other. If the object is symmetrical about an axis parallel to the relative wind direction, the point of maximum pressure for that particular attitude will manifestly be at the forward end of that axis. Thence derives the use of the pitot tube, with an open end facing the wind, as an air-speed meter.

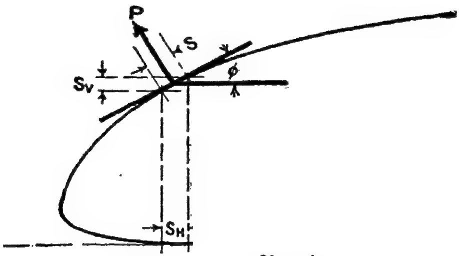


FIG. 39.—Geometry of the pressure-distribution diagram.

Such curves as those in Fig. 38 permit of a simple determination of total normal force by graphical integration. Figure 39 illustrates the conditions acting at a particular point on the surface of an airfoil section (shown only in part). At the point used for purposes of illustration a tangent to the surface of the section makes the angle ϕ with the chord, and s_H, the length of the projection on the chord line of the small element of surface, is equal to the product of s, the actual length of the element, and cosine ϕ. Similarly, $s_V = s \sin \phi$. Assuming consideration of a unit length of airfoil parallel to the span, the distance s is a measure of the area on which the pressure p acts, and the total force acting is ps, while the components normal and parallel to the chord are $ps \cos \phi$ and $ps \sin \phi$, respectively, or, substituting to eliminate the trigonometric functions, ps_H and ps_V. Integration of those two components around the whole boundary of the

surface would then give the total normal and longitudinal forces on the airfoil section. Manifestly, $\int p\,ds_H$ is equal to the area under a curve of which pressure intensity is the ordinate and distance along the chord the abscissa or, where the pressure function is considered as continuous and where the integration extends completely around the boundary, the area within the closed curve resulting.

It is equally evident that integration for longitudinal force on the section can be accomplished by taking the area within a similar curve, the ordinate again being pressure intensity, but the abscissa representing the distance above the chord of the point at which the pressure is taken, instead of its distance behind the leading edge.[1] To illustrate the method and certain general conclusions, a curve corresponding to one of those of Fig. 38, but plotted on distance above the chord as a base and on a greatly enlarged scale, is given in Fig. 40. To facilitate the comparison between the two figures, two points on the upper surface and two on the lower have been identified in each curve, (a given letter denoting the same point throughout), and the part of the curve corresponding to lower-surface pressures is dotted.

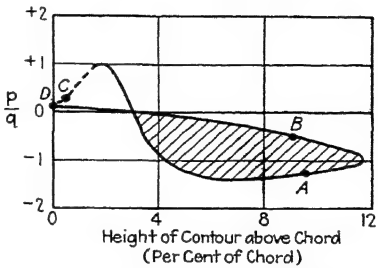


FIG. 40.—Determination, by pressure-distribution analysis, of the component of force along the chord of an airfoil.

It will be noted that the curve in Fig. 40 is made up of a large double loop, roughly of figure-eight form. If the airfoil selected for illustration had been one with a deeply concave lower surface, there would have been an additional, though a much smaller, loop. Of the figure-eight form, one branch is positive while the other is negative. The reason for the existence of a negative branch is indicated in Fig. 41, where a thick airfoil section is shown intersected by two lines parallel to the chord, the four points of intersection being identified as P and Q for the lower line, and R and S for the upper. At a small angle of attack the

[1] This method was first used by the British National Physical Laboratory, in 1912. ("Investigation of Distribution of Pressure over the Entire Surface of an Airfoil," by B. M. Jones and C. J. Paterson, *R. and M.* 73.)

forms of the streamlines would not have been fully accommodated to the shape of the upper surface when the point P was reached, and there would therefore be a positive pressure there, or at least a smaller pressure reduction below atmospheric than at point Q. As between these two points, therefore, the balance of force is directed backward along the chord, a reduction of pressure at Q tending to draw the airfoil backward, while the pressure drop at P would have a smaller component tending to pull it forward. As between R and S the reverse is the case, R being near the point of maximum suction on the airfoil surface. The lower part of the upper surface of a highly cambered airfoil

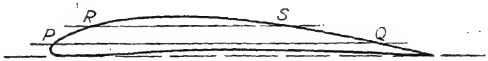


FIG. 41.—Geometry of the creation of a negative component of force along the chord.

therefore gives, in general, a positive longitudinal force or one directed backward along the chord, while the longitudinal force from the uppermost part of that surface, near the point of maximum camber, is negative. For good airfoil forms, the negative component overbalances the positive over a considerable range of angles, and the total longitudinal force is directed forward along the chord.[1] Mathematically stated, the condition for negative longitudinal force on an airfoil is that the ratio of lift to drag shall be in excess of the cotangent of the angle of attack, and that condition is likely to be satisfied over a range of angles approximately from +3 to +20 deg.

The method of graphical integration described has two uses. It serves to make it possible to determine the forces acting on a series of narrow strips of the airfoil and so to study the change of conditions along the span; also it permits the checking of the accuracy of the pressure-distribution experiments against direct measurements of total force on an airfoil model. For the normal force such a check can be direct, but for the longitudinal force it is necessary to add in another factor, the skin friction, or tangential component of force on the surface due to the friction between the particles of air.[2] As indicated in Fig. 39, there really act at every point two components of force, one normal to

[1] See p. 29, *supra*.

[2] Discussed in detail in Chap. VI.

the surface and the other tangential. The second is small in comparison with the first, and only the first is measured in pressure-distribution experiments, but since the tangential force acts very nearly parallel to the chord it has an appreciable effect on the longitudinal force on the airfoil, always increasing its magnitude if the force derived from pressure integration was positive and decreasing it if it was negative.

From the discussion just given and from examination of Figs. 38 and 40, it is evident that, although the lower surface may contribute appreciably to the lift of an airfoil, it has little effect on the longitudinal force or the relative efficiency as measured by the ratio of lift to drag. It is further evident that the largest negative longitudinal force and the smallest drag will in general be secured, at a given angle of attack and other things being equal, when the center of pressure is farthest forward on the chord, as a location of the center of pressure near the leading edge is indicative of large suctions on the upper surface near the leading edge, where they have a forward component along the chord, and smaller ones near the trailing edge, where the longitudinal component acts backward. The center of pressure itself can, of course, be located from the pressure-distribution diagram as the center of gravity of the area within the curve of pressure plotted against distance along the chord, as in Fig. 38.

Nature of Pressure Distribution.—A study of Fig. 38 reveals a general behavior in accord with what would have been predicted from the previous discussion of the air flow around airfoils. Thus, for example, the peak of maximum suction over the leading edge grows constantly sharper and higher and moves farther forward as the angle of attack increases. Whereas at an angle of 1 deg. the maximum reduction of pressure is found about one-fifth of the way back on the chord, with an actual positive pressure on the first 2 per cent of the chord behind the leading edge, at 20 deg. the maximum, about six times as high as at the angle previously cited, lies within 1 per cent of the leading edge. It will be noted, too, that despite the sharpness of the trailing edge of the section there is no very sudden change of pressure around that boundary, the apparent pressures falling off to values very close to zero on both the upper and lower surfaces as the trailing edge is approached. The distribution on the lower surface shows no very striking changes beyond an angle of 7 deg., even

at angles well beyond that of maximum lift. Beyond the burble point, the peak over the leading edge breaks down rapidly and the pressure over the upper surface becomes substantially uniform. Even at angles up to 30 deg., however, some traces of a maximum of suction on the first 1 or 2 per cent of the chord remain, as a reminder that separation is not instantaneous and that there is always some sign of initiation of streamline flow along the upper surface before the air breaks away into turbulence and the pressure drop subsides to the flat $0.5q$ that covers most of the chord.

The niceties of the subject, the variation of the pressure-distribution diagram along the span, its influence by various interferences, and the relation of its exact shape to airfoil form will be taken up from time to time through the next eight chapters.

CHAPTER IV

THE QUALITIES OF AN AIRFOIL

Methods of Plotting Airfoil Characteristics.—Since the forces acting on an airfoil change with every change of angle of attack, they can most easily be presented in graphical form, and a number of different expedients have been adopted in the endeavor to make a small number of curves so present an airfoil's characteristics to the eye that the essentials will be easily and quickly grasped.

The oldest of such devices employs three curves for each airfoil. The lift coefficient, the drag coefficient, and the ratio of lift to drag, which is useful under proper restrictions as a measure of the aerodynamic efficiency of the section, are separately plotted as ordinates against the angles of attack as abscissas. This is in addition to a curve of center-of-pressure travel or of some equivalent quantity such as the moment coefficient.

Another expedient, and the one now most generally employed, is to plot the lift coefficient as ordinate against the drag coefficient as abscissa (or, less commonly but with increasing frequency, the reverse), "spotting" the angles of attack along the curve. A single curve is thus made to take the place of the three used in the method previously described. The curve is called a polar, since if the lift and drag coefficients are plotted to the same scales the length of the radius vector from the origin to any point on the curve is equal to the coefficient of resultant force on the wing, while the tangent of the angle made by that radial line with the axis of abscissas is equal to the ratio of lift to drag. In actual practice, since the drag coefficients are much smaller than those of lift, different scales are used on the vertical and horizontal, the ratio of difference usually being 4 to 1 or 5 to 1. The length of the radius vector then has no physical meaning, but its slope still determines the lift/drag ratio, which is now equal to the product of the tangent of the angle previously described by the ratio between the scales used along the two

axes. A series of radial lines corresponding to integral values of the lift/drag ratio can then be drawn and marked off for direct reading of that quantity. The point corresponding to maximum lift/drag ratio is of course that at which a line drawn through the origin becomes tangent to the curve. Angles of attack are not read off so easily or clearly from the polar as when three separate curves are used, but the angle of attack is a variable of indirect and secondary importance in comparing the aerodynamic qualities of airfoils. It assumes real interest only after an airfoil has been selected, and when decision must be taken as to the angle at which the wings are to be mounted on the fuselage.

Figures 42 and 43 represent the characteristics of a given airfoil as plotted by the two methods.

A complete presentation requires in every case the inclusion of a curve of center of pressure or some equivalent. The commonest alternative to the center-of-pressure curve is a plot of coefficient of moment around some selected axis, plotted against either the angle of attack or the lift coefficient, usually the latter. Moments are commonly taken either about the leading edge or about a point one-quarter of the chord length back of the leading edge. The second practice has been gaining favor in recent years, as the moment about the 25-per-cent point is virtually the same for all angles of attack up to the burble point.[1] The mean value of the moment about that axis through the range of angles ordinarily used in flight, or the value of the moment about the leading edge at the angle of zero lift, can be used as simple numerical indices of the stability characteristics of an airfoil. If the moment is positive the center-of-pressure travel is "stable." The larger the negative value of the moment, on the other hand, the more unstable the airfoil and the greater the dependence that will have to be placed on the tail for stability. Although curves of center of pressure and moment are mutually interconvertible, if the magnitudes of the forces acting also are known, each has its particular uses in design calculation, and a

[1] This is in accordance with predictions from the mathematical theory of airfoil action, as explained in Chap. V. Actually for particular airfoils the "aerodynamic center," or axis of constant moments, may lie anywhere between 23 and 26 per cent of the chord from the leading edge. For present purposes, however, the axis will be treated as though it were constant at 25 per cent. See p. 109, *infra*.

report on an airfoil not infrequently includes both. Figure 44 shows the relation between the center-of-pressure and moment curves around the two typical axes for the same section for which the force coefficients were plotted. Where a curve of

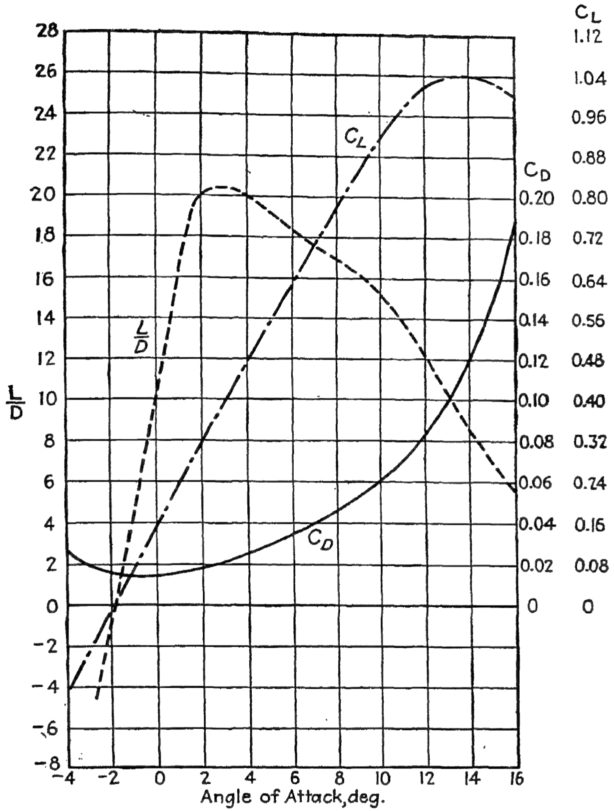


FIG. 42.—Characteristics of an airfoil, plotted against angle of attack.

moment coefficient against lift coefficient is used, especially if the moment is about the 25-per-cent point, it is very often superposed on the same chart with the polar for the section.

It will be noted that the center-of-pressure curve includes two branches, one of which was discussed in detail in the preceding

chapter. After the angle has been decreased slightly beyond that of zero normal force, the center of pressure turns up again in a very different position. There still is a diving moment around the leading edge; but whereas an upwardly directed force

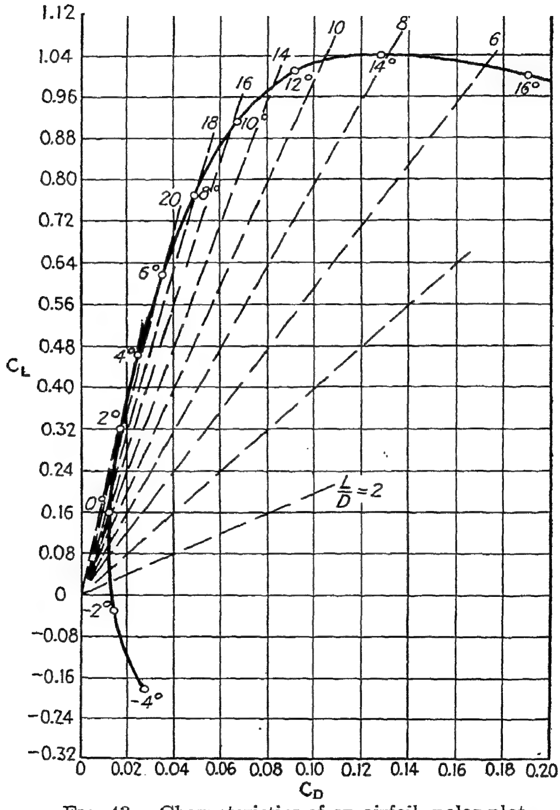


FIG. 43.—Characteristics of an airfoil, polar plot.

had to be applied far behind the leading edge to produce that moment, when the force is directed downward across the chord it must act correspondingly far forward of the nose of the section to produce the same result. As the angle decreases still further, the negative component of normal force grows larger, and the moment about the leading edge approaches zero and finally

changes sign. At the point of that change the center of pressure, of course, passes back across the leading edge and continues to move back as the angle becomes more and more negative. Since the center of pressure at negative angles moves backward as the lift coefficient increases, forward as it decreases, the typical airfoil, which displays unstable center-of-pressure characteristics

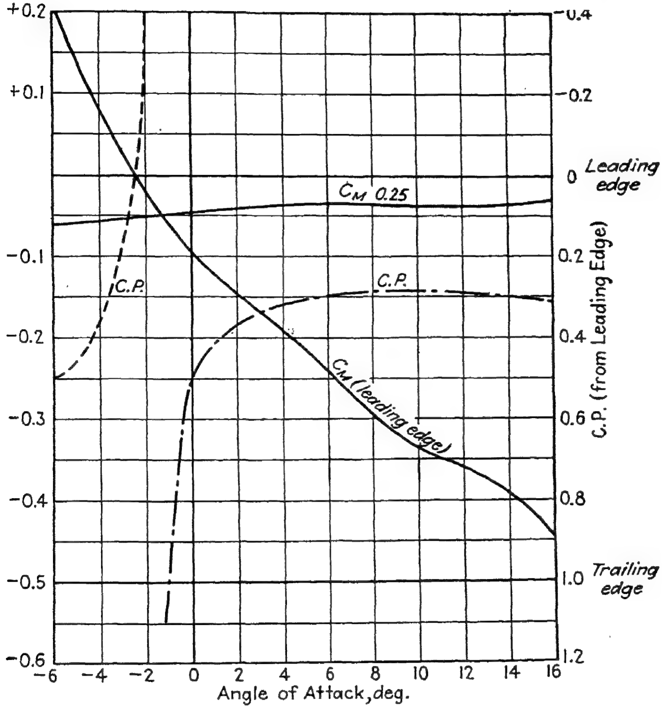


FIG. 44.—Pitching moment and center-of-pressure curves for a typical airfoil.

in the normal attitude, becomes stable when inverted. Its efficiency, however, is of course very inferior when in the inverted position.

Criteria of Airfoil Selection.—Whatever the means adopted for representing the characteristics of sections, some essential features of performance must be singled out to be used as a basis of comparison by the designer choosing a section for a new

machine. Such a choice cannot be made by looking at a great sheaf of curves as a whole. There must be specific points on which attention can be concentrated and figures tabulated for the various sections under consideration. Numerical criteria of airfoil performance must be found.

Among the most obvious of such criteria, though not by any means the most important, is the maximum value of the lift coefficient. It furnishes an index of the load which can be carried per unit of area for a given minimum speed or, conversely, of the minimum speed with a given wing loading. Since the lift of the wings of an airplane in steady horizontal flight must be approximately equal to the total weight of the machine (neglecting the small components of lift contributed by parts other than the wings), the general equation of lift takes the form for such flight

$$W = C_L \frac{\rho}{2} S V^2 \tag{13}$$

The weight of the machine and the area of the wings not being variable at the will of the pilot, the only factors subject to change are the lift coefficient and speed. It follows that when one of these has a maximum value the other must reach a minimum, or

$$W = C_{L_{max}} \frac{\rho}{2} S V^2_{min} \tag{14}$$

Solving for minimum speed and then for wing loading gives

$$V_{min} = \sqrt{\frac{W}{C_{L_{max}} \frac{\rho}{2} S}} \tag{14a}$$

$$\frac{W}{S} = C_{L_{max}} \frac{\rho}{2} V^2_{m} \tag{14b}$$

A high maximum lift coefficient is therefore to be desired in order that the minimum speed of flight at the landing speed may be low or, if that figure is held to a predetermined value, in order that the area of the wings may be reduced and the machine made less bulky, with a presumptive consequent reduction in structural weight.

A second figure useful as a criterion is that of maximum lift/drag ratio, acceptable as a general measure of airfoil efficiency. It is not the best of such measures, but it has the virtue of being the simplest. Better yet in most cases, even though as

yet somewhat less commonly employed, is the reciprocal, the minimum of drag/lift. Since the lift of the wings in normal flight is very nearly if not exactly equal to the weight, it is only necessary to multiply the drag/lift at any angle of attack by the weight to get the required propeller thrust at the corresponding condition. Obviously, other things being equal, the airplane which needs the least minimum thrust to sustain a given weight and drive it through the air is the best machine.

Instead of comparing minimum values of the drag/lift ratio, the comparison may be made between the ratios at some specified point, a point defined in terms of the lift coefficient. This is commonly done in choosing an airfoil with reference to its high-speed qualities. It may be known, for example, that a racing airplane of which the design is in prospect will probably have a maximum speed of about three times its minimum.[1] It then follows from Eqs. (13) and (14) that the lift coefficient when flying at maximum speed will be one-ninth that at minimum speed or one-ninth of the maximum coefficient. In the particular case of the airfoil treated in Figs. 42 to 44 this would give a lift coefficient at maximum speed of 0.12 and would require flight at an angle of attack of -0.6 deg. For an airfoil for service on such a machine the drag coefficient, or the drag/lift ratio, at one-ninth the maximum lift coefficient would be a factor of obvious importance.

The drag/lift ratio at one-ninth the maximum lift coefficient is of course equal to nine times the drag coefficient at the appropriate angle of attack divided by $C_{L_{max}}$. Most airplanes now have maximum speeds between two and three times their minima, and it will be found for the airfoil sections in common use that the angles of attack which correspond to maximum speed with such speed-range ratios lie very near to the angle of minimum drag. In most cases, therefore, no serious error will be made by assuming that the drag coefficient corresponding to the highest possible speed of horizontal flight is equal to the

[1] The minimum speed has sometimes been specified in racing rules in order to remove the temptation to use a tremendously high wing loading and clip the wings to dangerously small dimensions. Where no such specification has been imposed, the wing loading of racing airplanes has sometimes been raised to figures more than twice as high as would be considered safe in an airplane for commercial use.

minimum drag coefficient. If that assumption is made, the drag/lift ratio at some specified fraction of the maximum lift coefficient can be replaced as a criterion by the ratio of the maximum lift coefficient to the minimum drag coefficient, a more general figure not tied down to any particular speed range, although it follows that it is not rigorously accurate for any particular one. Such a ratio is commonly tabulated among the vital characteristics of airfoil sections and used in comparing them with reference to their high-speed performance.

In comparing the qualities of airfoil sections in climbing flight or for use in long voyages where the greatest possible economy is of the first importance, it is the consumption of power in driving the wings through the air that has to be watched, power being equal to the product of the force acting and of the distance through which it acts in unit time. In the particular case of the power requirement of an airplane wing it is the product of resistance and speed. Resistance is directly proportional to D/L for a given weight and area. Speed, the same factors again being held constant, is inversely proportional to the square root of the lift coefficient. Power therefore varies as $C_D/C_L^{3/2}$, and the minimum value of this expression is a fourth figure which should find a place in any tabulation of airfoil data.[1]

Structural Factors.—The criteria so far enumerated have been purely aerodynamic and have referred only to the magnitudes of the forces acting. There are others which relate more to structural properties.

It is evident, for example, that a thick wing section is desirable if thickness can be secured without sacrifice of other qualities. Wing spars have to carry a combination of lateral and direct loading, acting at once as beams and as columns. An increase in the depth of spar is therefore favorable to structural efficiency and to securing the necessary strength with a minimum of weight. Such an increase is to be had, in general, only by thickening the wing section. The thickness which is important is not the maximum distance between the upper and lower surfaces, but the separation of those surfaces at the points where the spars would usually be placed. Good general practice in a conven-

[1] A much more extended discussion of the bases of aerodynamic comparison of airfoil sections is contained in "The Choice of Wing Sections for Airplanes," by Edward P. Warner, *Tech. Note* 73.

tional two-spar wing, based on experience and capable of support by theory, locates the front spar at from 10 to 15 per cent of the chord and the rear one at from 55 to 70 per cent of the chord from the leading edge; the effective thickness of a wing may therefore be taken as the mean of its thicknesses at points 12 and 60 per cent of the chord from the leading edge or at some other location in those neighborhoods. Not only the mean spar depth but the depth of the shallower spar is usually important, and an airfoil section such as is shown in Fig. 45 would be structurally bad because of the inadequacy of the space in which the rear spar must be inserted. The only exception to this last conclusion is found in those cases where the two-spar arrangement has been abandoned in favor of a single heavy box or trussed spar or of a shell in which the skin of the wing acts as a unit to carry the

FIG. 45.—Airfoil section with thin trailing edge.

load. In single-spar wings, becoming increasingly frequent in recent years with the development of all-metal structures, the thickness of the section somewhere near the 35-per-cent point of the chord is of preeminent importance.

The highest efficiency in a structure can be realized, and the structure can be kept to a minimum weight for the support of a given total load, if the type of loading remains constant, so that the mode of its distribution among the several members and the relations of the stresses carried by the several members can continue always the same. In the structure of an airplane the distribution of load between the front and rear spars and between the several systems of trussing of the wing assembly varies with the position of the center of pressure, and the ideal from the structural point of view is therefore a section on which the center-of-pressure position remains constant for all angles of attack, or upon which the moment around the 25-per-cent point is zero. Airfoils answering substantially to that specification, and at the same time having good lift and drag characteristics, have been developed within the last few years and have been largely used, especially on machines having a very high speed range or intended for service requiring that they be dived to very

high speeds. As among the airfoils not actually having a stationary center of pressure there is a considerable difference in the rate at which the center of pressure travels backward as the angle of attack and the lift coefficients are diminished; the reduction of that rate of movement along the chord to a minimum is a desirable feature. It is particularly important in the single-spar wings, which are stout against flexure as long as it remains strictly in a single plane but have a minimum of facility for resisting torsion. It happens that such wings as that shown in Fig. 45, while they superficially appear ideal for a single-spar installation, have particularly unfortunate center-of-pressure characteristics. The rate of travel of the center of pressure has some effect, too, on stability, but that is less important than its influence on the load distribution in the structure.

In recapitulation, then, the principal characteristics sought in an airfoil are

1. High maximum lift coefficient.
2. High ratio of maximum lift coefficient to minimum drag coefficient.
3. Low minimum drag/lift ratio.
4. Low minimum value of $C_D/C_L^{3/2}$
5. Low drag coefficients at the particular lift coefficients at which the particular airplane under consideration will be operating under those conditions that make maximum performance most important (such as maximum speed, normal cruising, best rate of climb, and take-off, each of which corresponds to a particular speed and a particular lift coefficient for any given airplane).
6. Ample thickness of section at probable spar location.
7. Small center-of-pressure travel (or small value of 25-per-cent moment coefficient).

Number 5 is particular to a particular problem, requiring the reexamination of the available airfoils with the characteristics of a particular airplane in mind. The other six are general and can be tabulated once and for all for each new section (subject only to changes in these data with changing conditions of test in the wing tunnel and especially with changes of Reynolds' number). There are other qualities that may be sought, either universally or in selecting airfoils for some particular service, but they are as a rule unimportant in comparison with those listed above. The whole subject of airfoil selection becomes clearer in the light of the division of drag into its *induced* and *profile* parts, treated in the next chapter.

CHAPTER V

SOME APPLICATIONS OF AIRFOIL THEORY

The function of mathematical theory in engineering is not to supersede experiment, but to interpret and to extend it. Seldom if ever can theory supply the place of the fundamental data obtained in the laboratory or in the field, but it may, if the mathematicians have done their work well, stand on guard against our misapplication of laboratory or field results and double the range within which we can apply them with confidence. By that test, workers in the field of mathematical aerodynamics have succeeded. The wind tunnel remains an indispensable tool, but a large part of the burden of routine is lifted from the backs of wind-tunnel organizations. Studied in the light of theory, wind-tunnel work acquires a new coherence. Tests that once seemed to bear only upon a particular problem of limited and local interest broaden into a new significance. Experimental aerodynamics begins, thanks to its mathematical partner, to shed its drab uniform of persistent routine testing of design details and to assume the prouder panoply of true research. No aeronautical engineer or airplane designer, however completely he may be lacking in comprehension of the intellectual processes and the intricate technique by which Joukowski and Prandtl and Lanchester and Karman and Glauert and Theodorsen and Millikan and others like them have attained their results, can afford to ignore the results themselves.

It is with the results that we are concerned. The methods have been amply described elsewhere.[1] They will be touched

[1] There are many excellent volumes on aerodynamic theory and its particular application to airfoils. Most of the best work (though not by any means all) has been done in Germany, and most of the important primary sources are in German. The best general reviews in English are "Applied Wing Theory," by E. G. Reid, New York, 1932 (the simplest and most complete explanation for those not specializing in higher mathematics; contains a good bibliography of sources); "Principles of Flight," by E. A. Stalker, New York, 1931; "Applications of Modern Hydrodynamics to

upon here only very lightly, and from a purely physical point of view, with the sole object of giving some understanding of the foundations upon which the mathematicians build, and especially of the type of air flow that they have conceived and then shown actually to exist. Most of the formulas and conclusions that finally derive from the theory will have to be offered with no proof whatever, but most of them can and will be subjected to the acid test of check against the wind tunnel or against experience in flight.

Circulation.—The basic concept of the classic airfoil theory is that of circulation, or of a flow of air *around* the airfoil superposed

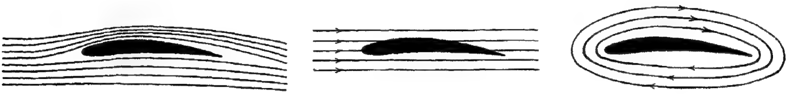


FIG. 46.—The circulatory flow around an airfoil.

on the rectilinear flow *past* it. Figure 46*a* shows the flow about an airfoil at a small angle of attack as we came to understand it in Chap. III, with the air rising ahead of the section and descending behind it, and traveling at increased speed above the airfoil and decreased speed below. Manifestly all these peculiarities of behavior can be taken care of, and the streamline pattern of Fig. 46*a* can be paralleled, if it is assumed that the flow is made up of the two components shown in Fig. 46*b* and *c*. Figure 46*c* is the circulation, and it can be shown[1] that wherever there is a circulation, there (and nowhere else) a lift is also.

Aeronautics," by L. von Prandtl, *Rept.* 116 (a classic treatment by the greatest of all contributors to the subject over the last thirty years, but now somewhat out of date); "The Elements of Aerofoil and Airscrew Theory," by H. Glauert, Cambridge, 1926 (considerably more complicated in its mathematics than Reid's and Stalker's volumes); and "Fundamentals of Fluid Mechanics for Aircraft Designers," by Max M. Munk, New York, 1929 (also limited to advanced students). An interesting, brief nonmathematical summary, by a pioneer in the field, is "The Vortex Theory and Its Significance in Aviation," by A. Betz, *Unterrichtsblatter für Mathematik und Naturwissenschaften*, No. 12, 1928 (translated as *Tech. Memo.* 576).

[1] Proved by W. M. Kutta, F. W. Lanchester, and N. Joukowski, between 1907 and 1910.

Even if the physical reasoning used in Chap. III to develop a typical air-flow pattern is laid aside entirely, there are two mathematical reasons for believing in the existence of a circulation. First, it can be shown that it is impossible for any lift to exist in a non-circulatory flow. Second, the flow itself would be physically impossible. In order that the air passing above the airfoil and that passing below may be at equal pressures when they meet behind the trailing edge, as they must be if they are to join smoothly and continue their joint way downstream, a non-circulatory motion would have to take the form indicated in Fig. 47, with the actual conjunction of the two sections some distance above the trailing edge. It needs no mathematics to demonstrate that no such flow, with the air having to make instantaneous turns around sharp corners, could actually exist.

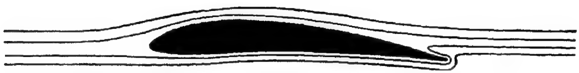


FIG. 47.—Fictitious flow, without circulation.

Obviously it is a fundamental condition that the two divisions of the flow must rejoin exactly at the trailing edge. That provides not only the proof that there must be a circulation of some kind but the means as well of calculating the actual strength of the circulation that will have to exist. The magnitude of the lift being, by the Kutta-Joukowski theorem,[1] directly proportional to the strength of the circulation that produces it, the lift can be directly computed for any angle of attack for any airfoil that is amenable to mathematical treatment. That limits it, if the treatment is to be rigorous, to the Joukowski and Witoszynski airfoils, obtained by the mathematical process of conformal transformation of a circular section.[2] A few of these mathematical sections are shown in Fig. 48, where it will be noted that their principal geometrical characteristics are that the curve of mean camber, or curve followed by the center line of the section, is always a circular arc and that in most cases the leading edge is

[1] Reid, *op. cit.*, p. 55.

[2] Although great progress has been made in the last few years, especially by Theodorsen, in the development of more generalized methods. See, for example, "Theory of Wing Sections of Arbitrary Shape," by Theodore Theodorsen, *Rept.* 411.

rounded on a very large radius and the point of maximum thickness is far forward on the chord.

The Kutta-Joukowski method of calculating the circulation, applied to the Joukowski sections, indicates that the slope of the curve of the lift coefficient against the angle of attack should be uniform for all the airfoils of the series, and numerically equal to 2π per radian (0.110 per degree) in the type of absolute units favored by the N. A. C. A.[1] and adopted as standard in this volume.

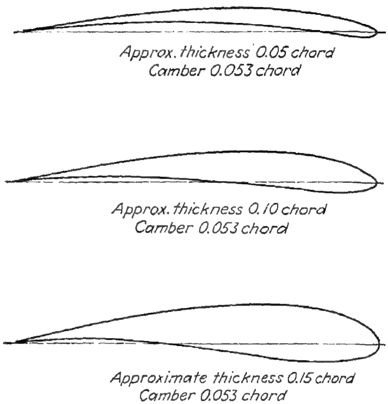


FIG. 48.—Typical Joukowski airfoil sections.

This particular prophecy of aerodynamic behavior is among the least successful of those for which the airfoil theory is responsible. It is verified by experience to the extent that it is true that practically all airfoil sections, whether they be of the Joukowski series or not, have lift coefficient curves of very similar slope over their straight sections (extending as a rule up to within about 4 deg. of the angle of maximum lift). The slope measured in the wind tunnel, however, proves to average about 12 per cent less than the slope as calculated by Kutta. The deficiency of slope ranges, for a considerable number of representative sections for which the comparison was made, from about 2 per cent up to 20 (indicating the spread of slopes among the sections studied). The difference might be presumed due at least in part to viscosity, the calculation having been made for a frictionless fluid, and Wieselsberger and Betz and Lotz, of Göttingen, have tried[2]

[1] Type 6 on p. 34.

[2] "The Most Important Results of the Wing Theory and Its Proof by Experiment," by C. Wieselsberger, from "Vorträge aus dem Gebiete der Hydro- und Aerodynamik," by von Karman and Levi-Civita; "Reduction of Wing Lift by the Drag," by A. Betz and J. Lotz; *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, May 28, 1932; translated and published as *Tech. Memo.* 681. See also "Reduction of Lift of a Wing Due to Its Drag" by

to establish a fixed relationship between the rate of deviation of the lift curve from its theoretical form and the amount of drag of the airfoil. They have had limited success.

What has been said about the necessity of a smooth joining of the two divisions of the air flow exactly at the trailing edge of course applies to the leading edge as well if it be sharp instead of rounded, the reasoning being in a slightly different form appropriate to its being a point of approaching instead of receding flow. Only one independent variable can be taken, however, and the trailing-edge condition supplies that one. The trailing-edge condition having determined the strength of the circulation the flow is fully prescribed, and it follows, as Theodorsen has pointed out,[1] that the flow of air on to a section with a sharp leading edge is likely to be smooth only at one particular angle. When the leading edge is rounded on a considerable radius, which is the common case in practical design, it is of course possible for the two groups of streamlines to separate at almost any point without undue shock and without requiring any individual particle of fluid to turn a sharp corner. The true "leading edge" in the strictly aerodynamic sense, or stagnation point where the upper-surface and lower-surface flows separate, then moves up and down the nose of the section as the angle of attack changes. Reasoning on that line led directly to the conclusion that the "effective form" of any section with a rounded leading edge must undergo a change with change of the angle of attack, and that the aerodynamically effective angle of attack was itself then changing at a rate different from that of the apparent or geometrical angle. It would follow that the slope of the lift curve ought properly to be referred to the truly effective angle if there is to be comparison with the mathematical prediction, and that, when the lift is plotted (as it always is and as a matter of practical convenience must be) in terms of the geometrical angle of the rigid section, deviations of the slope of its curve from the predicted values would have to be expected. Theodorsen's concept proves much more effective than that of Wieselsberger, and has enabled

J. Stüper, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Aug. 28, 1933; translated as *Tech. Memo.* 781.

[1] "On the Theory of Wing Sections, with Particular Reference to the Lift Distribution," by Theodore Theodorsen, *Rept.* 383.

him to produce a curve presenting $dC_L/d\alpha$ as a function of the radius of curvature of the leading edge of the airfoil and decreasing it from 2π for a sharp nose to 5.6 with a nose radius of $0.015c$ and 5.1 for a radius of $0.03c$ (c being the chord). His figures, well verified by experiment, obviate any necessity of departing from the ideal fluid and introducing frictional factors to account for the lift curve's behavior.

There have been many researches[1] on the direct measurement of the effect of circulation on the resultant air flow about an airfoil. Both the velocity and the direction of the flow have been investigated. In sweeping terms and to a rough approximation, such studies indicate a maximum air velocity over a typical airfoil, at an angle of attack near that of maximum lift, some 60 to 100 per cent greater than the velocity of the undisturbed relative wind. The maximum is of course located very close to the upper surface of the airfoil and some 10 per cent of the chord behind the leading edge, essentially at the point where the sharpest curvature of the streamlines determined by the form of the typical section would be anticipated. At a distance of 10 per cent of the chord above the surface, the increase in speed above the normal wind speed has fallen off to about 40 to 60 per cent, or about two-thirds of its value very near to the surface.

All this is best shown by an actual contour map of velocity distribution, with lines drawn like isotherms on a weather map to indicate the lines along which the velocity is uniform. Such a map, representing the flow at an angle of attack just below that of maximum lift is reproduced[2] in Fig. 49. In this particular case, over an airfoil of more than average abruptness of curvature of the upper surface, velocities as high as 2.2 times the normal wind speed were measured. The rapidity with which the excess

[1] As, for example, "The Two-dimensional Flow of Air around an Airfoil of Symmetrical Section," by T. Tanner, *R. and M.* 1353; "On the Flow of Air Adjacent to the Surface of an Aerofoil," by N. A. V. Piercy and E. G. Richardson, *R. and M.* 1224. A very complete analysis of the velocities of flow about a Joukowski section is given in "An Investigation of the Variations in the Velocity of the Air Flow about a Wing Profile," by Walter Repenthin, *Zeitschrift für Flugtechnik und Motorluftschiffahrt,* July 15, 1929, translated as *Tech. Memo.* 575.

[2] From "An Investigation of the Flow of Air around an Aerofoil of Infinite Span," by L. W. Bryant and D. H. Williams, *R. and M.* 989.

velocity falls off along a normal to the surface is striking, but it is as nothing compared with the rate of velocity change in the immediate neighborhood of the surface,[1] too close to be even suggested on the large scale of Fig. 49. Noteworthy, too, is the

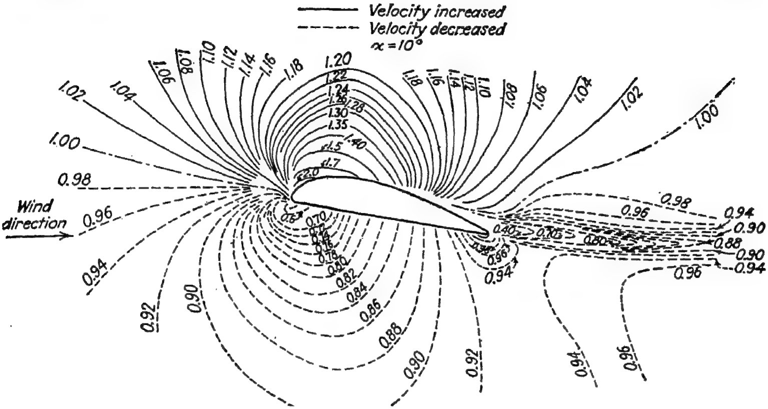


FIG. 49.—Contours of local velocity in the neighborhood of an airfoil section.

evidence of separation and of the existence of a region of dead air behind the after part of the upper surface. Better and more explicit evidence on that point, however, is provided by the variation of total head, or summation of pressure and velocity

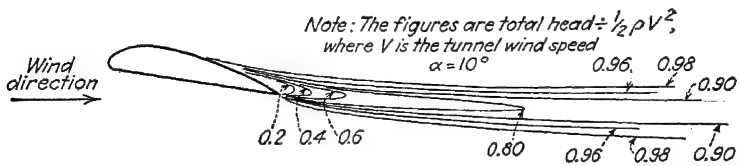


FIG. 50.—Variation of total head in the air flowing around an airfoil section.

heads, which according to Bernoulli's theorem[2] should be constant throughout a region of streamline flow. Figure 50,[3] where total head is plotted, shows how very faithfully the Bernoulli specification is met except immediately behind the upper surface, and how radically it is departed from there.

[1] Discussed in detail in Chap. VI.

[2] P. 38, *supra*.

[3] Also from *R. and M.* 989.

A scarcely necessary but nevertheless interesting confirmation of all these observations on speed distribution over the airfoil is found in wing-radiator experiments showing the average rate of heat dissipation per unit of surface to be about twice as large on the upper surface as on the lower, to increase as the angle of attack increases, to reach a maximum intensity near the leading edge of four or five times the mean upper-surface value.[1]

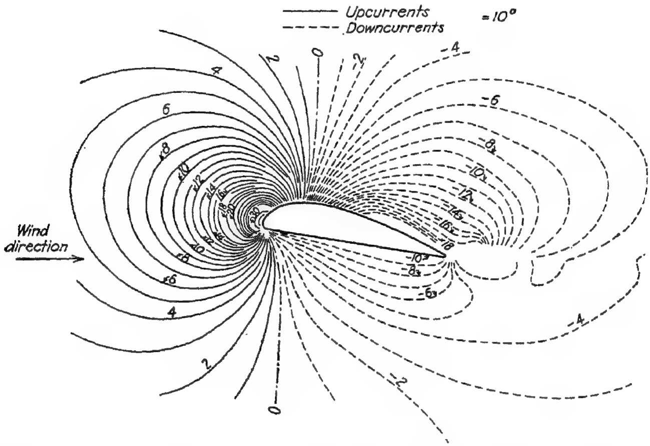


FIG. 51.—Contours of angular deviation (in degrees) of the streamlines in the neighborhood of an airfoil section.

Figure 51 represents the varia ions of local direction of flow in the same fashion as that used for speed variation over the same wing, and shows a general downwash angle decreasing to 5 deg. within one chord length behind the trailing edge of the wing, thereafter falling off gradually. Figures 49 and 51 together explain why instruments for measuring air speed and angle of attack, to give results reliable enough for serious test work, must be mounted well ahead of the leading edge (in the case of angle instruments at least two chord lengths).[2] A devia-

[1] "On the Convection of Heat from the Surface of an Aerofoil in a Wind Current," by L. W. Bryant, E. Ower, A. S. Halliday, and V. M. Falkner, *R. and M.* 1163.

[2] Where it is speed alone that has to be measured, satisfactory results can be obtained with an installation of an air-speed meter approximately

tion of the stream as great as 17 deg. was found one-quarter of a chord length ahead of the leading edge.

Magnus Effect.—Though the circulation around an airfoil is not at all difficult to picture, it is easier still to realize the existence of a circulatory flow and to form a mental image of its nature in the case of a rotating cylinder. Circulation around a cylinder or sphere, induced by the rotation of the body, explains the well-known Magnus effect and all its familiar physical expressions, the curving of a baseball or the "slice" of a golf-ball,

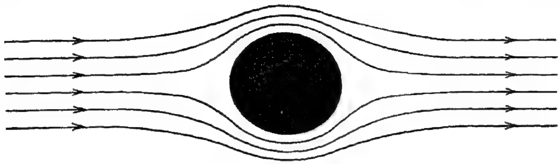


FIG. 52.—Idealized flow around a stationary cylinder.

the "drift" of a rifled projectile at long range, and the functioning of the Flettner rotor which had a brief but intense fame as a device for marine propulsion about 1924.

Figure 52 displays the idealized streamline flow around a stationary cylinder, assuming that the air closes in immediately behind the cylinder and ignoring the separation, analogous to the burbling of an airfoil, which must actually take place. The whole pattern, of object and flow, is symmetrical about a plane passed through the cylinder's center and parallel to the wind direction. The resultant force must therefore be strictly parallel to the wind, without a lift component. The closer spacing of the streamlines as they pass around the cylinder, as compared with the spacing in the free stream, is the geometrical indication of the higher speed and lower pressure in the flow in that region.[1]

Figure 53 centers upon the same cylinder, but it is no longer stationary. It is now being rotated about its own axis, and since the air is a viscous fluid the air that lies close to the surface

above the trailing edge, as the velocity in that region remains very nearly equal to the undisturbed wind velocity at all angles of attack. See "Full-scale Wind-tunnel Tests to Determine a Satisfactory Location for a Service Pitot-static Tube on a Low-wing Monoplane," by John F. Parsons, *Tech. Note* 561.

[1] See p. 38, *supra.*

of the cylinder is being dragged around with it by friction.[1] Circulation is being directly created by mechanical means, and circulation about a rotating cylinder has exactly the same effect as the circulation about an airfoil which is produced by

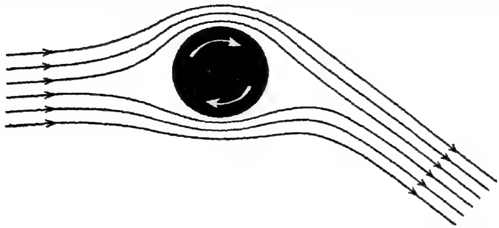


FIG. 53.—Idealized flow around a rotating cylinder, producing Magnus effect.

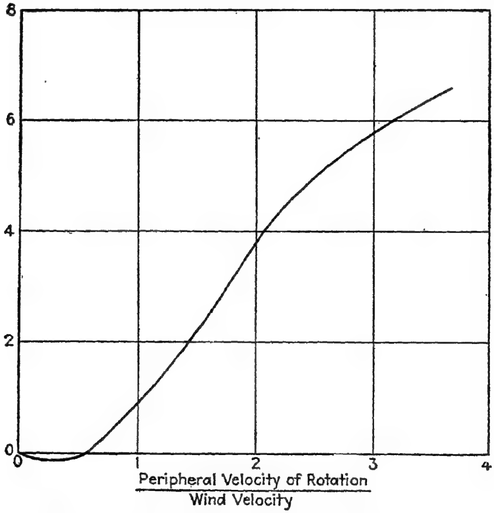


FIG. 54.—Lift produced by a rotating cylinder.

the form of the airfoil itself, without resort to mechanical manipulation. It creates a lift. It may, if the rotation is rapid, create a very large lift indeed. In Fig. 54 the lift coefficient for a rotating cylinder has been plotted against the ratio of the periph-

[1] For the mechanism of fluid friction and its magnitude, see Chap. VI, *infra*.

eral velocity due to the rotation to the translational, or wind, velocity. At the maximum the coefficient (the area used in calculating it being the simple product of the length and diameter of the cylinder) is considerably higher than can be attained with an airfoil, notwithstanding the unpromising aerodynamic form of the cylinder. With the cylinder capped with disks of three times its own diameter to prevent escape of air off its ends the maximum C_L rises another 80 per cent, to the amazing value of 9, allowing a theoretical transverse force or lift of 57 lb. per sq. ft. of projected area of the cylinder at a wind speed of 50 m. p. h. The top of the rotor of the original rotorship *Buckau* spread such a disk.

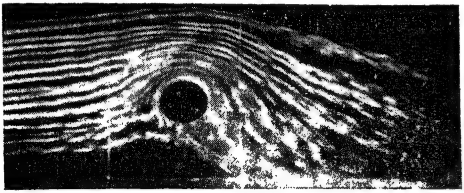


FIG. 55.—Air flow around a rotating cylinder.

That the sketch of the motion was in no way exaggerated is testified by Fig. 55, a photographic study of the flow in a wind tunnel.[1] The deviation of the streamlines from their normal course by the rotation of the cylinder was obviously extreme.

The lift curve's most striking characteristic is the abrupt change of form that appears at a certain magnitude of the ratio of the peripheral velocity of spin to the wind speed. The critical figure for a smooth cylinder is approximately 0.5 or a bit lower.[2] Below that level the spin is almost without lift-

[1] Furnished by the N. A. C. A., from the Langley Field smoke tunnel. See also "Experiments on the Flow behind a Rotating Cylinder in the Water Channel," by E. F. Relf and T. Lavender, *R. and M.* 1009.

[2] The subject has been studied in a number of laboratories, with essentially similar results in all cases. Good references are "Air Torque on a Cylinder Rotating in an Air Stream" (upon which Fig. 54 is based), by A. Thom and S. R. Sengupta, *R. and M.* 1520; "Application of the Magnus Effect to the Propulsion of Ships," by L. Prandtl, *Die Naturwissenschaft*, June 2, 1925, translated as *Tech. Memo.* 367; "Tests of Rotating Cylinders," by E. G. Reid. *Tech. Note* 209.

producing effect.[1] Above it, there appears a lift which thereafter increases in direct proportion to the rate of rotation.

The eccentricity so noted for a cylinder accords with the teaching of highly practical experience with spheres. When a baseball pitcher throws a ball that follows a substantially normal trajectory for a certain distance and then "breaks" into a curve, it is of this aerodynamic phenomenon that he is taking a presumably unwitting advantage. The ball leaves his hand with a definite velocity and a definite rate of rotation. Both are progressively reduced by air resistance, but the linear velocity falls off more rapidly than the rotational. The ratio between the two, originally below the critical value, accordingly rises in due course above it. A transverse force on the ball, corresponding to the lift on the cylinder in the wind tunnel, then develops, and the ball is diverted from its path. If the pitcher's fingers had given it a more vigorous spin the ratio of angular to linear velocity would have been above the critical value from the first, the deviation from the straight path would have been immediate, and the resultant curve would have been of the variety known to the fields of play as "roundhouse." A golf-ball often shows the same characteristic, a slice appearing to start after a hundred yards or more of straight travel, and for the same reason.

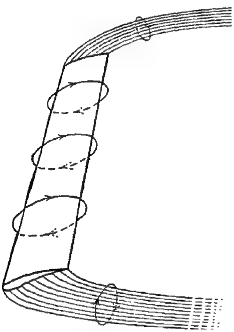


FIG. 56.—Flow from the tips of an airfoil (idealized).

Tip Vortices.—Both the rotating cylinder and the airfoil have been discussed as though they were uniformly two-dimensional, but in practice the wing has tips and they affect the flow. There is an excess of pressure above atmospheric below the airfoil,

[1] Presumably because the influence of a slower rotation fails to penetrate the boundary layer, for which see p. 121, *infra*. This theory is confirmed by the fact, observed in the British tests cited above, that when the cylinder is given a very rough surface, equivalent to sandpaper, a lift appears even at the very lowest rotational velocities and the lift curve rises in a straight line from the origin. The roughening of the cylinder would give it the power, so to speak, to project its influence through the boundary layer.

a drop to below atmospheric on its upper surface. Following the universal inclination to flow from high-pressure to low-pressure regions by every available path, the air escapes around the tips and sets up a rotational flow that persists far downstream behind the airfoil, and that is technically known as a tip vortex.

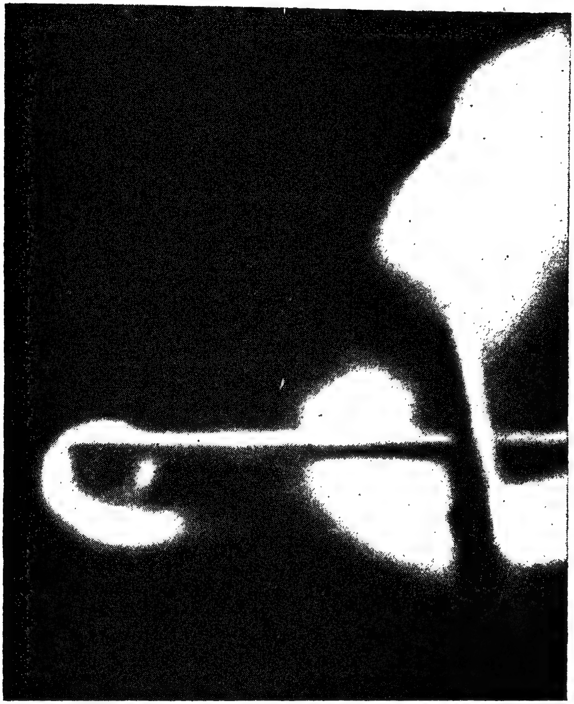


FIG. 57.—Tip-vortex form behind an airfoil (at left of illustration).

Crudely judged, without reference to the exact structure, the tip vortices appear, as is evident in Fig. 56, as a continuation of the circulation around the section. With the form of the flow made visible by various devices, they prove their reality as in Figs. 57[1] and 58.[2] The whole flow resolves itself into a rotation

[1] From "Wind Tunnel Studies in Aerodynamic Phenomena at High Speed," by F. W. Caldwell and E. N. Fales, *Rept.* 83.

[2] From the N. A. C. A.

around a U-shaped, or "hairpin," axis of which the wing is the base and the uprights or legs are imaginary lines extending backward from the wing tips.[1]

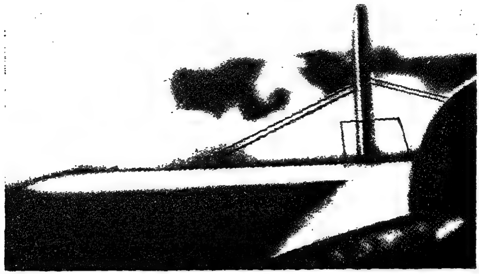


FIG. 58.—Tip-vortex formation, mapped in a smoke cloud through which an airplane has just flown.

Manifestly the air inside the axes of the tip vortices will be moving downward as a result of the composition of its straight motion away from the airfoil with its tangential velocity around

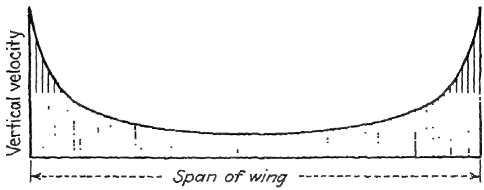


FIG. 59.—Variation along the span of the vertical component of velocity behind a rectangular airfoil.

the vortex axis, while the air outside the axes will have an upwardly directed component of velocity. The mathematical theory permits of calculating the whole flow pattern in the vortices, and the variation of the vertical components of velocity across the span, a short distance behind the airfoil, is found to be as plotted in Fig. 59. Perhaps a more helpful representation,

[1] Here as elsewhere in this chapter, the term "rotation" is used with the physical connotation that it carries in common parlance, to help in giving a picture of the nature of a flow, and without reference to its specialized technical employment by mathematicians. Mathematically, for example, a circulatory motion is irrotational.

carrying a closer physical concept of what happens in and around the tip vortex, is that of Fig. 60,[1] where lines of equal vertical velocity component are the basis of the plot. Along the zero contours the air in this case was moving in a horizontal plane parallel to the undisturbed wind direction. The actual line of motion of course was not necessarily parallel to the undisturbed flow, and in general there would be a very substantial transverse component along the zero line, as points on that line would lie

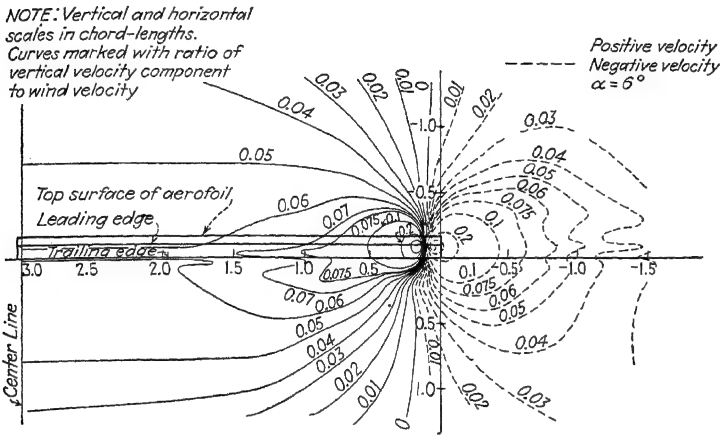


FIG. 60.—Variation of the vertical component of velocity close behind an airfoil.

immediately above or below the vortex axis. On other contours than that of zero the flow had a vertical velocity component, upward or downward as the case might be and of magnitude proportional to the figures attached to the contours (which represent the ratio of the vertical component to the normal wind speed). The neat conformity of the representation to the deductions from the theory is obvious.

A simple physical explanation of the general downflow, or downwash, immediately behind the wing has already been given in a previous chapter.[2] The vortex theory supplies a rigorous

[1] Taken from "An Inves[illegible] Aerofoil of Finite Span," by A. Fage and L. F. G. Simmons, *R. and M.* 951.

[2] Chap. III, *supra*.

mathematical treatment and a basis for computation and for further analysis.

Tip Vortices and Pressure Distribution.—So pronounced a modification of flow along the span cannot, and does not, fail to leave a record of itself in the distribution of the forces on the wing. The flow up around the end of the wing onto the upper surface impairs the mechanism whereby a high pressure drop

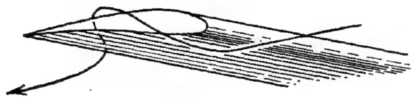


FIG. 61.—Motion of a particle of air around the tip of an airfoil (idealized).

is built up in the forward part of the upper surface in a two-dimensional motion, and it would be expected that the peak of the curve would be appropriately lowered as the tip is approached. On the other hand, the same flow around the tip and inward along the upper surface to the trailing edge creates a downward curvature of the streamlines at all points back to the trailing edge, as will be apparent from the diagrammatic trace of the motion of a particle in Fig. 61. A substantial depression of pressure ought then to be maintained all the way back to the

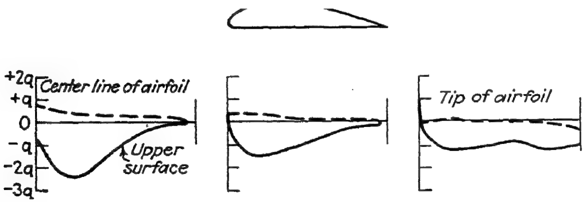


FIG. 62.—Change of pressure distribution along the span.

trailing edge, in place of the customary steady drop of pressure difference to a virtual zero at the trailing edge.

Everything happens according to prophecy, especially (as would be anticipated) at large angles of attack. Figure 62[1] shows the pressure distribution at an angle of 10 deg. about three sections of a square-tipped airfoil, the sections being at the center of the wing, 90 per cent of the way out to the tip, and 97 per cent of the way, respectively. Obviously (again in

[1] From "Pressure Distribution over Thick Aerofoils—Model Tests," by F. H. Norton and D. L. Bacon, *Rept.* 150.

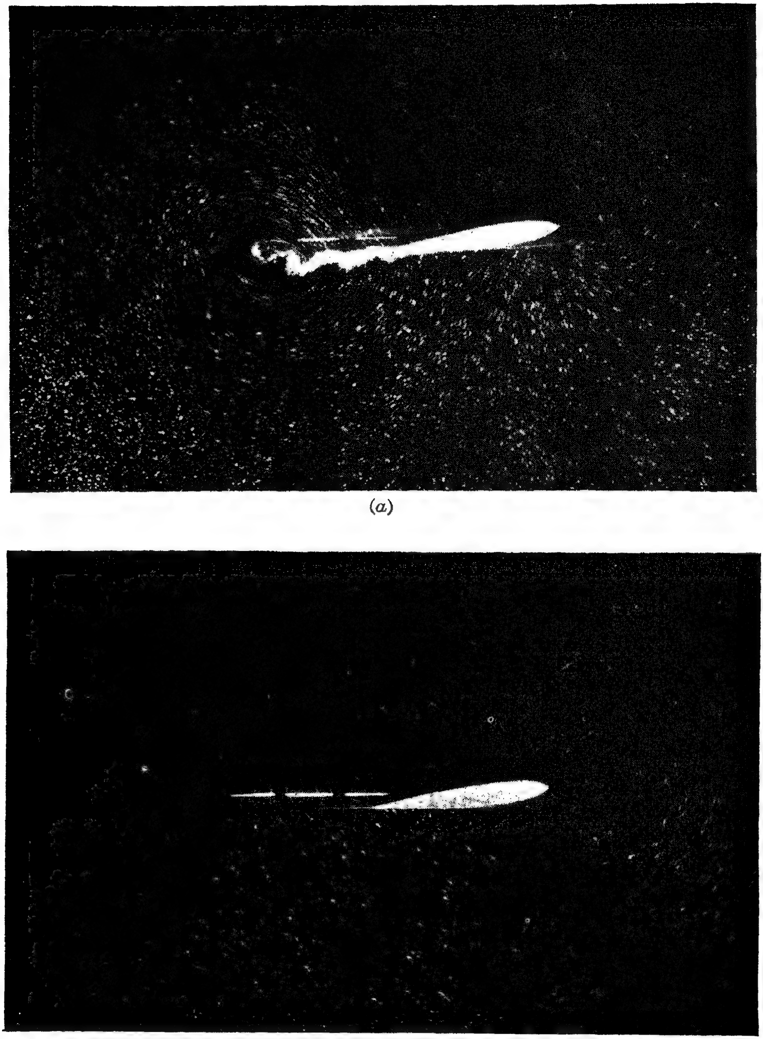
accordance with prediction from the presumed nature of the flow) almost all of the effect is concentrated in the last fraction of the span. The same sort of reasoning accounts very readily (so readily that it may be left to the reader's imagination in place of being developed here in full detail) for the reduced positive pressure on the lower surface at the tip.

For obvious reasons, since the supply of air to every portion of the upper surface near the tip is being constantly replenished by flow around the end from the lower surface, separation and burbling do not play the part that they do in the two-dimensional motion. At very high angles, indeed, the intensity of loading per unit of span at the extreme tip is higher than on any other part of the span, and even at moderate angles the span-loading curve often shows a hump at the tip after falling off steadily most of the way out from the center of the wing. Figure 154[1] illustrates characteristic span-loading curves for several angles of attack.

The Development of Circulation.—Obviously the tip vortices cannot be created absolutely instantaneously, and cannot attain their full development until the wing has moved at least two or three chord lengths relative to the air. Furthermore, Prandtl has demonstrated that the U-shaped pattern including the circulation cannot, so to speak, be developed directly from zero. In a perfect fluid a vortex pattern would be immortal, the vortex going on to an infinite distance, and the pattern can only be brought into being from a state of rest in a fluid of small friction if it starts as a completely closed system. The U-shaped axis of the circulation and vortices must start as a closed figure, and the circulation about the wing must, at the instant of its development, be counterbalanced by a vortex of equal strength and opposite rotation thrown off at the trailing edge of the airfoil and gradually passing downstream. There must be a similar formation and dissipation of a vortex (or a succession of them) at the trailing edge whenever the circulation is changed in strength, as when the angle of attack of the airfoil has just been changed. Wind-tunnel measurements indicate[2] that the circulation, and so the lift, attain 55 per cent of their ultimate full values

[1] P. 228.

[2] "Growth of Circulation about a Wing, and an Apparatus for Measuring Fluid Motion," by P. B. Walker, *Rept.* 1402.



(a)

(b)

FIG. 63.—Flow around an airfoil. (a) immediately after start of motion, showing counter-circulation just behind trailing edge; (b) after motion is well developed.

by the time the wing has moved one chord length from a stationary position, and 85 per cent after a total movement of five chord lengths. Predictions from theory are in very close accord with experiment on this point. For the practical discussion of airplane performance the aerodynamic forces on a wing can as a general rule be considered as adjusting themselves instantly to the full effects of changes in angle of attack and speed, though time lag in the adjustment often has an influence on control and maneuvering characteristics.

All this may well be a little difficult to comprehend from words alone. It becomes much clearer from Fig. 63, where two successive stages in the growth of circulation about an airfoil, one taken immediately after the start of the motion and the other at a time when the airfoil had moved six chord lengths from a standing start and the development of the permanent flow pattern was virtually complete, are shown photographically. It must be noted in interpreting these pictures that they show the flow about an airfoil moving through a stationary body of fluid, whereas all air-flow illustrations previously used represented the case of air flowing past a stationary airfoil. Obviously the same relative flow will present quite different apparent patterns under the two conditions. For the particular purpose of showing and analyzing circulation the method of Fig. 63 is the more useful, as the translational component of the relative motion is entirely eliminated and the circulatory component is shown directly in its proper relation to the airfoil.

Induced and Profile Drag.—Most important of all the contributions made by airfoil theory to the practical pursuits of the airplane designer is the concept of induced drag. The division of airfoil drag into two parts, induced and profile, was first proposed by Prandtl about 1915. He and his coworkers at Göttingen, Betz and Munk and others, developed and extended the theory until by the end of the war they had made it possible to calculate the change of airfoil characteristics with changing aspect ratio, the mutual influence of the wings of a biplane upon each other, and a variety of other bits of aerodynamic information of less fundamental importance. The wind tunnel, which had been used prior to that time for many long series of tests designed to supply aspect-ratio corrections and biplane-interference corrections and the like, was freed for other work. In 1916

the typical airplane designer was unlikely ever to make even the slightest contact with airfoil theory. The engineer and the mathematician lived in wholly different worlds. Since 1922 or thereabouts, on the other hand, no one can pretend to be educated in aeronautical engineering unless he has at least a working acquaintance with the standard induced-drag formulas and the manner of their application.

The induced-drag theory is based on the assumption, mathematically supported to be sure, that the lines of flow of the air about the section of an airfoil depend only on the form of the section and its attitude relative to the air and are independent of the aspect ratio and (within reasonable limits) of the proximity of two or more wings to each other. If that assumption is correct, the forces acting on the airfoil should depend only on its section and on its attitude with respect to the air flowing immediately past it, and should be referred to the mean direction of the flow of the air in the immediate neighborhood of the airfoil as the fundamental datum line.

The forces of lift and drag are normally referred to axes perpendicular and parallel to the direction of undisturbed motion of the relative wind at a considerable distance from the airfoil, outside the immediate zone of its influence. However, since the lines of flow of the relative wind are turned downward on passing the airfoil it would be more scientifically exact, although certainly far less convenient for the engineer, to refer the forces to axes perpendicular and parallel to a mean relative wind line, a line bisecting the angle between the direction of the relative wind before entering the immediate zone of airfoil influence and its direction after passing over the airfoil. Such a mean line is shown dotted in Fig. 64, and the components of force referred to it may be called the *profile lift* and *profile drag*, since, if the assumption previously stated is correct, they are functions only of profile and attitude and not of plan form. The profile lift and drag are denoted in the diagram by L_o and D_o; the lift and drag referred to the relative wind at a distance, as ordinarily measured, by L_a and D_a.

L_a and D_a can of course be found by taking the components of profile lift and profile drag parallel to the ordinary lift and drag axes. Since the profile drag is much smaller than the lift, its component parallel to the lift axis is insignificant, and the

profile lift is therefore substantially equal to the lift as ordinarily measured. That, however, is far from being true of the drag. As indicated in Fig. 64, the component of profile lift parallel to the drag axis is likely to be of considerable importance. The drag is then made up of two parts, the true or profile drag and a component of lift which is called the induced drag, and the second component is dependent upon aspect ratio and mutual interferences and varies with them. Plainly its magnitude can be determined whenever the magnitude of ϵ is known.

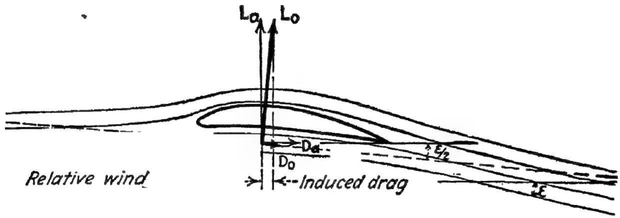


FIG. 64.—Geometry of induced angle and induced drag.

That Fig. 64 falls a considerable distance short of literal representation of the flow around an airfoil is evident if it is compared with Fig. 51, but as an idealized version it fits reality nearly enough to suit our present purpose. There is no pretense of rigor in this discussion or in the derivations that are to follow, but only a search for a convenient relationship between the induced-drag phenomena and a simple and straightforward line of geometrical reasoning.

Sweep Area and the Calculation of Downwash.—The angle of downwash, or the angle through which the streamlines have been turned when they leave the wing, is calculated from a detailed analysis of the motions of the air in the tip vortices. In the case of a rectangular wing, the amount of downwash and the ratio of induced drag to local intensity of lift vary as indicated in Fig. 59. It instantly strikes an engineer as probable from analogy with other mechanical problems that the total induced drag for the wing will prove to be a minimum, other things being equal, if the downwash can be made uniform from tip to tip. Prandtl and Munk have demonstrated that that is indeed the case, and have furthermore shown[1] that the condition

[1] "The Minimum Induced Drag of Airfoils," by Max M. Munk, *Rept.* 121.

of uniform downwash will be satisfied by an airfoil so shaped that the curve of lift distribution along the span has the form of a semiellipse.[1]

For any other distribution the analysis of the flow and the calculation of the induced drag are considerably more complex than for the elliptic-lift-curve case, but Betz and Glauert have accomplished it for a number of typical wing forms. Fortunately for simplicity, it happens that the increase of induced drag above its ideal minimum is small within the range of wing-plan forms commonly used in airplane design. The simple formulas based on minimum induced drag are generally employed in design calculations, and then if a very high degree of accuracy is desired the induced drag can be multiplied by a coefficient somewhat greater than unity (but never exceeding 1.2 in the case of a rectangular wing, for example) depending on the precise plan form and consequently on the form of the curve of lift along the span.

The mathematical processes involved in the development of the induced-drag formulas from the vortex theory are too complex even to be summarized, but there is a less rigorous and an incomplete method of derivation which involves only elementary mechanics. It depends upon the concept of *sweep*,[2] or of the bounding of a definite area of the air stream within which the effect of the airfoil on the air flow is presumed to be constant, while outside that area it is assumed that there is no effect at all. Having determined the sweep area, and ignoring entirely the fluid beyond its boundaries, we can apply the theorem, already discussed,[3] that the force exerted upon a solid obstacle placed in a fluid stream of finite extent is equal to the momentum communicated to the fluid in unit time and that any particular component of force is equal to the component of momentum change in exactly the opposite direction. The lift is then equal to the downward momentum.

The momentum theory is rigorously true for a stream of particles striking a wall. While it cannot hold with the same exactitude

[1] An alternative proof of this fact, considerably simpler than the classic German form, has been given in "A Proof of the Theorem Regarding the Distribution of Lift over the Span for Minimum Induced Drag," by W. F. Durand, *Rept.* 349.

[2] Due to Lanchester.

[3] P. 21, *supra*.

when the moving stream is surrounded by a limitless ocean of the same fluid, as in air, or when it is subject to some exterior interference, it still serves as a useful basis of calculation. Neglecting changes in the speed of the air along the streamlines, which would connote changes in pressure, the downward momentum communicated to the air in unit time is equal to the mass of air affected in that time, multiplied by the downward velocity imparted. That component of velocity, in turn, is equal to the product of the speed of the flow and the sine of the angle through which the streamlines are turned. The mass of air affected is the product of sweep area, velocity, and density.

Of course the sweep area is a fictitious concept, without direct physical significance. The effect of the airfoil on the air flowing past it actually extends to a great distance above and below, gradually becoming less and less marked and approaching zero asymptotically. The fiction is, however, a very convenient one. It gives a basis for reconstructing the induced-drag formulas, should we lose them, with nothing to work from except a simple mechanical theorem and a simple geometrical assumption.

If S' be the sweep area the mass of air handled in unit time is $\rho S'V$, ρ as usual being air density. The downward momentum communicated to the fluid is then the product of this quantity by $V \sin \epsilon$, where ϵ is the final angle of downwash beyond the immediate zone of influence of the circulation around the wing. Since this must be substantially equal to the lift, and since the downwash is always small enough so that the value of ϵ in radians can be substituted for $\sin \epsilon$ without appreciable error,

$$L = \rho S'V^2\epsilon \tag{15}$$

The angle between the undisturbed stream of air ahead of the wing and the mean relative wind, or the angle between the axes of profile lift and drag and those to which the forces are referred in ordinary measurement, is $\epsilon/2$, and

$$\frac{\epsilon}{2} \qquad \overline{2\rho S'V^2} \tag{16}$$

Prandtl has demonstrated that in the ideal case of uniform downwash, or elliptic distribution of lift along the span, the

sweep area is bounded by a circle having a diameter equal to the wing span.
Then

$$S' = \frac{\pi b^2}{4} \tag{17}$$

b being the span.

The induced drag, as is clear from Fig. 64, is equal to the product of the lift by the mean relative wind angle. Then substituting the value just given for S' in the formula for the angle,

$$D_i = \frac{L^2}{\frac{\rho}{2}\pi b^2 V^2} \tag{18}$$

It is often desirable to deal with coefficients rather than with total forces. The equation for induced-drag coefficient now becomes

$$C_{D_i} = \frac{D_i}{\frac{\rho}{2}SV^2} = \frac{L^2}{b^2 S V^4 \pi} = \frac{C_L{}^2 S}{b^2 \pi} \tag{19}$$

or

$$C_{D_i} = \frac{C_L{}^2}{\pi R} \tag{20}$$

where R is the aspect ratio of the airfoil.

Although the formula naturally derives in terms of absolute coefficients, it is of course an easy matter to convert it into any other form desired. In the particular case of m. p. h., lb. per sq. ft. coefficients, it becomes

$$D_{c_i} = \frac{L_c{}^2}{0.00255\pi R} = \frac{125 L_c}{R} \tag{21}$$

It may appear strange that induced drag should be tied up so closely and exclusively with aspect ratio, as the flow across the central part of an airfoil will in any case be two-dimensional in its nature and free from transverse component.[1] However,

[1] The presumption of two-dimensional flow at the plane of symmetry, though it seems obvious enough, is not rigorously true. There is a certain amount of periodic oscillation that gives to the flow at the center line of the wing a transverse velocity component first in one direction and then in the other. The common supposition that a thin plate inserted into a wing

the depth of the sweep area at the mid-section of the airfoil depends on the span alone and increases indefinitely with increasing span. A clearer physical picture is given by Eq. (18), where the induced drag appears as proportional to the square of the lift per unit length of leading edge (a quantity often denoted *span loading*) and independent of the chord. Equation (20) follows from that as a necessary mathematical consequence. The practical application of the formulas in the making of aspect-ratio corrections will appear in Chap. VIII.

Lift-curve Slope and Aspect Ratio.—Downwash has another effect, in changing the angle of attack at which the aerodynamic forces reach particular specified values. As is evident from Fig. 64, the true angle of attack at which the airfoil meets the air is inferior to the apparent angle, or the angle between the airfoil chord and the undisturbed flow at a distance, by the amount $\epsilon/2$. The angle of zero lift is the same for all airfoils of a given section, but the slope of the curve varies with aspect ratio, the airfoils with small aspect ratio and so with large downwash angles having the largest relative displacement to the right of the points on the upper part of the curve of lift against angle of attack. The theoretical figure derived by Kutta for the slope of the lift curve[1] holds good only for the plot against true angle of attack, or for the airfoil of infinite aspect ratio which has no downwash and no induction, and any finite airfoil must be corrected accordingly to make the figures comparable. With a little manipulation, the correction can be thrown directly in terms of the lift curve.

If α_1 is the angle of attack at which the lift coefficient attains a certain value on an airfoil of aspect ratio R_1, and if α_∞ is the angle at which the same value is attained with another airfoil of the same section but of infinite aspect ratio, the two differ by the amount of the induced angle, or

$$\alpha_1 = \alpha_\infty + \frac{\epsilon}{2} = \alpha_\infty + \frac{L}{\frac{\rho}{2}\pi b^2 V^2} \tag{22}$$

at its plane of symmetry, and in a plane perpendicular to the wing's own, can have no effect on the flow around the wing is therefore inaccurate, though it may be nearly enough true to serve as a first approximation.

[1] P. 72, *supra*.

Since the zero-lift angle is the same on the two curves,

$$\frac{\left(\frac{dC_L}{d\alpha}\right)_\infty}{\left(\frac{dC_L}{d\alpha}\right)_1} = \frac{\alpha_1}{\alpha_\infty} = 1 + \frac{L}{\frac{\rho}{2}\pi b^2 V^2 \alpha_\infty} \tag{23}$$

Since the lift curve in every case approximates to a straight line over a great part of its length,

$$C_L = \alpha_\infty \left(\frac{dC_L}{d\alpha}\right)_\infty \tag{24}$$

Substituting,

$$\frac{\left(\frac{dC_L}{d\alpha}\right)_\infty}{\left(\frac{dC_L}{d\alpha}\right)_1} = 1 + \frac{\left(\frac{dC_L}{d\alpha}\right)_\infty}{\pi R_1} \tag{25}$$

or

$$\left(\frac{dC_L}{d\alpha}\right)_1 = \frac{\pi R_1}{1 + \frac{\pi R_1}{\left(\frac{dC_L}{d\alpha}\right)_\infty}} \tag{25a}$$

The angle of attack must of course be given in radians in calculating the slope. If the Kutta value of 2π is substituted as a general approximation for the slope, the equation takes the form

$$\frac{(dC_L/d\alpha)_\infty}{(dC_L/d\alpha)_1} = 1 + \frac{2}{R_1} \tag{26}$$

$$\left(\frac{dC_L}{d\alpha}\right)_1 = \frac{2\pi R_1}{2 + R_1} \tag{26a}$$

Since the actual slope of the lift curve conventionally falls about 10 per cent below Kutta's figure, better results are obtained by substituting 1.8 for 2.0 in the numerator of the right-hand term of (26), and in both the numerator and the denominator of (26a).

The introduction of induced drag furnishes a new argument in favor of using the polar type of curve for presenting the results of wind-tunnel tests, as the induced drag is a function of lift alone and has nothing to do with the angle of attack except to modify it through the introduction of an induced angle. The polar of induced drag is then the same for all airfoils of the same

plan form, varying only with the aspect ratio. It has the form of a parabola, as shown in Fig. 65, and since the drag under any condition is the sum of induced drag and profile drag the profile-drag coefficient at any value of the lift coefficient can be measured off directly as the horizontal distance between the induced-drag parabola and the polar of the airfoil at the appropriate ordinate.

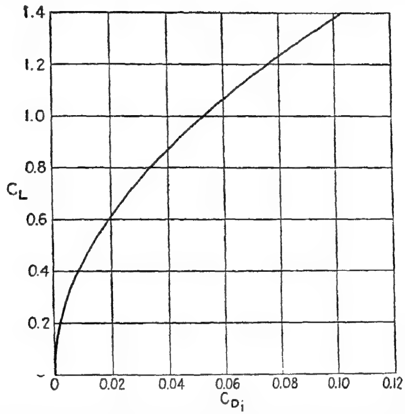


FIG. 65.—Polar of induced drag for an ideal airfoil of aspect ratio 6.

The theory introduces another innovation into the plotting of airfoil characteristics. The normal type of polar plot has to be drawn on a comparatively small scale for the drag in order that the drag coefficients at the higher angles of attack may not be crowded off the sheet. As the drag at high angles is mostly induced, and as profile drag therefore varies with angle much less rapidly than does total drag, a polar plot of profile drag as abscissa against lift as ordinate allows the use of a large scale without stretching the curves out to unwieldy dimensions. A study of the detached values of profile drag and of the form of its curve, too, gives a clearer picture of the real nature of an airfoil's aerodynamic performance than can be gained from curves in which the figures truly characteristic of the section are masked behind components dependent only on aspect ratio. Much use will be made of profile drag, and of a further refinement giving a still more open scale, as a basis of plotting in the airfoil-data comparisons that occupy Chap. VII.

Biplane Theory.—The reasoning about sweep area makes it obvious, if it was not before, that the wings of a biplane combination cannot operate like independent monoplanes. They cannot because, unless the gap were to exceed the span, their sweep areas would intersect and overlap. The combined sweep area of a biplane must therefore be less than twice that of either wing acting separately. Munk has suggested that it be approximated by drawing the circles on the two wings and subtracting from the combined areas the area of their overlap, giving the type of figure shown in Fig. 66. A simpler approximation is secured by assuming the area between the wings to be filled in solidly, giving a sweep area, in the case of a biplane with wings of equal span,

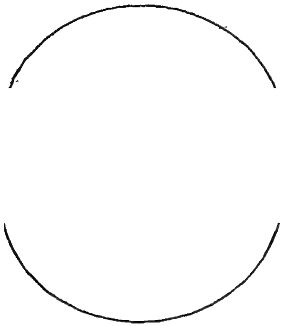


FIG. 66.—Sweep area for biplane (idealized).

$$S' = Gb + \frac{\pi}{4}b^2 \tag{27}$$

Following exactly the same procedure as in the case of the monoplane, the induced drag becomes

$$D_i = \frac{L^2}{\frac{\rho}{2}V^2b^2\left(\pi + 4\frac{G}{b}\right)} \tag{28}$$

$$D_i = \frac{2C_L{}^2S_1}{b^2\left(\pi + 4\frac{G}{b}\right)} \tag{29}$$

S_1 being the area of a single wing and G the gap. Attempts have been made to develop similar simple constructions for the equivalent sweep area in the case of a biplane with wings of unequal span, but they have been relatively unsuccessful. No method has been found that will give induced-drag coefficients remaining close to the true values throughout the possible range of biplane proportions, and it is necessary to fall back directly upon the factors that have been calculated from the basic theory by Munk and others[1] and tabulated.[2]

[1] "The Induced Drag of Multiplanes," by L. Prandtl, *Technische Berichte* (technical reports from the German air service), vol. III, p. 309, 1918,

To Munk, also, we owe the proof that induced drag is a minimum when the relation of the upper and lower wings to each other is such that their lifts are equal, if they are of the same span. The analogy with the conclusion that the induced drag of a monoplane is a minimum when the downwash is uniform all along the span is obvious. Deviations from the ideal are even less important than in the case of the monoplane, however. Even when the lift of one wing exceeds that of the other by 50 per cent, the total induced drag is increased only 2 per cent as a result.

The effect of an unequal lift distribution appears directly from the expanded expression for biplane induced drag:

$$D_i = \frac{1}{\pi\frac{\rho}{2}V^2}\left(\frac{L_u}{b_u^2} + 2\sigma\frac{L_uL_l}{b_ub_l} + \frac{L_l^2}{b_l^2}\right) \qquad (30)$$

where L_u and L_l are the individual lifts of the two wings, b_u and b_l their spans, and σ a computed quantity depending on the ratios $G/[(b_u + b_l)/2]$ (G being the gap of the biplane) and b_l/b_u. The first and third terms of (30), of course, represent the individual induced drags of the two wings. The second term is the total of their mutual interference; and since the effect of the upper wing on the direction of the mean flow past the lower is exactly equal to the reciprocal effect of the lower wing upon the upper, the second term is really the sum of two equal parts.

By making appropriate assumptions regarding the relation between L_u and L_l (the commonest assumption, and one that substantially represents the truth in most cases, being that the lifts of the two wings are in the same proportion as their

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translated in *Tech. Note* 182; "Isoperimetrische Aufgaben aus der Theorie die Fluges," by Max M. Munk, doctoral dissertation, Univ. Göttingen, 1918; "General Biplane Theory," by Max. M. Munk, *Rept.* 151; "Aerofoil Theory," by H. Glauert, *R. and M.* 723; "Elements of Aerofoil and Airscrew Theory," by H. Glauert, Chap. XIII, Cambridge, 1926; "Theoretical Relationships for a Biplane," by H. Glauert, *R. and M.* 901; "An Extended Theory of Thin Airfoils and Its Application to the Biplane Problem," by Clark B. Millikan, *Rept.* 362 (presenting a new method of analysis, differing in some of its essentials from the earlier procedure of Prandtl and Munk).

[2] The numerical values of the biplane interference coefficients will be given in Chap. IX.

areas) Eq. (30) can be thrown into other forms of greater simplicity and readier practical application. It may conveniently be put either into the shape

$$D_i = \frac{L^2}{\pi \frac{\rho}{2} V^2 k^2 b^2} \tag{31a}$$

or

$$C_{D_i} = \frac{C_L{}^2 S}{\pi k^2 b^2} \tag{31b}$$

which is the one most generally used by specialists in aerodynamic theory, or alternatively

$$C_{Di} = \frac{n C_L{}^2 S_1}{\pi b^2} = \frac{n C_L{}^2}{\pi R_B} \tag{32}$$

which is on the whole a little better for many practical design problems. In these expressions S is the total area, S_1 the area of an individual wing (the larger of the two if they are of unequal size), b the span (again the larger of the two, if there is a difference), and R_B the aspect ratio of the biplane (or, yet again, of the larger of the two wings), and n and k are computed from (30) after the values of σ are known. For a biplane with wings of equal span and chord,

$$n = 1 + \sigma = \frac{2}{k^2} \tag{33}$$

Where spans, chords, or areas are unequal, values of n and k can still be computed from (30) and from the values of σ originally provided by Munk, but the relations are less simple.

n is the factor by which the aspect ratio of a biplane must be divided to find the aspect ratio of a monoplane having the same induced-drag coefficient as the biplane at the same lift coefficient, and likewise the factor by which the induced-drag coefficient of a monoplane having the same aspect ratio as the biplane must be multiplied to get the biplane coefficient. k is known as the "span factor" and is the quantity by which the span of the biplane must be multiplied to determine the span of a monoplane which would have the same total induced drag as the biplane when sustaining the same gross lift. As previously noted, and

as in the approximation of (29), the coefficients expressing biplane interference are functions of the ratio of gap to span. It is conceivable that airplanes might be built with G/b lying anywhere between 0.02 and 0.40, but in practice in present-day design it is almost always found between 0.11 and 0.22. Comparing (29) and (33), the approximation for a biplane with wings of equal area and similar form becomes

$$n \approx \frac{2\pi}{\pi + 4\frac{G}{b}} \tag{34}$$

Comparison of the approximate value of n and the exact one[1] reveals the approximation as in error by amounts up to 8 per cent, always in the direction of overestimating the induced drag. Though it would be hard to produce any physical or geometrical reason for such a modification, the sweep-area method would give more exact results if the area were subjected to a still further increase of a perfectly arbitrary order, and if $1\frac{1}{3}$ times the area between the wings, instead of the actual area, were considered as lying within the sweep. Thereupon the approximate formula would become

$$n \approx \frac{2\pi}{\pi + 5.33\frac{G}{b}} \tag{35}$$

or, with substantially equal accuracy,

$$n \approx 2 - 3.4\frac{G}{b} + 4\left(\frac{G}{b}\right.$$

With that modification arbitrarily introduced, the approximate values lie within 4 per cent of the true ones throughout the range of common design practice.

The slope of the lift curve can be computed, for the biplane as it was for the monoplane, in terms of the slope for infinite aspect ratio. The formulas for slope and for induced drag are not, however, so readily deducible from each other in the biplane's case. The curvature produced in the streamlines past the upper wing by the influence of the lower one, and vice versa, modify

[1] Tabulated on p. 256, *infra*.

the lift and must be allowed for.[1] When the allowance is made, the resultant influence of biplane interference on slope is found to be most readily and most accurately presented as a function of gap/chord, rather than of gap/span as with induced drag. The slope calculation is too elaborate for explanation here, and the numerical results will be held for presentation with biplane test data, and with the discussion of the cases of unequal spans and unequal chords, in Chap. IX.

Triplanes fit into the same pattern as biplanes under the theory, though the coefficients are of course different. It again develops that the sweep area must be given an empirical increase (in this case of about one-half the area between the wings) to make the simple calculation based on sweep area square with the exact one. In other words, n for the triplane combination is approximately (always within 5 per cent) given by

$$n \approx \frac{3\pi}{\pi + 12\frac{G}{b}} \approx 3 - 10.5\frac{G}{b} + 20\left(\frac{G}{b}\right) \tag{36}$$

The induced-drag technique can be applied as well when two or more wings are placed one behind the other, or when they are laid side by side, as when they are superposed. It lends itself to the prediction of the performance of tandem combinations and of wings with slits cut through them parallel to the chord—a practical problem frequently arising where separable panels of the structure are held together by fittings and the gap that has to be left to reach and adjust the fitting is left unprotected by a cover plate. Where two wings lie tip to tip with a small opening between, it is clearly impossible for either one to show a full development of its normal vortical structure at its inner tip. The combined behavior must be intermediate between that of a single uninterrupted wing and that of two wings totally independent in their action. If the combination is considered as being a single wing with a longi-

[1] "The Elements of Aerofoil and Airscrew Theory," by H. Glauert, Chap. XIII, Cambridge, 1926; also, for a somewhat different method leading to different values and to an entirely different form of presentation, "Effect of Streamline Curvature on Lift of Biplanes," by L. Prandtl, "Ergebnisse der aerodynamischen Versuchsanstalt zu Göttingen," vol. III, translated as *Tech. Memo.* 416.

tudinal slit, the effective aspect ratio must be reduced by the flow of air through the slit and the formation there of a tip vortex of sorts, but obviously the reduction can never be as much as 50 per cent.

The numerical values both for the tandem and for the effect of the slit will be reserved for Chap. VIII, but immediate mention must be made of the extraordinary readiness with which the phenomena normally associated with a wing tip develop through even the narrowest opening. A slit at mid-span with a width of but 0.001 of the span theoretically diminishes the effective aspect ratio by 24 per cent, increases the induced drag by 31.[1]

Another problem capable of theoretical treatment is that of the normal airplane case, a wing with a fuselage interposed at its center. The fuselage distorts the curve of lift distribution along the span, distorts the induced-drag predictions to suit. With a proper relationship between fuselage and wing the increase in drag may be very small, and in any event the variations in fuselage form are so innumerable and so little subject to mathematical expression that it still appears better to place dependence on direct wind-tunnel testing for the study of interference.

Ground Interference, Wind-tunnel Corrections and Induced Drag.—There are many other applications of the theory, and specifically of the process of computing induced drag. It can be used, for example, to explain "ground effect," or the apparent reduction in drag of an airfoil operating in close proximity to a large plane surface parallel to the relative wind and below the airfoil. The ground interferes with the free downward movement of the air behind the wing; the effective angle of downwash at a given lift is reduced; the reduction of induced drag, and so of total drag and power required for flight at a given speed, is an automatic consequence. Hence the possibility, often observed by test pilots, of flying somewhat faster at a very low altitude than at any greater one.

The treatment of the problem resolves into considering the wing near the ground as one-half of a biplane combination, the other member of which is imaginary and situated as far below

[1] The analysis of this problem, due principally to Drs. R. Grammel and K. Pohlhausen, is effectively summarized in "Applications of Modern Hydrodynamics to Aeronautics," by L. Prandtl, *Rept.* 116.

the ground as the real member is above it.[1] The subsurface wing is assumed inverted, so that the surface of the ground becomes a plane of symmetry through the middle of a facing-both-ways biplane. The interference between the two wings is the same as in a normal biplane of like gap, but since one element is inverted it is of opposite sign. The imaginary "downwash" of the imaginary wing below the surface becomes an upwash with respect to the real wing above it and partially nullifies the downwash and so the induced drag that it would have, due to its own lift, if it were operating in complete isolation. All of this, it will be observed, adds up qualitatively to the same sort of declaration about the resultant air-flow structure as was arrived at more directly in the previous paragraph, but has the advantage of providing a basis for numerical calculation. In the case of a biplane flying near the ground this mirror-image method produces a quadruplane, half of which is below the surface and inverted with respect to the other half. The numerical results of ground-effect computations are gathered in Chap. XI.

The ordinary wind-tunnel test is a particular case of ground effect, for the model in the tunnel is seldom separated from the tunnel walls, or alternatively from the boundary of an unconfined stream, by more than the model's own span. The induced-drag concept permits the calculation of the effects of such proximity and allows the correction of the tunnel data to give true free-air results. As may readily be realized by anyone who has read carefully in the previous pages of this chapter, the interference effect is to make an airfoil perform better in an enclosed stream, worse in the open-jet type without fixed boundaries, than it would in free air. In some cases the corrections to drag may amount to 10 per cent of its total value or more, and the major aerodynamical laboratories now make them, as a rule, before publication of their data. Further detail is a matter for the specialist in experimental aerodynamics and need not concern the student of design enough to justify a more elaborate discussion here.[2]

[1] "Wing Resistance near the Ground," by C. Wieselsberger, *Zeitschrift für Flugtechnik und Motorluftschiffahrt,* No. 10, 1921; also, for a general and comparatively simple review, "Applied Wing Theory," by Elliott G. Reid, pp. 166–176, New York, 1932.

[2] Those who desire it may refer to "Aerofoil and Airscrew Theory," by H. Glauert, *op. cit.* Chap. XIV; or to "The Theory of Wind-tunnel Wall Interference," by Theodore Theodorsen, *Rept.* 410.

Similarly, the same basic theory can be used to determine (though not with entirely satisfactory accuracy) the effect of stagger on the distribution of lift between the wings of a biplane. It can be used to investigate the theoretically interesting, though practically unimportant, problem of the airfoil with shielding plates set up at its tips in planes perpendicular to its own.

The use of end plates as illustrated in Fig. 67 obviously obstructs the normal formation of the tip vortices and so reduces the downwash. In the extreme case of plates of infinite extent the flow must become truly two-dimensional and the "mean downwash" and the induced drag must be zero. If the complete disappearance of induced drag on a wing of finite chord is a little difficult to follow from a physical image of the circulation and flow pattern without resort to a simple faith in the indications of analysis, the difficulty clears up with the application of the sweep-area concept. With end plates in position it is reasonable to suppose the sweep increased by the area intercepted between the plates, and with plates of infinite extent the sweep area becomes infinite and the downwash, calculated as in (16), becomes nil. That very simple rule for computing the effect proves to give a fair accord with experiment.[1]

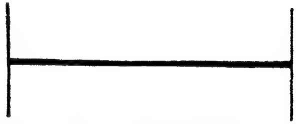

FIG. 67.—Airfoil with end plates.

Modification of the Formulas with Changes of Lift Distribution.—The simple formulas so far developed apply, it will be recalled, only to the case in which the curve of lift distribution along the span of the airfoil has the form of a semiellipse—a condition, incidentally, that is substantially realized when the airfoil itself has an elliptical plan form. They serve as a fair approximation for the wing forms most commonly used in airplane design; but to make them rigorously correct for rectangular wings or wings with a uniformly tapering chord, correction factors[2] have to be introduced. Glauert shows that the particular modifica-

[1] For a discussion of the theory of the effect of end plates, coupled with a record of some experiments on their use, see "Drag of Wings with End Plates," by Paul E. Hemke, *Rept.* 267.

[2] The calculation of the corrections is exceedingly complex, and each value is the development of an infinite series. The simplest process of calculation has been developed by Glauert and is reprinted, together with full tabulations of the results in many cases, in his "Aerofoil and Airscrew Theory," *op. cit.*, pp. 145–155.

tion required substitutes for the formulas (20) and (25) the more elaborate forms

$$C_{Di} = \frac{C_L^2}{\pi R}(1 + \delta) \tag{37}$$

and

$$\alpha_1 = \alpha_\infty + \frac{C_L}{\pi R}(1 + \tau) \tag{38}$$

where τ and δ are computed coefficients which unfortunately prove themselves to depend upon the aspect ratio of the airfoil as well as upon its general shape, and also upon the slope of the lift curve. However, the slope of the lift curve can be assumed, with but little error, to be 5.5 absolute units per radian, and τ and δ can then be tabulated, for airfoils of any particular shape, as functions of aspect ratio. They are plotted, both for rectangular wings and for straight tapered ones, in Chap. VIII, where it becomes quite obvious that the tapered wing with a tip chord from one-quarter to one-half that at the center must have a lift distribution very closely approaching the ideal elliptical form. That might have been anticipated from the geometry. In no case within the range of ordinary design experience is the effect of lift distribution of very great importance, though it is certainly too large (running up to about 5 per cent of the total airfoil drag) to be neglected in close work. The effect on the slope of the lift curve is of much greater relative magnitude, but the lift-curve slope is itself of somewhat secondary importance in design work. In practically all cases, sufficiently accurate results can be secured by assuming the induced drag of a rectangular wing to be 5 per cent higher, and the change of angle of attack due to aerodynamic induction to be 15 per cent greater, than the values determined by (20) and (25), respectively. Applying the angle of attack correction to (26), the numerator of the fraction in the expression for the ratio of the lift-curve slopes is raised from 1.8 for the elliptical distribution to 2.05 for a rectangular wing, the same value that it would have had for the elliptical curve if it were possible to use the Kutta formula for slope at infinite aspect ratio directly.

Glauert has also developed[1] a method for dealing with another variation in wing form, the washing out of angle or the giving of

[1] *Op. cit.*, p. 153.

a twist to the wing so that the angle of attack varies uniformly from a maximum at the center to equal minima at the two tips. In that case the values of τ and δ vary with angle of attack and lift coefficient as well as with the geometrical characteristics of the wing itself, for manifestly the relative effect of a given twist will be less at a high lift coefficient than at a small one. In the particular case of an airfoil with an aspect ratio of 5.5, and with a twist from center to each tip of 5.7 deg., δ ranges from 0.22 at one-quarter of the maximum lift coefficient down to 0.025 at one-half the maximum and 0.008 at three-quarters. The application of all these figures to practical design, and to the actual choice of wing arrangements, will be reserved to a later chapter.

Experimental Verification of the Induced-drag Formulas.—However sound the bases of airfoil theory may appear, and however well the mathematicians may agree among themselves, they cannot be given full confidence until they have been through the acid test of check against the direct results of wind-tunnel tests. There is enough of assumption underlying the theory, and enough difference between an imaginary perfect fluid and real atmospheric air, to make that an inescapable necessity. If the test is applied at a few selected points, however, and if it shows good agreement there, the use of the formulas derived from the theory can safely be extended over a great area without further check. They can, indeed, be applied to certain problems upon which no direct wind-tunnel test would be practicable.

There have been many comparisons of data obtained by direct measurement in the wind tunnel and those computed from the induced-drag formulas, the most extensive series being due to Glauert.[1] His work, covering the study of four series of wind-tunnel tests on aspect-ratio effect, made in three different countries, and two series on biplane interference factors, showed the theory in a very favorable light. It develops that the deviation of the drag coefficient from a single mean curve, when correction is applied by the formula for any aspect ratio from 3 to 9, and at any lift coefficient within the range where the values of drag are of first-class practical interest, is very unlikely to be

[1] "Experimental Tests of the Vortex Theory of Aerofoils," by H. Glauert, *R. and M.* 889. See also "Note on the Application of the Vortex Theory of Aerofoils to the Prediction of Downwash," by L. F. G. Simmons and E. Ower, *R. and M.* 914.

more than 0.0012—a little more than 10 per cent of the minimum-drag coefficient for a good airfoil. It is not at the minimum-drag angles that the largest irregularities are likely to be found, however, and these seeming deviations from harmony have hardly exceeded 5 per cent of the drag at any point in the tests so far made. The apparent uncertainty in predicting the angle of attack at which any specified lift coefficient will be found, similarly, seems not to exceed 0.3 deg. Pitching-moment coefficients for monoplanes should, according to the theory, be a function of the lift coefficient alone and independent of the aspect ratio, and Glauert's experiments show the moment coefficients for a given lift coefficient actually to lie always within 0.005 of a mean value.

The check on theory is equally good in the case of the biplane. Biplane combinations of many different gaps, staggers, and aspect ratios, all corrected back to equivalence with a standard monoplane by the induced-drag method, show deviations of drag from the experimental curve for the basic monoplane of less than 0.0016.

The apparent errors are small, but they are not absolutely negligible in all cases. Though it is extremely dangerous to generalize in dealing with small and somewhat irregular differences in small quantities of which the experimental error is itself an appreciable part, the calculated aspect-ratio corrections both to drag and to angle of attack seem to have a slight tendency to run larger than they should. Whereas the uncorrected drag is of course lowest for airfoils of high aspect ratio, after several sets of data to a common basic aspect ratio have been corrected, there is a tendency for the drag of the high aspect-ratio cases to run above the mean curve while that for very low aspect ratios (less than 3) runs distinctly below it. The persistent differences certainly do not exceed 2 per cent of the drag at any point and are by no means so regular as to justify introducing an empirical correction factor, but they are just apparent enough to deserve mention and a hope that the whole subject may be reexamined in a more modern wind tunnel and with a more refined technique than were available for the early tests on which Glauert's study was based.

The main effect of such checks as have so far been made has been to stabilize our confidence in the wing theory and in the

induced-drag formulas that result from it, and to establish the formulas in an unchallengeable position as practical working tools for the designer's daily use.

Airfoil-characteristic Predictions.—The possible resources of the basic theory are not by any means exhausted with the correction of wind-tunnel data from one set of conditions to another. Its sponsors and their students have probed progressively deeper into the fundamental mechanism of airfoil action; analyzed more and more completely the vortical structure out of which the resultant flow is built; drawn progressively nearer to the exciting achievement of being able to predict an airfoil's performance completely from its geometry without need for wind-tunnel

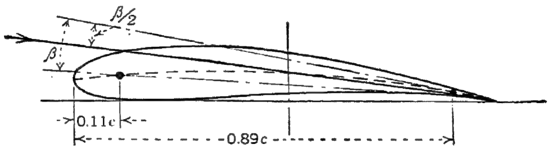

FIG. 68.—Graphical method for the determination of an airfoil's zero-lift angle.

intervention. They may never fully attain that power, for the actual performance is influenced by the difference between the ideal fluid to which the theory relates and the real one in which the airfoil has to move, and in some cases the practical application of the theory gives results disappointingly at variance with experience, but already they provide the formulas for certain predictions of a very useful order.

The conclusion that the slope of the lift curve should be 2π per radian, and the degree in which it fails of apparent realization and the apparent reasons for the discrepancy have already been discussed.[1] If the slope is assumed known, a complete definition of the lift curve up to within a short distance of the point of maximum lift will require only the determination of the angle of zero lift. For that there is a method due to Munk.[2] The processes of integration from which it derives need not even be summarized, but they resolve finally into a simple graphical process. The wind direction for zero lift, Munk concludes, can

[1] P. 72, *supra*.

[2] "General Theory of Thin Wing Sections," by Max M. Munk, *Rept.* 142; "The Determination of the Angles of Attack of Zero Lift and of Zero Moment, Based on Munk's Integrals," by Max M. Munk, *Tech. Note* 122.

be found by drawing a line from the trailing edge of the section through the intersection of the mean camber line with the mid-point of the chord. A more accurate process, shown in Fig. 68, fixes the zero-lift line as the bisector of the angle between lines from the trailing edge to the mean camber line at points located 11 per cent of the chord from the leading edge and the trailing edge, respectively. A still more accurate one makes use of five points, but for that anyone who may be interested to pursue the matter further can turn to the original source.[1]

It is more convenient to handle these determinations arithmetically than to make the careful graphical layout that would be required for each section. The first, or one-point, method obviously reduces by simple trigonometry to the formula

$$\alpha_0 = -114.6h_{0.5} \tag{39}$$

where α_0 is the angle between the selected geometrical chord and the zero-lift line and $h_{0.5}$ is the mean camber at the mid-point of the chord, stated as a fraction of the chord. Trial on a number of sections shows it better to make an empirical reduction of the coefficient to 110, and when that value is used the formula gives results seldom in error by more than 0.5 deg.[2] The second similarly reduces to the form

$$\alpha_0 = -(32h_{0.11} + 420h_{0.89}) \tag{40}$$

The only remaining feature of the lift curve that needs to be predicted is its maximum—its value and the angle at which it occurs. That is totally beyond the reach of the ordinary theory, for the theory postulates a streamline flow and the very existence of a maximum lift is a consequence of separation and breakdown of the flow. It will have to be reserved for later discussion[3] in connection with the phenomena that influence separation.

Profile drag, too, is so largely frictional that little can be learned of it from ideal-fluid studies. That too will come in for further treatment in the next chapter.

There remains the distribution of pressure over the wing's surface, and especially its collective expression by pitching-moment or center-of-pressure coefficient. The first conclusion

[1] *Tech. Note* 122, *cit. supra.*

[2] For further data on zero-lift prediction by formula, see p. 193, *infra.*

[3] P. 143, *infra.*

on that point that we owe to mathematical analysis is that the coefficient of moment about an axis 25 per cent of the chord from the leading edge[1] should be independent of the lift coefficient and so of the angle of attack. That works out reasonably well in practice, at least up to 80 per cent of $C_{L_{max}}$, the moment coefficient ($C_{M_{0.25}}$) seldom varying more than 0.010 from a mean value for the section. The moment proves for most airfoils to be even more uniform for an axis at 24 per cent than for one at 25, the mean aerodynamic center, or axis of most nearly uniform moments, lying for various airfoils at from 23.0 to 25.5 per cent of the chord behind the leading edge and about 1 per cent on the average above the chord drawn connecting the extreme leading and trailing edges.[2] The vertical coordinate has but little effect, and in most of the generalized discussions in the present volume the moment axis has been assumed at the 25 per cent point and a mean value taken for C_M, in preference to having to struggle with a separate determination of the aerodynamic center and a slightly different set of moment conversion factors in each new paragraph. For refined design calculations, however, where the data for a particular airfoil are being used, it is necessary either to use the true location of the aerodynamic center as determined for that airfoil or (since the moment coefficient at best is not exactly constant) to keep to the 25 per cent point and use at each angle of attack the true value of $C_{M_{0.25}}$ as determined for that angle.

The $C_{M_{0.25}}$ figures for various sections of course differ widely among themselves, and the best approach to the determination of their actual magnitudes is made by Munk's method[3] for computing the angle at which the moment at an axis at mid-chord ($C_{M_{0.5}}$) is equal to zero (or, in other words, the angle at which the center-of-pressure coefficient is 0.5).

$$C_{M_{0.5}} = 0$$

when

$$\alpha = -63.25(h_{0.0425} - h_{0.9575}) \tag{41}$$

[1] See p. 60, *supra.*

[2] Values for a large number of sections are tabulated in "The Characteristics of 78 Related Airfoil Sections from Tests in the Variable-density Wind Tunnel," by Eastman N. Jacobs, Kenneth E. Ward, and Robert M. Pinkerton, *Rept.* 460.

[3] *Rept.* 142, *Tech.* 122, both *cit. supra.*

where α is referred to the geometrical chord of the section and $h_{0.0425}$ and $h_{0.9575}$ are, in accordance with the convention previously used, the ordinates of the median camber line at 4.25 and 95.75 per cent of the chord, respectively. Since the geometrical characteristics of a section are usually specified by its ordinates at 5 per cent intervals it is more convenient to use the 5 and 95 per cent points, rather than those so exactly defined in the formula, and $h_{0.05}$ and $h_{0.95}$ may be substituted in (41). Strictly speaking, (41) applies only in the case of infinite aspect ratio, as the moment is a function of lift rather than of angle of attack and a given moment will continue to appear in conjunction with a given lift coefficient even after the slope of the lift curve has been changed by a change of induced angle. The error due to applying the formula directly to a wing of aspect ratio 5 in the case of a deeply cambered section, with a considerable difference between the angle of zero lift and that where $C_{M_{0.5}} = 0$, may be as much as 0.025 in $C_{M_{0.25}}$—far too large to be neglected.

To transform the angle so obtained into a determination of the moment-coefficient characteristic of the section requires only a combination of (41) and (39) or (40). Under any given condition, the moments about two different axes on the chord obviously differ by the product of the distance between the two by the lift. If coefficients rather than total forces and moments are being used the distance between the axes is, of course, stated as a fraction of the chord. $C_{M_{0.25}}$ should then be one-quarter of the value of C_L at the point where $C_{M_{0.5}}$ is zero. Allowing 0.098 per deg. as the average slope of a lift curve, and determining the angle of zero lift by (39) as empirically modified, the formula for standard moment coefficient for any section appears as

$$C_{M_{0.25}} = -0.024(110h_{0.5} - 63.25[h_{0.05} - h_{0.95}]) = -2.70h_{0.5} + 1.55(h_{0.05} - h_{0.95}) \quad (42)$$

Comparison with wind-tunnel results for a number of sections shows this formula only as approximation. It gives results differing from the experimental ones, in some cases, by as much as 0.015. The moment by formula has, in most cases, too large a negative value, and substantially better results can be obtained if the formula be modified empirically to the simpler form

$$C_{M_{0.25}} = -2.5h_{0.5} + 2.0(h_{0.05} - h_{0.95}) \quad (43)$$

So modified, it will give C_M correct to within 0.005 in practically every case except for sections of more than 15 per cent thickness.

Such generalized predictions as these are developed from the analysis of sections peculiarly subject to mathematical treatment. Great progress has been since 1920, however, in extending the range of mathematical action. Particularly notable is the work done since 1930 by Theodorsen,[1] who has reached the point of being able to compute the pressure distribution around a perfectly empirical section with an agreement with experimental result that lies virtually within the experimental error everywhere except on the last 20 per cent of the chord—where separation is likely to be taking place. Pressure distribution both along the chord and along the span of a wing, like zero-lift and moment coefficient and aspect-ratio and multiplane corrections, now appears to lie entirely within the mathematician's grasp. Only the maximum lift and profile drag elude him.

Compressibility Effects.—The pressure around an airfoil at an angle of attack near that of maximum lift ranges from a definite maximum of $\frac{1}{2}\rho V^2$ at the stagnation point to a minimum that at a high angle of attack may attain to $-2\rho V^2$, or even lower, 3 or 4 per cent back of the stagnation point along the upper surface. At 100 m.p.h. and in a standard atmosphere $2\rho V^2$ equals 102 lb. per sq. ft., a pressure drop sufficient to produce a drop of 5 per cent in the density of the air if the temperature remained constant. Actually the flow is adiabatic instead of isothermal, and the density change is approximately $3\frac{1}{2}$ per cent instead of 5, a drop in temperature counterbalancing the remainder of the decrease in pressure. At higher speeds there would be a larger effect, but the condition of simultaneous high speed and angle of attack near that of maximum lift is, of course, an abnormal one, approached only in abrupt recoveries from dives. Full-scale experiment shows the possibility of a pressure drop of 300 lb. per sq. ft., a local decrease of air density of some 10 per cent, in the course of such recoveries. The density decrease may be as much as 5 per cent in steady flight at maximum speed for a racing

[1] "On the Theory of Wing Sections, with Particular Reference to the Lift Distribution," by Theodore Theodorsen, *Rept.* 383; "Theory of Wing Sections of Arbitrary Shape," by Theodore Theodorsen, *Rept.* 411; "General Potential Theory of Arbitrary Wing Sections," by Theodore Thedorsen and I. E. Garrick, *Rept.* 452.

plane. Around a propeller blade, it becomes much too large to calculate by any such simple method.

Even on a wing it begins to be too large to overlook, and Prandtl[1] and Glauert[2] have restudied the effect of circulation with due regard to the density variations that would occur in a perfect gas. As might be foreseen, the major change revealed is in the slope of the lift curve, since any effect that compressibility may show on the total force on the airfoil would obviously be likely to increase in direct proportion to the magnitude of the pressure intensities existing and the density changes undergone around the airfoil, and so to the magnitude of the total force itself. It might be expected, in other words, that its first effect would be simply to change the scale of the curve. The formula for the magnitude of the change is

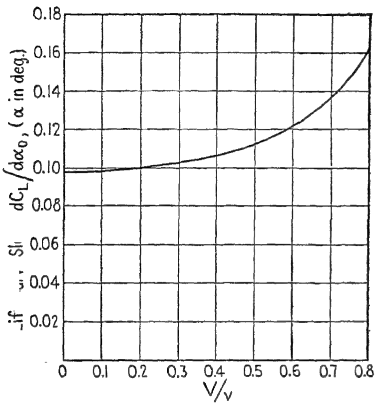

FIG. 69.—Variation of slope of lift curve with speed.

$$\frac{(dC_L/d\alpha)}{(dC_L/d\alpha)_0} \qquad \frac{1}{\sqrt{1 - (V/v)^2}} \tag{44}$$

where the subscript zero denotes conditions prevailing at speeds definitely below the compressibility range and where v is the speed of sound in air. Figure 69 is the curve of lift-curve slope against V/v, the ordinates reduced by 12 per cent from the theoretical values throughout to fit the facts of experience and Theodorsen's theory.[3] The results of such experiments as have been undertaken at exceedingly high speeds[4] are in fair

[1] "Handbuch der Physik," 1927 ed., vol. VII, p. 340.

[2] "The Effect of Compressibility on the Lift of an Aerofoil," by H. Glauert, *R. and M.* 1135.

[3] P. 73, *supra*.

[4] "The N. A. C. A. High-speed Wind Tunnel and Tests of Six Propeller Sections," by John Stack, *Rept.* 463; "Tests of Sixteen Related Airfoils at High Speeds," by John Stack and Albert E. von Doenhoff, *Rept.* 492.

agreement with the Prandtl-Glauert formula up to a speed of about 75 per cent that of sound, or about 580 m. p. h. Above that point there is an abrupt change.

The Prandtl-Glauert analysis applies only to the first phase of compressibility, where reduced air density is playing a part in determining pressure but where the fundamental flow structure and pattern of streamlines have undergone no essential change. The second phase, much more catastrophic in its effects, is entered at the point where the velocity of the air along certain streamlines actually reaches the speed of sound.

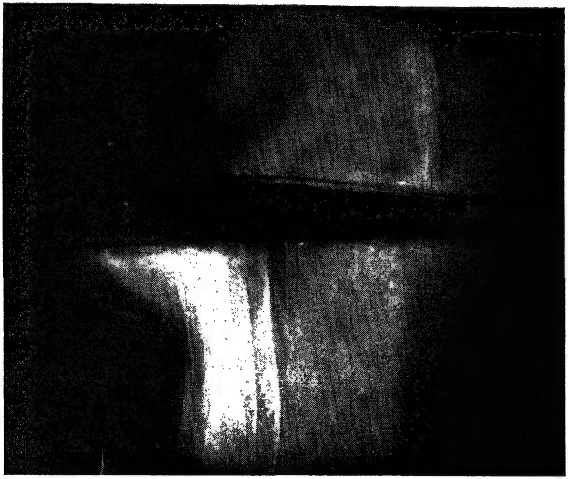

FIG. 70.—The air structure around an airfoil at very high speed.

It will be recalled[1] that the maximum speed of flow observed above an airfoil's leading edge ranges from $1.5V$ to $2.2V$. If the flow pattern actually remained unchanged the maximum would reach the speed of sound, at high angles of attack, at a speed of flight of about $0.5v$, or 380 m. p. h. Actually it does so, at least for thick sections, and there ensues a sudden collapse of the flow pattern and the formation of a permanent compression wave diverging from the surface of the airfoil at a sharp angle. This phenomenon has been named *compressibility burble* and is illus-

[1] See p. 75, *supra*.

trated in Fig. 70,[1] where the lines leading away from the airfoil are traces not of air flow but of pressure gradient. They follow the pattern made familiar in flight photographs of bullets (as in Fig. 71[2]), and the existence of such sound waves diverging from objects in motion at very high velocity is a matter of familiar experience to everyone who has stood under rifle fire and heard the sharp crack, entirely distinct from the report of the gun and materially preceding it, with which the bullet passes.

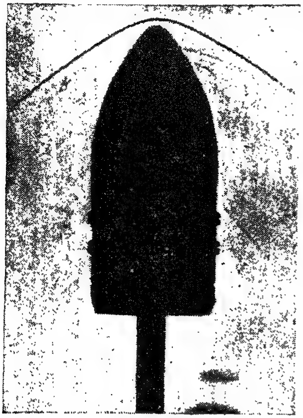

FIG. 71.—Photograph of a bullet in flight, showing the compression wave around the nose.

What high speeds do to the airfoil appears from Fig. 72,[3] a group of pressure-distribution diagrams for an airfoil of moderate thickness and with a flat lower surface. Even at half the speed of sound, the pressure drop above the leading edge has already begun to fall off, but the general form of the diagram remains unchanged, at an angle of 12 deg., until V/v exceeds 0.8. At higher angles of attack the break comes at a somewhat lower speed. After passing the critical combination of speed and angle the collapse is sudden and complete. The almost constant pressure drop over the whole extent of the upper surface, the disappearance of the peak of "suction" at the leading edge, and the existence at the trailing edge of a pressure substantially below atmospheric provide direct and conclusive evidence of separation immediately behind, or in fact almost immediately at, the nose. At very high speeds, above that of sound, there is a compression wave on the lower surface as well, with effects equivalent to a separation at

[1] Supplied by the N. A. C. A., from their high-speed wind tunnel; for discussion, see "The Compressibility Burble," by John Stack, *Tech. Note* 543.

[2] Reproduced from "High-velocity Wind-tunnels," by E. Huguenard, *La technique aéronautique*, Nov. 15 and Dec. 15, 1924, translated as *Tech. Memo.* 318.

[3] From "Pressure Distribution over Airfoils at High Speeds," by L. J. Briggs and H. L. Dryden, *Rept.* 255.

mid-chord—as is evidenced by the last of the four diagrams of Fig. 72.

As we noted in the previous paragraph, the second or burbling phase of compressibility effect is entered at lowest speeds when lift coefficients are highest, since it is then that the maximum speed of flow over the surface is in highest ratio to the undisturbed air speed. More specifically, it makes its appearance at a lower speed at high angles of attack than at low, and with thick airfoils or airfoils having a sharp curvature of the upper surface near the leading edge than with thin ones or those with the maximum camber farther back and a more gentle form. At a

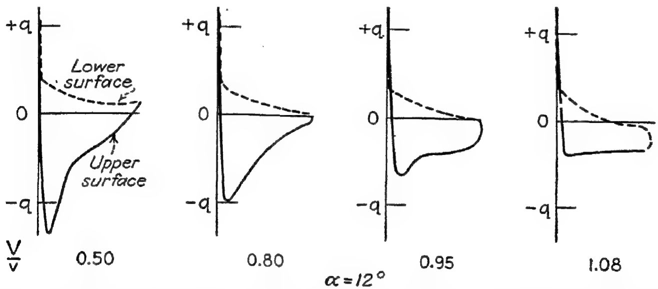

FIG. 72.—Change of pressure distribution on an airfoil with increasing speed and growing influence of compressibility.

minimum, on an airfoil having a thickness of 12 per cent of the chord, it may manifest itself at $0.6v$ or even a bit lower. On the same airfoil at an angle corresponding to $C_{L_{max}}/4$ it will be delayed to $0.8v$. On an airfoil of 6 per cent thickness and with its point of maximum camber farther back along the chord the appearance of the burble and the consequent break of the lift curve comes only at $0.84v$. The region around $0.8v$ is exceedingly sensitive, and it takes a pronounced change in section form or in angle to retard the appearance of burble for another $0.02v$. To hold it off to beyond $0.85v$, or 650 m. p. h., seems practically impossible.[1] It is already common for propeller-blade tips to operate at speeds above that level, and the probable consequences of doing so are of very real practical interest. Specific numerical data for particular sections will be considered in the next chapter

[1] *Repts.* 463, 492, *cit. supra;* also "Wind-tunnel Tests on High-tip-speed Airscrews," by A. S. Hartshorn and G. P. Douglas, *R. and M.* 1438.

but one,[1] but, as a general picture of what has been described at some length in the previous paragraphs, Fig. 73 presents curves of lift coefficient against speed for a typical section at a medium and a low angle of attack. First the steady increase of lift, at an accelerating rate as the speed increases, then the sudden break at the burble.

The strictly supersonic region too has been the subject of mathematical treatment[2] and of a limited amount of experi-

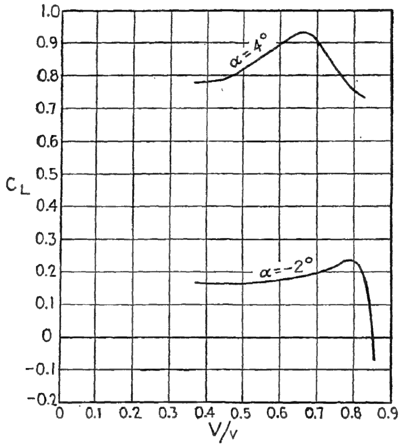

FIG. 73.—Variation of lift coefficient with speed, at a given angle of attack.

ment.[3] The analysis provides an explanation of the burble as a sound wave of predictable properties and gives reason to expect a shifting of the zero-lift angle at high speeds toward 0 deg. measured from the chord of the lower surface. That follows from the diminishing importance of the upper surface's role after the flow pattern across it has broken down. From the same diminu-

[1] P. 221 *et seq.*, *infra.*

[2] "Luftkräfte auf Flügel die mit grösserer als Schallgeschwindigkeit bewegt werden," by J. Ackeret, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, No. 16, 1925; "Applications to Aeronautics of Ackeret's Theory of Aerofoils Moving at Speeds Greater than That of Sound," by G. I. Taylor, *R. and M.* 1467.

[3] "A High-speed Wind Channel for Tests on Aerofoils," by T. E. Stanton, *R. and M.* 1130.

tion there is reason to expect that the performance of an airfoil at extremely high speeds will be largely independent of the precise form, dependent principally on the sharpness of the leading edge. There is reason to expect that, with the flow along the upper surface playing a diminishing part in controlling the lift, the center of pressure will move back toward mid-chord as the speed increases. All this finds confirmation from the work of Briggs and Stack and others at speeds of from $0.5v$ to $0.9v$ and from that of Stanton at $1.7v$.

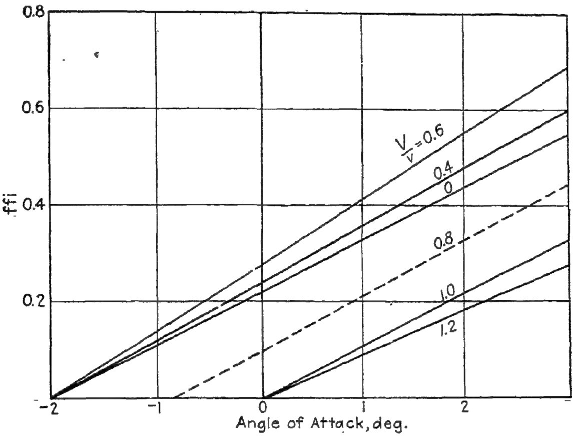

FIG. 74.—Variation of lift-curve slope with speed.

Confirmed by experiment, too, are G. I. Taylor's[1] calculations of the energy dissipated in sound waves, producing a proportional drag on the airfoil. His calculations show a wave-making drag coefficient of 0.045 for a symmetrical biconvex airfoil with a thickness ratio of 0.10, and experimental values run a little higher still, owing to factors obviously present but not taken into account in the calculation. The maximum of L/D for such an airfoil at $1.7v$ was found to be 4.4, scarcely one-quarter what would be expected of it at the same Reynolds' number at a moderate speed.

As a final expression of the typical effects of compressibility on lift, first in increasing the slope of the curve, then in shifting the

[1] *R. and M.* 1467, *cit. supra.*

zero angle as the burbling régime is entered and definitely esta lished, and finally in reducing the slope as the behavior becom less and less that of an airfoil and progressively more like that a sharp-nosed two-dimensional bullet, Fig. 74 presents defini sections of the lift curves for a number of speeds in a typic case.

CHAPTER VI

BOUNDARY LAYERS AND SKIN FRICTION

The classic airfoil theory treats of streamline flows in frictionless fluids. When upper-surface separation begins to be an important factor the foundations of the theory are shaken, but it is comparatively easy to examine the flow of smoke about an airfoil and see that separation takes place, at the angles of attack that ordinarily prevail during flight, so near the trailing edge as to be palpably unimportant. It is not so simple to reach offhand conclusions about the importance of friction.

Air like every other fluid has a certain amount of internal friction. Its degree as a property of the fluid is measured by the coefficient of viscosity,[1] a quantity that concerned only physicists

[1] The coefficient of viscosity may be casually defined as the shearing stress in the fluid corresponding to a unit rate of displacement, or more rigorously as the force required to move one of two parallel plates immersed in the fluid, each plate being of unit area, the two separated by unit distance, and the relative velocity between them also being of unit intensity. The coefficients for some common substances, in English (ft.-lb.-sec.) units and at 60° F. and normal atmospheric pressure, are

	Coefficient of viscosity (μ)	Kinematic viscosity (ν)	ν, relative to air
Air	0.00000035	0.000154	1.00
Carbon dioxide	0.00000030	0.000084	0.55
Water	0.000037	0.000019	0.12

The kinematic viscosity of certain other fluids being so much lower than that of air, permitting the attainment of a desired Reynolds' number at a much lower VL than air would demand, it has been suggested that a "wind tunnel" might advantageously be filled either with water or with carbon dioxide ("Nouvelle méthode d'essai de modèles en souffleries aérodynamiques," by William Margoulis, *L'Aérophile*, January, 1921). The practical difficulties of handling either a liquid or the necessary volume of any gas other than air being intense, the only attempts to raise the Reynolds' number by lowering ν have been made through the variable-density tunnel

until a few years ago but that has now made its way into popular speech and understanding as among the important determining characteristics of a motor oil. To become conscious of it not merely as a number specifying an oil but as a very real physical property, move a thin metal plate rapidly edgewise, first through water and then through a heavy-bodied lubricating oil at a low temperature, and note the difference in the force required for the purpose in the two cases. The force, be it large or small, is called *skin friction*, and experiment leaves no doubt that it is a function of viscosity.

All of the aerodynamic forces so far examined would still exist in a fluid completely lacking viscosity, and in fact they have been studied here on the hypothesis that there is none. Skin friction, on the other hand, defined as the component of force acting tangentially at the surface of the body, is the offspring of viscosity and could not exist without it. It may be more practically described as the component of resistance which would be present on a thin flat plate placed edgewise to the wind. Clearly, a plate so thin as to offer no perceptible direct obstruction to the movement of the air and to set up no eddies and produce no curvature of the streamlines passing it would have no excuse for experiencing any aerodynamic resistance whatever of any of the types discussed in previous chapters. That even such a plate does experience a resistance is a matter of common knowledge. The type is new to our study, and the source is frictional.

Boundary Layer.—The ideal case of viscous motion is that in terms of which the coefficient is defined, the relative motion between two parallel plates. In that case, as indicated in Fig. 75, there can be assumed a uniform velocity gradient between the two surfaces, the fluid conceived as arranged in strata or laminae that slip uniformly over one another. The assumption actually lies in reasonable proximity to the physical facts where

mentioned on p. 25. ν varies in air approximately as $p/T^{1.5}$, where p is the pressure and T the absolute temperature.

In round numbers, Reynolds' number under ordinary atmospheric conditions is $6{,}000VL$ where V is in ft. per sec. and L in ft., or $9{,}000VL$ where the velocity is in m.p.h.—two very useful constants to remember. They are in error by a little over 5 per cent, the true values of the multipliers being 6,490 and 9,520, but the approximations are good enough to satisfy practically all ordinary engineering purposes.

the separation of the plates is small and the relative velocity low. Exactly at the surface the velocity of the fluid relative to the plate is presumed to be zero, so that with a surface free from irregularities the motion is entirely one of slippage of fluid over fluid. A simple physical demonstration of the essential truth of what may at first sight be a rather startling assumption[1] is afforded by the collection and undisturbed continuance of layers of dust on the walls of a wind tunnel through which a steady current of air is flowing at speeds of 60 m. p. h. or more much of the time.

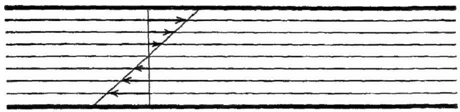

FIG. 75.—Motion of viscous fluid between two parallel plates.

Obviously in the type of flow that most concerns students of aeronautics the mechanism must be somewhat different. The surface of a wing or body furnishes one boundary. No other is physically apparent, and the stream stretches away to infinity. Like many other problems of hydromechanics, this one owes its solution to Prandtl, who was the first to propose[2] that the free stream itself be considered as supplying a second boundary, analogous to the second plate in a viscosity experiment, and that the effects of viscosity could be regarded as confined in their entirety to the space between the boundaries thus established. The thin stratum of fluid so segregated is called the *boundary layer*.

The impression must not be left that the boundary layer is a mathematical *tour de force* to simplify the analyst's problems. It is a physical fact, proved by measurements without number, that the velocity in a fluid stream increases very rapidly outward along a normal to a solid surface, until it attains the full free-stream value at what is usually a distance of only a fraction of an inch. Since viscosity is a phenomenon of shear, its effects are

[1] Discussed and proved by T. E. Stanton, *Proc.* Roy. Soc., Sec. *A*, vol. 97, p. 413, 1920.

[2] *Proc.* Third Inter. Mathematical Cong., Heidelberg, 1904; translated as "Motion of Fluids with Very Little Viscosity," by L. Prandtl, *Tech. Memo.* 452.

important only where there is an appreciable velocity gradient along a normal to the direction of flow. Only in the boundary layer does such a gradient attain a substantial value. Only in the boundary layer, therefore, as Prandtl declared, need viscosity be reckoned with.

A sample velocity curve is offered in evidence in Fig. 76.[1] This particular one happens to have been taken adjacent to the surface of a symmetrical airfoil and 5 per cent of the chord from the leading edge, but the velocity distribution adjacent to the surface of a flat plate near its leading edge or near the nose of almost any body of smooth form would show essentially the same characteristics at some Reynolds' number. The Reynolds' number, as will appear in due course, is exceedingly important. In this case it was 1,200,000.

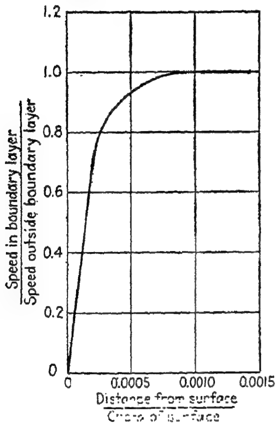

FIG. 76.—Velocity distribution in the boundary layer, near the leading edge of an airfoil.

The extreme thinness of the boundary layer and the virtual constancy of the velocity gradient across the greater part of its breadth are apparent from the curve. A velocity two-thirds that of the undisturbed stream is attained, in a steady rise from zero, within 0.008 in. of the surface of the airfoil. The gradient of velocity in that region is over 80,000 ft. per sec. per ft., and at such a rate of shear in the fluid even the smallest of coefficients of viscosity will obviously correspond to a very considerable viscous stress. The "stress" is, of course, the skin friction, which can indeed be computed from the geometrical characteristics of the boundary layer and especially from the velocity gradient that a plotting of its cross section shows.

Laminar and Turbulent Flow.—Figure 76 illustrated one type of boundary-layer distribution. Figure 77 repeats it, on a different scale, and combines with it the representation of

[1] Based on "An Experimental Determination of the Intensity of Friction on the Surface of an Aerofoil," by A. Fage and V. M. Falkner, *R. and M.* 1315.

three others. The added curves are taken from the same test of the same airfoil, but for points spaced along the entire length of the chord.

Obviously the boundary layers in the added curves are much thicker than in the first. Obviously the velocity gradient immediately adjacent to the surface is much lower. Obviously the uniform gradient terminates at a much lower velocity, and the outer part of the layer, in which the gradient is falling off and the velocity is approaching its limiting value so gradually

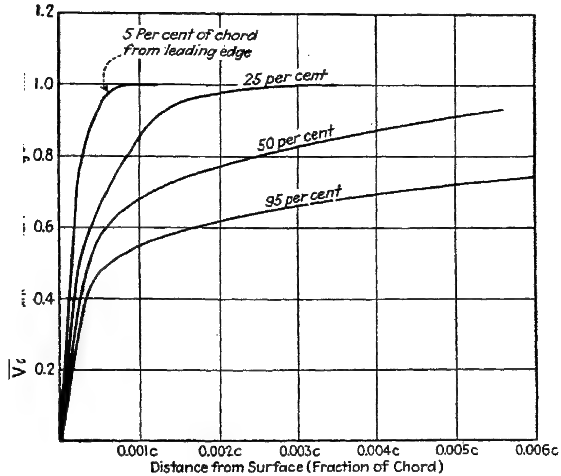

FIG. 77.—Change of velocity distribution in the boundary layer with distance from the leading edge.

as to seem almost asymptotic, is relatively much thicker than it was near the leading edge. But all these differences are not minor matters of degree alone. They signalize the appearance of an entirely different type of flow.

A boundary layer may be either of two things. It may be, as so far pictured, a group of infinitely thin strata moving one over the other in purely laminar flow. It may, on the other hand, be a substantially isolated region containing a turbulent flow of its own, the individual particles circulating back and forth through the thickness of the layer instead of moving on orderly paths parallel to the surface to which it adheres. Of

the reality of the distinction the flow photographs of Fig. 78, showing the motion of smoke over a flat plate under laminar and turbulent conditions, respectively, give ample proof. Even more positive is the evidence of records of instantaneous variation of velocity. Such records[1] are given, first for the free wind-tunnel stream and then for two points very close to the surface

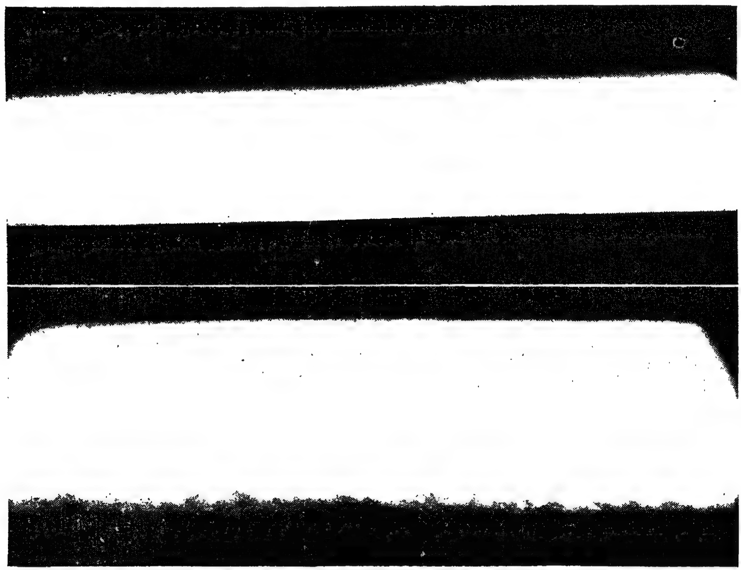

Fig. 78.—Non-turbulent and turbulent boundary layers.

of a streamline body, in Fig. 79. The flow at the first boundary-layer point, near the nose of the model, is not only infinitely smoother than that at the second one, well back along the contour, but considerably smoother than the flow in the free stream. The laminar flow, where it exists, actually damps out fluctuations already present in the wind. Where it gives way to the turbulent type the fluctuations reappear—magnified.

The transition from one type of flow to the other is commonly presumed, for the purposes of mathematical treatment, to be

[1] From "Experimental Investigation of Boundary Layer Flow," by L. F. G. Simmons and A. F. C. Brown, *R. and M.* 1547.

perfectly abrupt. As a matter of fact it is not,[1] and over a substantial part of the surface there is likely to be a superposition of two boundary layers, a turbulent zone overlying a laminar one. The refinement of a separate consideration of the transitional region is not, at least for our present purposes, justified. One type of flow or the other can be assumed as predominant in influence at every point, and the mathematical fiction of an

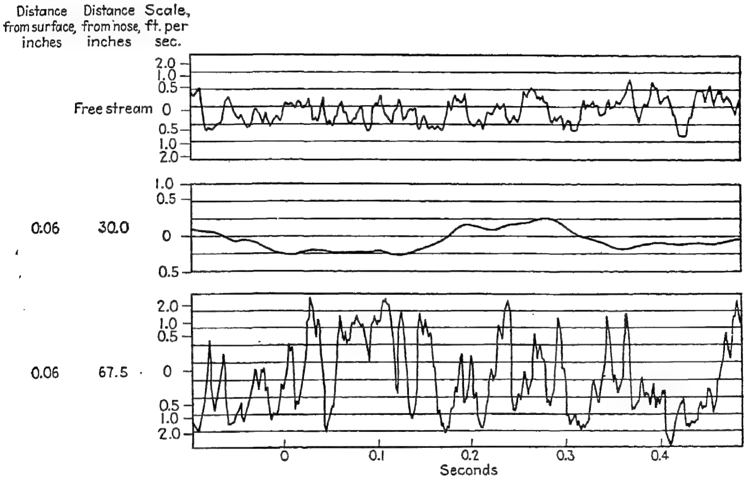

FIG. 79.—Short-period variations of velocity in a free stream and in a laminar and a turbulent boundary layer, respectively.

abrupt substitution can be preserved. The laminar and turbulent regions can be treated as mutually exclusive.

They may exclude each other from a given spot in the boundary layer, but the whole extent of the boundary layer need not by any means be consigned to a single type of flow. The laminar and the turbulent alternatives may coexist, not at the same point but in tandem. Each may have its own zone of regency, the available territory parceled out between the two. When that happens—and it is indeed the typical case—the parceling is not fortuitous. It is controlled by definite law.[2]

[1] "Quelques problèmes actuels de l'aérodynamique," by Theodor von Karmán, Chambres Syndicales des Industries Aéronautiques, Paris, 1932.

[2] This general subject of turbulent and laminar motion was given its aerodynamic application and first critically discussed in that connection

It has been recognized for many years that the flow in pipes, smoothly laminar at low speeds, became turbulent as the speed increased. Necessarily, as the theory of dimensions[1] enables us to prove, any phenomenon of fluid flow that undergoes change with speed must in fact be a function of Reynolds' number. It has been recognized that there was a critical range of Reynolds' numbers below which the flow could be relied on to be laminar, above which it would be turbulent despite anything that could be done. The range has been shown by Schiller[2] to extend in the case of pipes approximately from $N = 2{,}300$ to $N = 20{,}000$, and it has been shown that the exact value at which the transition from one type of flow to the other occurs depends very largely on the initial condition of the fluid stream. Practically speaking, no such thing as completely still air or a completely non-turbulent stream exists. For obvious reasons, any turbulence initially present in the stream enhances by just so much the disposition to turbulence in the boundary layer, and so transfers the transition from one type of flow to the other to lower Reynolds' numbers. The point is one of great practical importance in interpreting wind-tunnel tests, and will be further discussed in that connection.

As fluid flows along a flat plate placed edgewise to the stream a laminar boundary layer builds up its thickness gradually,

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by J. M. Burgers, of Delft, in "The Motion of a Fluid in the Boundary Layer along Plane Smooth Surface," *Proc.* First Inter. Cong. for Applied Mechanics, Delft, 1924. Experiments on flow on pipes have a pronounced bearing, among the most interesting of earlier references being to Theodor von Karman and to L. Schiller (both in the *Zeitschrift für angewandte Mathematik und Mechanik* for 1921, pp. 233 and 436, respectively). Excellent general discussions and summaries of the mathematical treatment and of experimental data accumulated are contained in "Effect of Turbulence in Wind Tunnel Measurements," by H. L. Dryden and A. M. Kuethe, *Rept.* 342; "Skin Friction," by L. Bairstow, *Jour.* Roy. Aero. Soc., January, 1925, p. 3; and "Skin Friction and the Drag of Streamline Bodies," by B. M. Jones, *R. and M.* 1199; "Modern Aerodynamical Research in Germany," by J. W. Maccoll, *Jour.* Roy. Aero. Soc., August, 1930, p. 649; "The Behavior of Fluids in Turbulent Motion," by A. Fage, *Jour.* Roy. Aero. Soc., July, 1933, p. 573. Many papers of a highly mathematical order by Prandtl, von Karman, Pohlhausen, Millikan, and others also demand notice from the student of the subject. A complete bibliography of some 500 references has been compiled by A. F. Zahm and C. A. Ross, published by the Division of Aeronautics, U. S. Library of Congress.

[1] P. 23, *supra*.

[2] Schiller, *loc. cit.*

approximately in accordance with the formula mathematically developed by Blasius[1]

$$d = 5.5\sqrt{\frac{\mu x}{V\rho}} = \frac{5.5x}{\sqrt{N_x}} \tag{45}$$

where d is the boundary-layer thickness, x the distance from the leading edge of the plate to the point under examination, V the speed of the undisturbed stream, and N_x the Reynolds' number obtained by taking x as the linear dimension in the familiar form $VL\rho/\mu$. To establish analogy between the flow along the plate and the flow through a small pipe, since the flow in the pipe displays a velocity gradient from zero at the walls to a maximum at the center, the thickness of the boundary layer is taken as the equivalent of the radius of the pipe. On that basis of comparison it would be logical to expect that the transition from laminar to turbulent flow would appear at a Reynolds' number, calculated by using the thickness of the layer as the linear dimension, of about one-half the critical values specified for pipes (since the diameter of the pipe has been replaced by a quantity equivalent to the radius). If improbable extremes of turbulence in the stream are omitted, the critical range of N_d(d being the boundary-layer thickness as before) might be expected to lie between 1,500 and 5,000, and, if Blasius' formula (45) is applied to make it possible to replace d by x, those values of N_d prove to correspond to 75,000 and 800,000, respectively, for N_x.

The appearance of turbulent flow may be detected photographically, by the change in the form of the velocity-distribution curve as indicated in Fig. 77, by direct measurement of the degree of pulsation of velocity as in Fig. 79, or, most simply, by the fact that the Blasius formula ceases to apply under tur-

[1] *Zeitschrift für Mathematik und Physik*, vol. 56, p. 1, 1908. The analysis has been subsequently extended to cover the transition point and the distribution of velocity and change of thickness in the turbulent portion of the boundary layer. Though there have been many studies of notable significance in this connection, of particular interest and of ready availability to an English-speaking public are "The Boundary Layer and Skin Friction for a Figure of Revolution," by Clark B. Millikan, Applied Mechanics Section of *Trans*. Am. Soc. Mech. Eng., Jan. 30, 1932; and "Computation of the Two-dimensional Flow in a Laminar Boundary Layer," by Hugh L. Dryden, *Rept.* 497.

bulent conditions and that a plot of boundary-layer thickness against distance from the leading edge of the plate will show a sudden increase in the rate of thickening at the transition point. Experiment clearly shows the thickening, as in Fig. 80,[1] and shows also that in the particular case for which that illustration is drawn the transition took place at $N_x = 320{,}000$, well within the critical range prophesied by computation from pipe-flow data.

There remains one more important relationship, that between boundary-layer conditions and separation.

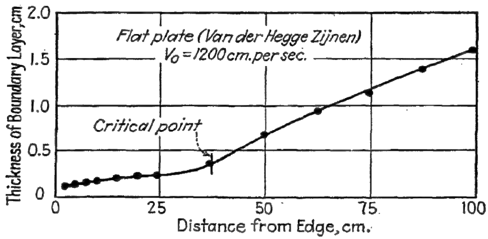

FIG. 80.—Variation of thickness of boundary layer with distance from the leading edge of a flat plate.

Separation.—The boundary layer serves, from the point of view of the streamline-flow pattern, to increase the effective thickness of the object immersed in the stream. Even the thinnest flat plate acquires a certain effective thickness and forces the streamlines to diverge and the velocity and pressure to undergo certain changes as the air passes over its surface, for the slow-moving air immediately contiguous to the plate acts in defining the flow very much as though it were a part of the plate itself.

The thickness of the boundary layer is tiny, however, and velocity and pressure changes in the streamline flow past a flat plate edgewise to the wind are small in proportion to the smallness of the disturbance that the boundary layer entails. In dealing with objects of greater solid thickness, such for example as

[1] Taken from "Measurements of the Velocity Distribution in the Boundary Layer along a Plane Surface," by B. G. van der Hegge Zijnen, Thesis from Delft Technical High School, 1924. To be cited also in this connection is "The Motion of a Fluid in the Boundary Layer along a Plane Smooth Surface," by J. M. Burgers, *Proc.* First Inter. Cong. for Applied Mechanics, Delft, 1924.

struts and streamline bodies, the changes of flow condition are much more important and finally react drastically upon the stability of the motion. Take, for illustration, a circular cylinder.

The pattern of air flow around a cylinder obviously calls for an increase of speed and a drop of pressure along each streamline. The maximum speed would be expected, from purely physical considerations, abreast of the cylinder's median plane perpendicular to the stream, or of the vertical diameter. Measurements in fact show little change of speed after passing an angle of 65 deg. around the cylinder from the wind direction as an axis, a velocity of just over 1.5 times the undisturbed wind speed being attained in that region. At some point, just past the midsection if the flow retains its perfect symmetry, the velocity must begin to fall off appreciably and the pressure to rise.

Normally the particles of a fluid move, like objects impelled by gravity, from a region of high pressure to one of low. Precisely as the kinetic energy of a projectile fired vertically into the air enables it to a limited degree to overcome gravity, exchanging kinetic energy for potential, so the kinetic energy of a moving fluid enables it to make headway into the teeth, so to speak, of a rising pressure. But the particles in the boundary layer are in a different situation. They have the same positive pressure gradient to overcome, and they have less kinetic energy or none at all with which to overcome it. The increase of pressure becomes an insurmountable barrier, and the stream finds itself stopped in its tracks. Having nowhere to go, the particles pile up on each other at that point. The boundary layer thickens abruptly, and the streamlines are forcibly pushed away from the cylinder contour. Separation ensues. The forcible detachment of the streamlines from their normal paths, in fact, is separation. It leads directly to so general a spread of turbulence from the stagnated boundary layer outward into the general stream that any subsequent restoration of smooth streamline motion is out of the question.

This is, of course, an oversimplified picture of a phenomenon so complex that even the most elaborate treatments make no pretense of a full explanation.[1] If there were nothing more

[1] The closest approach to it is to be found in "On the Theory of Laminar Boundary Layers Involving Separation," by Theodor von Karman and Clark B. Millikan, *Rept.* 504.

to it than this, separation in the case of the cylinder would take place at an angle from the nose of something over 90 deg., whereas experiment proves that under the conditions that now engage us it actually occurs at about 73 deg. What happens, of course, is that the increase of pressure projects its influence back upstream, that the boundary layer begins to stagnate and pile up before the actual positive pressure gradient under perfect streamline-flow conditions is encountered, and that the point of separation is correspondingly advanced. In the first instant after the

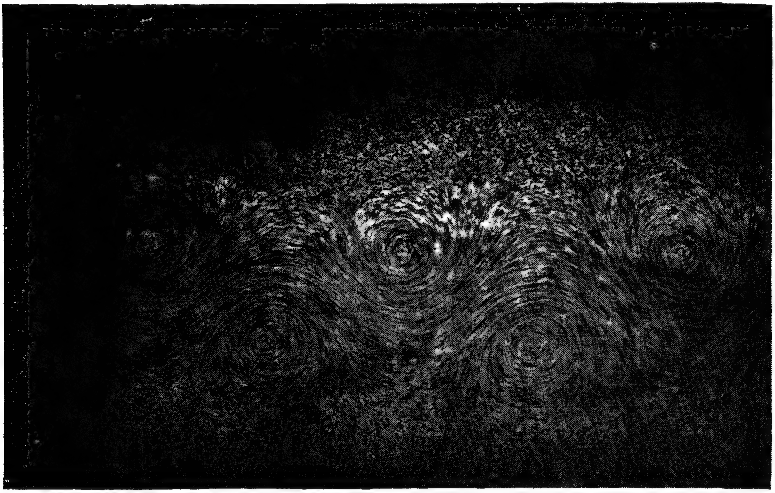

Fig. 81.—Vortex pattern formed behind a moving cylinder.

motion is started separation might indeed be at the 95-deg. point or thereabouts, but separation feeds upon itself. Once existent, the very fact of its existence so changes the pressure distribution and the entire flow pattern as to push the conditions that stagnate the boundary layer back toward the nose of the contour. Separation can be pictured as creeping gradually upstream, each advance providing the material for a further progress, until stable conditions are finally attained at the 73-deg. angle already mentioned.

Oversimplified, too, is the concept of a large and indiscriminate field of fine-textured turbulence following after separation. The actual mechanism is a detachment of the fluid from the

stagnated portion of the boundary layer and its discharge downstream in a band of vorticity or *vortex sheet*. That in turn quickly resolves itself into a chain of distinct vortices, of a structure of which the smoke ring forms the commonest and most familiar example. The vortices derive first from one side of the cylinder and then from the other in neat alternation and with an almost perfect periodicity. Figure 81[1] shows the flow pattern in a water tank through which an elliptic cylinder with a major axis six times its minor one was moving. This particular illustration,

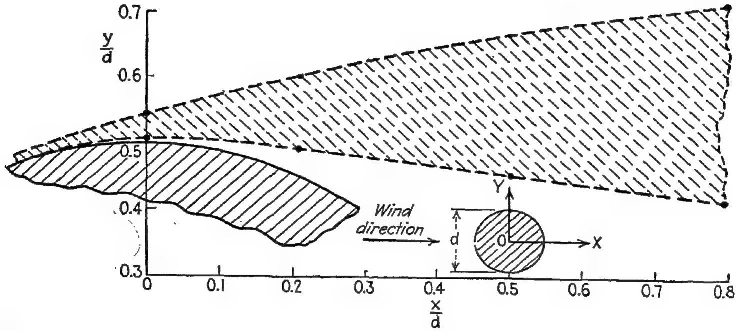

FIG. 82.—Formation of vortex sheet at point of separation of flow on a circular cylinder.

referred like Fig. 63 to an initially stationary body of fluid and a moving object, shows the conditions at a distance behind the cylinder of seven times its major axis—approximately the point of strongest and clearest manifestation of the vortex system. Figure 82 sheds further light on the matter with an enlarged section[2] of the part of a circular cylinder near the separation point and of the vortex sheet immediately behind it. At 0.1 of the cylinder's diameter downstream from its aftermost point the total spread of the pattern, from the outer boundary of one

[1] From "An Experimental Investigation of the Wake behind an Elliptic Cylinder," by G. J. Richards, *R. and M.* 1590.

[2] Taken from "The Structure of Vortex Sheets," by A. Fage and F. C. Johansen, which also provides one of the most complete and simplest of physical explanations of the whole subject (*R. and M.* 1143). Another interesting reference, with many photographic illustrations of the flow in the wake of all manner of objects, is "Air Flow," by W. S. Farren, *Jour. Roy. Aero. Soc.*, p. 451, June, 1932.

vortex sheet to the outer boundary of the other, proves to be 1.35 times the diameter. The breadth of the deadwater region between the inner boundaries of the two vortex sheets, at the same downstream coordinate, is $0.30D$. Another dimension of some interest is that between successive vortices after the alternating-chain pattern, or *vortex street*, has developed. It is, roughly speaking, six times the diameter in the case of a cylinder, $4D$ in that of a sphere.[1] A cylindrical pipe an inch in diameter, exposed to the wind on an airplane flying 150 m. p. h., must then cast off about 440 vortices per second. This periodicity helps to explain the characteristic sound often emitted as simple bodies are moved rapidly through the air, the pitch of the sound dependent on the body's size, form, and velocity.

All this may seem somewhat remote from airplane design, but no aeronautical engineer ought to be without a general appreciation of the dynamics of the fluid in which his products have to move and a general understanding of how it will behave under any conceivable set of conditions.

Now we return to a question immediately and intimately affecting the magnitude of aerodynamic resistance—the question of the conditions under which separation occurs. All the reasoning on that point to date has been limited to the case where the boundary layer is laminar. When it becomes turbulent there is a greater measure of intermingling of the particles from the inner and the outer strata and of actual interchange between the boundary layer and the stream beyond. The facilities for a dragging along of the boundary layer by the friction of the free-flowing air, and so of continuing to make headway against a rising pressure, are immensely improved. Stagnation is delayed if not entirely averted, and so is boundary-layer stagnation's companion and offspring, separation.

We must dwell on that point, for it is at once of great practical importance and contrary to casual anticipation. Neglecting analysis entirely and leaping at the obvious, one would naturally expect the general turbulence of separation to be promoted by the limited and local turbulence of a boundary layer in which N_d exceeds the critical value.[2] The revelation that, on the other

[1] "Vortex System behind a Sphere Moving through Viscous Fluid," by H. F. Winny, *R. and M.* 1531.

[2] See p. 127, *supra*.

hand, the turbulence at the boundary acts as a positive preservative for streamline motion beyond the boundary layer, and effectively discourages its breakdown, requires some mental readjustment.

Turbulence in the boundary layer is not, of course, a definite separation preventive. There comes a point where the positive

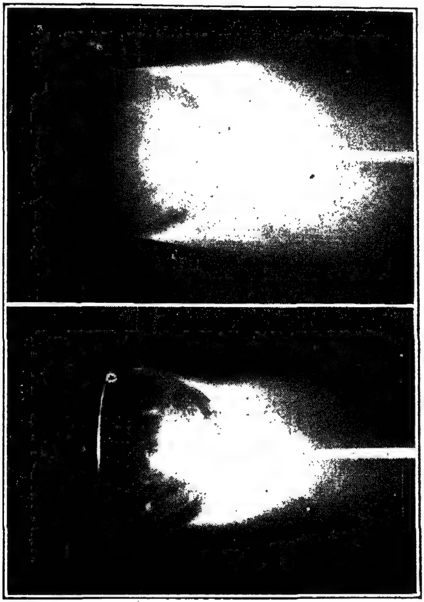

FIG. 83.—Flow of air past spheres, below and above the critical speed. (Air moving from left to right).

pressure gradient is too steep to be surmounted even under these conditions, but its coming is delayed. Specifically, it is delayed on a circular cylinder from an angle of about 75 deg. to one of from 90 to 105 deg. to the wind direction.[1] Figure 83[2] shows the difference of flow in the two cases for a sphere.

[1] "The Airflow around a Circular Cylinder in the Region Where the Boundary Layer Separates from the Surface," by A. Fage, *R. and M.* 1179.

[2] From the Göttingen aerodynamic laboratory. Furnished by Dr. Prandtl.

Naturally so large a difference of flow pattern has an effect on resistance. Specifically it has, as shown in Fig. 84, the effect of considerably increasing the intensity of the pressure drop over the forward portion of the object and of carrying a smooth curve of declining pressures back to points farther removed from the nose, and down to lower minimum values,

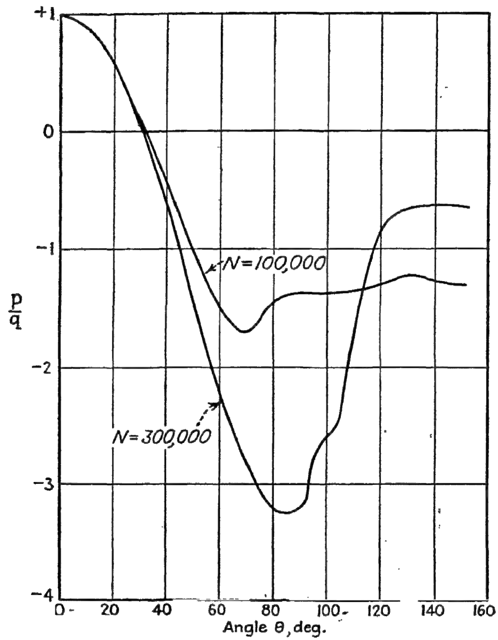

FIG. 84.—Pressure distribution over a sphere at two speeds, below and above the critical value, respectively.

than in the laminar-flow, early-separation case. The effect on total force, the product of an integration of the pressure curve over the whole area, is substantial. In the case of the sphere, the resistance when the boundary layer is turbulent is approximately one-third of what it is in the laminar case.

The type of flow existing in the boundary layer at a given point on an object's surface, it will be recalled, has already been presented as dependent on two things—Reynolds' number and

turbulence in the free stream. A third has now to be added—the surface condition of the model itself. Each may be treated in turn.

Dependence on Reynolds' number is evidenced in Fig. 85,[1] where the resistance coefficient for a circular cylinder is plotted against N. Clearly (and this is confirmed by direct observation of the air flow and otherwise) the transference of the separation point from one extreme position to the other is not abrupt. The transition is rapid, but it is a transition.

The mid-point of the transition in this case is at a Reynolds' number of 320,000. If it is assumed that that corresponds to a

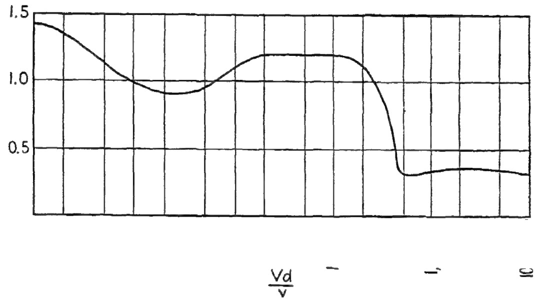

Fig. 85.—Variation of drag coefficient with Reynolds' number for circular cylinders.

change from laminar to turbulent flow at the 80-deg. point, then since the distance along the surface of the cylinder to that point equals $^{80}\!/_{360} \times \pi D$, or $0.7D$, $N_x = 225{,}000$. This figure, however, is calculated for the speed in the undisturbed stream and not for the speed locally prevailing. The speed past a cylinder at 80 deg. from the nose being approximately $1.6V_0$, the true N_x for true local conditions is 360,000. It thus lies well within the range determined by the Blasius formula and the experiments on pipes to determine critical values of N_d. Of course there is no particular reason to expect that the experimental figure would agree with any accuracy with the one so calculated, since the Blasius formula is by specification limited to flat plates. It is pleasant to discover so good an agreement,

[1] "New Data on the Laws of Fluid Resistance," by C. Wieselsberger, *Tech. Note* 84.

but it must be written down as scarcely more than a coincidence. Actual measurement of the boundary layer around a cylinder[1] has shown it substantially (as much as 40 per cent) thinner on the forward part of the cylinder than the flat-plate theory would allow, but for reasons already indicated it thickens rapidly as the region of maximum pressure drop on the surface is approached.[2]

If there is logical reason to expect agreement between flat-plate and cylinder results in any particular it would be in the critical value of the Reynolds' number based on boundary-layer thickness locally prevailing, N_d, rather than on N_x. Fage has shown[3] that the transition from laminar to turbulent motion on the surface of the cylinder actually took place in a particular series of experiments at values of N_d ranging from 1,620 to 4,650, the higher values occurring at the higher speeds. Once again the range is in the neighborhood of that determined[4] for pipes and flat plates, but so wide a variation among the figures for the several speeds of test evidences a change of the determinant mechanism. The concept of a transition in flow depending on nothing except the attainment of a critical value of N_d, which in turn depends only on the initial turbulence of the stream, must be abandoned; or retained only as effective within carefully specified limits or as an interesting qualitative approximation helpful in understanding what the boundary layer is and does.

Stream Turbulence.—The effect of turbulence initially present in the stream could be foreseen from the vantage point of the ground already covered. It facilitates the change from laminar to turbulent flow, and the change takes place at a lower Reynolds' number as a result. The critical zone within which the separation point is moving backwards along the surface as Reynolds' number increases ought then also to be transported to a lower range of N, and in general the curve of resistance coefficient

[1] "Viscous Layer Associated with a Circular Cylinder," by J. J. Green, *R. and M.* 1313.

[2] For a simple theoretical discussion of the effect of curvature of the boundary, together with the results of many experiments, see "Behavior of Turbulent Boundary Layers on Curved Convex Walls," by Hans Schmidbauer, a thesis submitted to the Technische Hochschule in München, 1934; translated as *Tech. Memo.* 791.

[3] *R. and M.* 1179, *cit. supra.*

[4] See p. 127.

against Reynolds' number ought to be transported bodily to the left.

Figure 86[1] records the proof that it is so in fact. Its three curves all relate to the same cylinder. All were taken in the same wind tunnel, but in one case the tunnel maintained its normal state and in the other two its section was crossed, upstream from the model, by a net of heavy twine stretched from wall to wall. No more effective device for producing turbulence could be found, and to vary the amount of turbulence introduced at the point of mounting the model the net was moved to a suc-

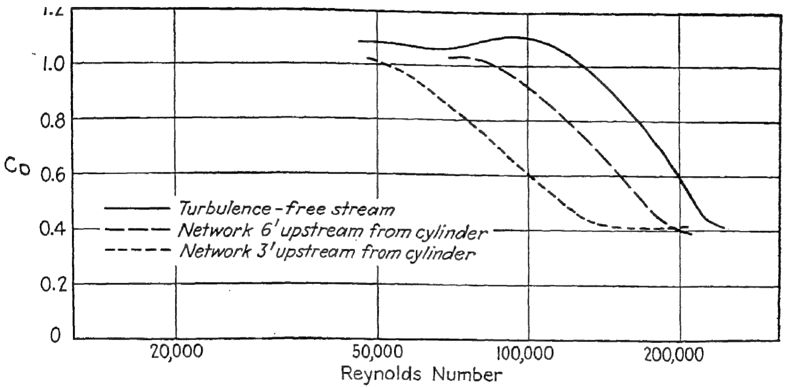

FIG. 86.—Effect of turbulence in the wind-stream on the drag characteristics of cylinders.

cession of locations along the tunnel axis. Turbulence in the stream was, of course, a maximum near the net, gradually dying out as the flow progressed downstream. In strict accordance with prediction, the curves in Fig. 86 are spread out horizontally, the curve for the minimum-turbulence condition of the open tunnel at the extreme right, that for the maximum turbulence, obtained with the net stretched at minimum distance upstream from the model, at the extreme left. Over a considerable region there results the condition, at first sight astonishing, that air resistance is highest in a smooth stream, falling steadily lower as the stream becomes more turbulent.

[1] From *R. and M.* 1283, by Fage and Warsap, *cit. supra*. Reference should be made also to H. L. Dryden's studies of wind-tunnel turbulence and its effects, *Repts.* 342, 392, *cit. supra*.

The maximum degree of turbulence so introduced shows the median point of the critical range at a Reynolds' number, based on diameter, of 80,000, a value of N_x as defined on page 135 of 90,000. Though the experiments provide no data for the computation of N_d, it is highly probable that it would be found in some cases below Schiller's minimum critical value for pipes,[1] the value at which he found that any preexisting turbulence, however great, would be damped out and a laminar boundary motion established.

The conditions in the free tunnel, corresponding to the curve at the extreme right of Fig. 86, were of course by no means entirely turbulence-free. Adjustment for the varying turbulence characteristics of the tunnels is one of the great problems of wind-tunnel testing, and the impossibility of making the adjustment perfectly complete is the major factor accounting for the differences between the results obtained from various tunnels when all test the same model.[2] The best approximation to an adjustment is made by determining for each tunnel a turbulence factor[3] and then plotting the tunnel's results for purposes of comparison not against the true Reynolds' number but against a fictitious one obtained by multiplying in the turbulence factor at each point. If some phenomenon is observed at a Reynolds' number of 200,000 in a tunnel that has been found to have a turbulence factor of 2, for example, it would be assumed that the same phenomenon would have appeared at a Reynolds' number of 400,000 in a turbulence-free stream, and the curve would be plotted accordingly. The correlation so obtained is by no means perfect in general, but in bringing all tunnels together on the point of occurrence of critical phenomena such as sudden changes in the values of aerodynamic force coefficients it is very nearly so. The turbulence factor is determined by a process inverse to that of applying it, the point of manifestation of some well-known and easily duplicated critical phenomenon being recorded as a characteristic of the tunnel. Dryden defines it approximately as $300{,}000/N_{0.3}$, where $N_{0.3}$ is the Reynolds' number that corresponds to a drag coefficient of 0.3 for a sphere

[1] See p. 126, *supra*.

[2] *Rept.* 342, *cit. supra;* "Joint Report on Standardization Tests on R. A. F. 15 Airfoil," by W. S. Diehl, *Rept.* 309.

[3] *Repts.* 342 and 392, by H. L. Dryden, *cit. supra*.

(0.3 being approximately intermediate in the transition from the higher to the lower drag regime). For tunnels in actual service the factor ranges from very nearly unity in the National Advisory Committee's full-scale tunnel up to 2.5 in the variable-density installation at the same laboratory, or to a maximum of 3.5 in a tunnel that has had nets or other turbulence-producing accessories deliberately introduced.

In all this work it is assumed that the turbulence in free air is zero. The assumption is far from being true. The earth itself has a boundary layer, and a generally turbulent one, of a mean thickness found[1] to be thirty-eight times the wind speed per second. With a wind of 20 m. p. h., or 29 ft. per sec., the boundary layer extends to a height of about 1,100 ft. Even when there is no general wind the air is in restless local motion. Still the fine-patterned turbulence that most affects the boundary structure is so slight as to be negligible, at least by comparison with the differences that exist among tunnels. Analytical allowance for the effects on aircraft performance of changing fine-scaled turbulence in the atmosphere with changes in wind or altitude or other variables belongs to the future.

Easily foreseen, with a little reflection, is the result of roughening the surface of a cylinder. Increased roughness thickens the boundary layer, and so increases N_d. It acts directly, too, to destroy laminar flow, for the difficulties of maintaining a strictly stratified motion on an irregular uphill-and-down course along a rough surface's miscellaneous protuberances are obvious. Hence there is an effect exactly akin to that of increased stream turbulence. Figure 87,[2] with several curves for cylinders covered with sandpaper of varying degrees of coarseness, shows it. One of the neatest of its contributions is the comparison between the two curves farthest to the left, representing two cylinders covered with the same sandpaper but differing in diameter. The individual abrasive particles being of the same actual size in the two cases, the *relative* roughness is greater for the smaller cylinder, which accordingly shows its critical region at lower Reynolds' numbers.

[1] "The Scale of the Boundary Layer in the Atmosphere," by Robert H. Goddard and Arnold M. Kuethe, *Jour. Aeronautical Sci.*, p. 115, May, 1935.

[2] From *R. and M.* 1283, *cit. supra.*

Figure 87 indicates also, by the varying steepness of the transitional parts of the curves, a more abrupt change in the type of flow with rough cylinders than with smooth ones. That too might have been anticipated. Still farther, it shows that not all of the curves stabilize at the same resistance coefficient at high values of N. There is in fact a steadily ascending scale toward the left, with final coefficients for the roughest cylinder more than twice as high as those for the perfectly smooth one. That will be analyzed later.[1] For the moment, let it merely be noted.

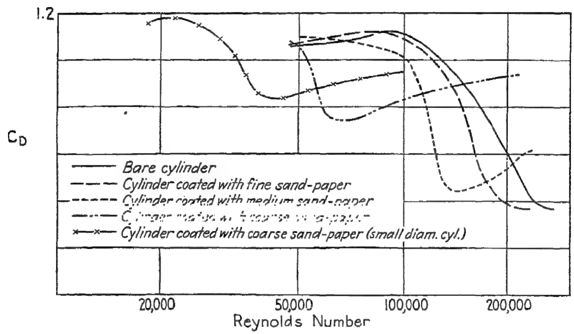

FIG. 87.—Effect of surface roughness of cylinders on their drag characteristics.

Despite the difference in final values of the coefficients, it is true for particular ranges of Reynolds' number that a rough surface is helpful in reducing the resistance. When $N = 70{,}000$, for example, the drag of a 6-in. cylinder coated with an exceedingly coarse sandpaper is fully a third less than that of the same cylinder with its surface carefully polished. The reason is clear enough now, but to one starting with no knowledge of boundary-layer structure and of the true mechanism of separation the result would seem opposed to all common sense. Though it is true that these eccentricities are most commonly encountered in the wind tunnel, they are by no means entirely without practical interest. A wire or pipe or turnbuckle barrel anywhere up to two inches in diameter, and exposed to a wind at any speed up to 200 m. p. h., will have its drag reduced by a certain amount of roughening of its surface.

[1] P. 153, *infra*.

Another, and a more detailed, illustration of the effect of surface condition on flow is afforded by Fig. 88. Its curves of pressure distribution show how the pressure drop just forward of the meridian of a circular cylinder can be increased, the resistance-producing pressure-drop behind the cylinder reduced, by laying a fine wire along the side of the cylinder just forward of the separation point to introduce turbulence into the boundary layer.

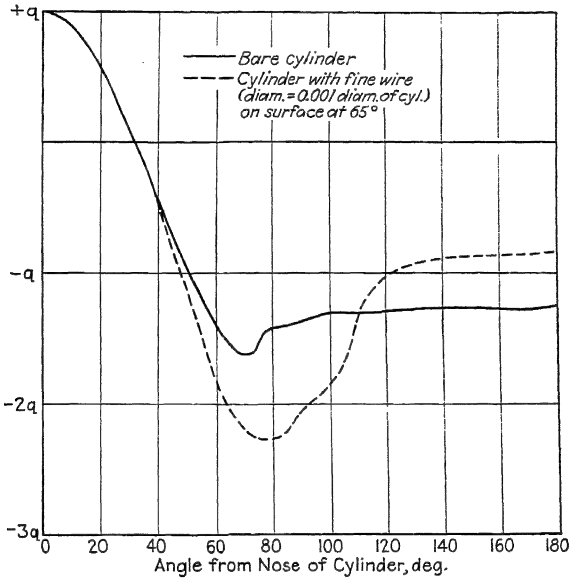

FIG. 88.—Effect of an obstruction on the surface of a cylinder upon the pressure distribution around it.

Such a disturbing element of course shows beneficial results only at Reynolds' numbers low enough so that the boundary layer of the smooth cylinder would have been of the laminar type.[1]

Boundary-layer Control.—The boundary layer is the source of separation, and if the stagnation of the boundary layer could be prevented separation might cease to be. Various wing forms[2] owe their efficiency to their creation of flow conditions

[1] For a practical application of this precise form of disturbing element in airplane design, see p. 431, *infra*.

[2] Particularly the slotted wing, for which see Chap. X, *infra*.

that discourage separation. Going beyond the simple modification of airfoil geometry, there are two mechanical devices for accomplishing the same end, and much more effectively.

Stagnation in the boundary layer can be prevented either by accelerating the fluid that is threatening to come to rest or by removing those particular particles from the scene before they have had a chance to do harm. A velocity profile through the boundary layer normally has the form indicated in *a* of Fig. 89.[1] The curve happens to have been drawn for laminar flow, but much the same reasoning can apply in a turbulent motion. With stagnation impending, the profile is changed as in *b*. To pre-

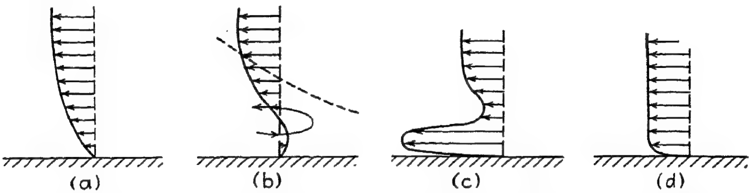

FIG. 89.—Natural and artificially-controlled boundary-layer flows. (*a*) Normal laminar flow; (*b*) stagnated flow, with separation impending; (*c*) boundary-layer control by pressure; (*d*) boundary-layer control by suction.

vent the change from *a* to *b*, the form *c* can be introduced by blowing air at high speed out of a backwardly directed slit on the surface, or *d* can be substituted by sucking the inner portion of the boundary layer through a similar slit into the interior of the object exposed in the stream. The first method was invented by Prof. A. Baumann. The second, which seems likely to prove of the greater ultimate practical utility, was proposed by Prandtl in the original boundary-layer paper of 1904. The two systems present the seeming anomaly of seeking a common goal by the diametrically opposite procedures of applying positive pressure, on the one hand, and negative pressure, on the other. The greater the intensity, either of suction or of pressure, the more marked the effects become.

[1] Based on "The Boundary Layer as a Means of Controlling the Flow of Liquids and Gases," by Oskar Schrenk, *Die Naturwissenschaften*, p. 663, Aug. 23, 1929, translated as *Tech. Memo.* 555. An admirable non-mathematical summary of the whole subject of boundary-layer control, either by mechanical or by purely aerodynamic means, up to the time of its publication.

Both systems work. Both in fact succeed, given the expenditure of enough power in speeding up or removing the air, in preventing separation. Practical applications will be discussed in more detail elsewhere,[1] but an illustration that may be cited at once is the success of the National Advisory Committee in maintaining up to an angle of attack of 58 deg. a steady and unbroken flow over the upper surface of a thick airfoil that normally would burble at around 15. Boundary-layer control by suction did all that, and it served too to produce the flow shown in Fig. 90,[2] drawn from the thirty-year-old original work of Prandtl. On one side of the cylinder in the photograph the

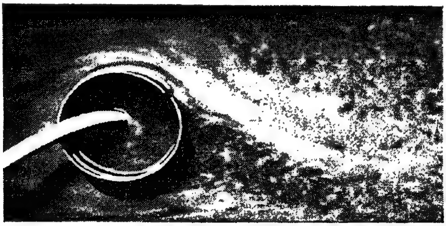

FIG. 90.—Prevention of separation (on upper part of cylinder's section) by boundary-layer control. (The flow of air is from left to right.)

customary separation point appears just forward of mid-section, the customary turbulence in the wake from that point back. On the other side, where a slit in the cylinder about 45 deg. behind mid-section allows the drawing off from the boundary layer of a small but steady stream of fluid, separation is retarded to a point just behind the slit. No contrast could be more vivid, and the enormous possibilities of such a system of streamline-flow preservation in reducing the drag are obvious.

Maximum Lift of Airfoils.—The lift curve of an airfoil follows a substantially straight course, as a rule, up to an angle of attack of from 10 to 15 deg. Somewhere in that region it begins to bend, and within a few degrees of further increase of angle it reaches and passes a maximum. Sometimes the peak of the curve is gently rounded; sometimes sharp; sometimes so abrupt that there is no longer a continuous curve at all, but a sudden break from one lift coefficient on the virtually straight portion of the curve

[1] Chap. X, *infra*.

[2] From *Tech. Memo.* 452, *cit. supra*.

to another and much lower one. In every case, however the maximum may be defined, it is separation that produces it.

Like other separation phenomena, then, it is a satellite of boundary-layer structure. Even at small angles there is separation near the trailing edge except for very thin airfoils.[1] As the angle increases, other things being equal, the point of separation moves forward, the departure of the still-rising lift curve from a straight line being the first sign that the forward motion is becoming rapid. At a given angle, a change from laminar to turbulent boundary-layer motion ought to retard separation—ought to increase maximum lift. Maximum lift ought to rise with Reynolds' number. That it would be unwise, on the basis of the reasoning so far available to us, to expect the rise to be regular and continuous can however be demonstrated.

Maximum lift is the resultant of two contending influences. As the angle of attack increases, the curvature of the streamlines and the velocity of flow over the upper surface increase, which would tend to increase the lift almost without limit; but at the same time the separation point moves forward, which tends to decrease it. As usual in such a case, the resultant attains a maximum at an intermediate value of each of its factors. As has been neatly shown by direct measurements of the part of the upper surface over which general turbulence of the wake exists,[2] the maximum in this particular case occurs, for typical airfoil sections of about 10 per cent thickness ratio, with separation fully developed at about mid-chord. With a small Reynolds' number the boundary-layer flow is laminar throughout, separation is determined by the conditions appropriate to laminar flow, and minor changes in Reynolds' number should have no great effect until it has been increased to a level that produces a change from laminar to turbulent flow in the boundary layer at a point ahead of the separation point for maximum lift. When N has been increased up to that point separation is retarded, precisely as on a cylinder or sphere, and not only is there a direct increase of lift at the angle of attack under examination but it becomes possible to effect still further increase by increasing the angle of attack. The increase of angle makes the separation point move

[1] See, for example, Fig. 26, p. 41, *supra*.

[2] "An Experimental Study of the Stalling of Wings," by B. M. Jones and others, *R. and M.* 1588.

forward again, and a new maximum of lift is found, at a larger angle than before but with approximately the same point of separation.

The procedure can be repeated with an additional gain of the same order, but it cannot be repeated indefinitely. After the point of transition from laminar to turbulent boundary flow has moved very far ahead of the separation point, still further movement has little or no effect on separation. Just as in the case of the cylinder, we may expect to find a critical range within which, and nowhere else, the effect of Reynolds' number is pronounced.

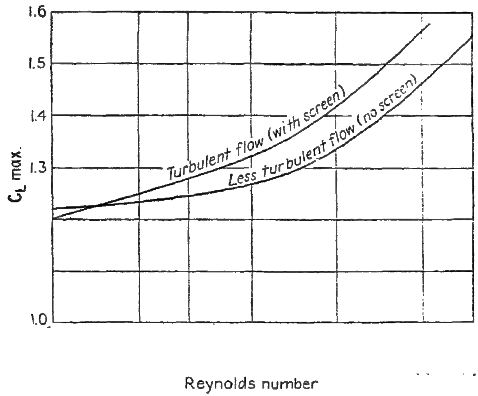

FIG. 91.—Variation of maximum lift with Reynolds' number, for a typical airfoil.

All this squares with experience in a general way. Figure 91[1] shows the variation of maximum-lift coefficient for a typical airfoil over a wide range of Reynolds' numbers, both in an open tunnel and with the turbulence of the stream deliberately increased. From $N = 1{,}000{,}000$ to $N = 3{,}000{,}000$ the increase of lift is rapid. Elsewhere it is slow. The increase of turbulence shifts the curve to the left, precisely as for sphere or cylinder drag.[2] The angle of maximum lift is found to increase from 15 deg. when $N \doteq 178{,}000$ to 19 deg. when it surpasses 3,000,000,

[1] From "Tests in the Variable Density Wind Tunnel to Investigate the Effects of Scale and Turbulence on Airfoil Characteristics," by John Stack, *Tech. Note.* 364.

[2] See p. 136, *supra.*

just as was foreseen. Other airfoil sections generally show a similar change of maximum lift at some point, though the critical values of N vary widely with the section. In some cases the rapid increase of maximum lift does not even begin until Reynolds' number exceeds 4,000,000.

It is there that prediction, except in a very mild and qualitative form, breaks down. If the change from laminar to turbulent flow is to be accomplished when $N_x = 500{,}000$ in a stream of very low turbulence,[1] the transition at the mid-point of the chord ought to come at a Reynolds' number for the airfoil of 1,000,000. It ought to come even lower than that if account were taken of the local increase of speed of flow over the forward part of the upper surface and of its effect on the true local value of N. Conversely, at a Reynolds' number of 2,000,000 the transition point ought to be only 25 per cent of the way back on the chord, at 4,000,000 only 12 per cent, far ahead of the apparent points of separation for airfoils in their high-lift regimes.

The explanation of the apparent discrepancy probably lies in the rather ill-defined location of the point of separation on airfoils and certain other objects. For a sphere or a cylinder, separation is quick and complete, and uncertainty about its position is limited to 10 or 15 deg. at most. For airfoils it is much less so, developing out of a gradually accelerating thickening of the boundary layer. It is easy to comprehend, though it might not have been foreseen, that the place and degree of this thickening and the extent of disturbance of the streamline flow along the curvature of the upper surface may depend on the boundary-layer conditions at points far ahead of the separation point as ordinarily regarded.

Reconsidering the matter in that light, one would expect to find thin sections sensitive to conditions even nearer the leading edge than would affect the thicker ones. A thin airfoil or one with the maximum camber unusually far back forces a comparatively sharp change of curvature of the streamlines directly over and immediately behind the leading edge. Beyond that region the curvature of path and the liability of separation are less than with a section of greater thickness or one with a bluffer form at the nose. The region of partial separation should then start closest to the leading edge for thin sections, for those with the

[1] See p. 127, *supra*.

maximum upper-surface camber well back on the chord, and for those with small leading-edge radii, and the critical range in respect of maximum lift for those groups would be expected to come at exceptionally high Reynolds' numbers. In a general way, with some apparent exceptions but not enough to destroy one's confidence in the validity of the reasoning, all this is borne out in practice.

There are other variations of maximum lift, generally of a more minor order, that are less tolerant of rational development. The characteristic, which some sections extend over a wide range of Reynolds' numbers, of decreasing the maximum lift as N increases has no obvious physical explanation. In a general way, however, most of the behavior of a lift curve near its peak and of the scale effects to which the lift is subject can be understood in terms of boundary-layer and separation phenomena.

There have been many attempts to bring this whole subject within the domain of mathematical treatment. So far it has resisted them, and its complexities appear to exceed anything else with which the student of mathematical aerodynamics has to deal. Partial treatments of a very suggestive order have been accomplished, but all have required assumptions of somewhat limited validity as a starting point.[1]

Skin Friction.—According to the classical theory of streamline motion, fluid resistance ought to exist only in company with separation. An object so smooth in curvature, and so perfectly accommodated to the necessities of the fluid in which it moves that its streamline pattern remains unbroken throughout, should have no drag. That drag in fact exists even in such a case, and that it is known as skin friction, has already been the subject of remark. That it is intimately connected with the state of the boundary layer ought by now to be obvious, for a frictional drag of the fluid on the immersed object must be exactly counterbalanced by the drag of the object on the fluid, with a resultant deceleration of the fluid particles and loss of total head. The loss of total head integrated across the wake ought in fact to be a

[1] The most interesting and useful of this work has been done by Clark B. Millikan and Theodor von Karman, in "A Theoretical Investigation of the Maximum Lift Coefficient," *Jour. of Applied Mechanics*, Am. Soc. Mech. Eng., 1935. Also, by the same authors, "On the Theory of Laminar Boundary Layers Involving Separation," *Rept.* **504**.

measure of the drag (except the induced drag), and the equivalence has been very closely checked in particular cases.[1]

In the same fashion, the skin friction alone can be determined at each point by analyzing the motion of the fluid immediately adjacent to the object and by determining from the velocity gradient in the fluid the frictional forces that must be acting on it.[2] For a particular fluid, of known viscosity characteristics, and for a known type of flow adjacent to the surface of the immersed object the friction ought to be mathematically predictable.

It has been found so, subject to some differences of opinion about the precise physical assumptions to be made, and formulas have been developed for skin friction for both the laminar-boundary-layer and the turbulent-boundary-layer cases.[3] They give results agreeing with those of experiment closely enough to be used interchangeably, and as the analytical expressions can be indefinitely extended, to cover an infinite range of Reynolds' numbers, they may be used in place of experimental curves for our present purposes.

The same study that led Blasius to a formula for the thickness of a laminar boundary layer[4] led also to the conclusion that the frictional coefficient under those conditions should be $1.34/\sqrt{N}$. His figure can hardly be checked by direct measurement except at very low values of N, since the boundary layer cannot be relied on to remain laminar over an entire surface except for Reynolds' numbers of 200,000 or less. Check in that region shows experimental values generally a little above the Blasius values, but coming within 25 per cent of them.[5] In view of the rarity in practice of problems involving such low Reynolds' numbers, such discrepancy as exists will be allowed to pass without further comment.

The turbulent regime shows, as might have been expected, more frictional drag. It shows also a less rapid decrease of

[1] "On the Drag of an Aerofoil for Two-dimensional Flow," by A. Fage and L. J. Jones, *R. and M.* 1015.

[2] "The Skin Friction on a Circular Cylinder," by A. Fage, *R. and M.* 1231.

[3] An excellent general review, brief and non-mathematical, is contained in Theodor von Karman's "Some Aerodynamic Problems of Airships," *Pub.* 1 of the Daniel Guggenheim Airship Institute, p. 45, Akron, 1933.

[4] See p. 127, *supra*.

[5] "Skin Friction and the Drag of Streamline Bodies," by B. M. Jones, *R. and M.* 1199.

coefficient with increase of N, so that the spread between the laminar and turbulent friction values widens as Reynolds' number increases. Prandtl has developed[1] the expression $0.074/\sqrt[5]{N}$ for the frictional coefficient when the boundary layer is turbulent—the case of most frequent practical application in aeronautics.

Here again the agreement with experiment has in general been good—well within 10 per cent. There is a disturbing factor in some of the National Advisory Committee's recent work on thin airfoils at high Reynolds' numbers, showing as it does minimum drag coefficients slightly below the anticipated level for the skin friction alone, but it is too early to generalize on the cause of those apparent discrepancies—fortunately of very minor degree. To complete the study it is necessary to consider the range in which the boundary layer is laminar on the forward part of the body, turbulent on the after part and with the dividing line between the two shifting with every change of Reynolds' number. A simple method of calculating a transition curve requires nothing more than the assumption that the type of flow changes at some fixed value of N_x, a calculation from that assumption of the precise point of change for a series of Reynolds' numbers, and the taking of a weighted mean of the frictional coefficients for the two parts into which the body is thus divided for each value of N in turn. Thus, if the critical value of N_x is assumed to be 300,000, then a flat plate at a Reynolds' number of 900,000 would be expected to have laminar flow over the forward third of its extent, turbulent flow over the remainder. The frictional coefficient would be the sum of one-third of the laminar-flow coefficient for a Reynolds' number of 300,000 (since N for this portion of the resistance is figured as though the plate terminated at the transition point) and two-thirds of the turbulent-flow value for $N = 900,000$.

All this is plotted in Fig. 92—the laminar-flow coefficients by formula, those for turbulent flow, and two alternative transition curves based on assumed critical values of N_x of 200,000 and 500,000, respectively. The second transition line, that starting at 500,000, corresponds approximately to free-atmosphere conditions and has been very closely checked in experiments by

[1] "Ergebnisse der aerodynamische Versuchsanstalt zu Göttingen," vol. III, 1927.

Gebers[1] in which the turbulence of the stream was carefully kept to a minimum. The other curve represents more nearly the conditions in a high-turbulence tunnel.

As a general rule, aerodynamic coefficients can be applied only to objects of strict geometrical similarity with those on which the original measurements were taken, and under those conditions it makes no great difference what particular linear dimension is selected for use in computing Reynolds' number.

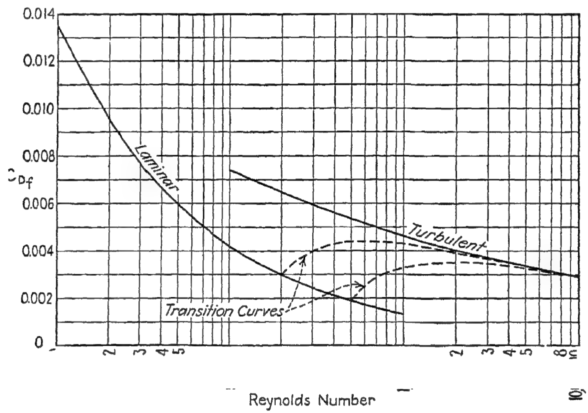

FIG. 92.—Variation of skin-friction coefficients with Reynolds' number.

Skin friction, however, is a special case. Obviously the distance along the surface in the direction of flow is what matters. Changes of a plate's breadth at right angles to the wind direction have no effect, within wide limits, on the friction per unit of area. The equation for friction on a rectangular flat plate under the laminar condition then takes the form

$$D_f = C_{D_f}\frac{\rho}{2}S'V^2 = \frac{1.34}{\sqrt{N}}\frac{\rho}{2}\cdot 2blV^2 \qquad 2.68\sqrt{\nu}\,\frac{\rho}{2}\,bl^{0.5}V^{1.5} \tag{46}$$

where S' is used in lieu of S as a reminder that skin friction operates over the entire exposed surface and that it is the entire

[1] "Law of Similitude for the Surface Resistance of Lacquered Planes Moving in a Straight Line through Water," by Friedrich Gebers, First Inter. Cong. on Applied Mechanics, Delft, 1924, reprinted as *Tech. Memo.* 308.

exposed surface that must be used in calculating it. For an airfoil or for a flat plate that includes both sides, and gives twice the area as ordinarily calculated. If the coefficients of skin friction on an airfoil are to be compared directly with the drag coefficients as directly measured, then, they must be doubled to put the two quantities on equal terms before the comparison begins.

It is easy to anticipate what such a comparison will show, at least at the angle of minimum drag. On the flat plate, which is itself a species of airfoil, the skin friction and the total drag at 0 deg. angle of attack are identical by definition. On a thin biconvex airfoil, there is still substantial identity.[1] As the airfoil thickness increases, or as it loses its symmetry and acquires a positive mean camber, the identity vanishes and the minimum airfoil drag not only rises well above the skin friction but sometimes begins to show a different mode of variation with Reynolds' number.

The precise fashion in which minimum drag varies with airfoil characteristics will be reserved for the next chapter,[1] but there need be no delay in mentioning the principal peculiarity of its dependence on Reynolds' number. Following the turbulent-boundary skin-friction curve closely, though at a greater or less distance above it, at high Reynolds' numbers, it generally continues to follow the same curve down to values of N far below those at which the change of boundary conditions would have been expected to appear. The superficial indication is that on thick airfoils the change from laminar to turbulent flow must take place at values of N_x well below those predicted from the flat-plate tests of Van der Hegge Zijnen and others,[2] but that is in emphatic contradiction to the readings of the maximum-lift characteristics, which suggest if anything a delayed transition. The behavior might, of course, be different at high angles of attack and at low. Such actual measurements as have been taken lend no very strong support to theories either of accelerated or of delayed transition, British experiments[3] on the upper surface of an airfoil with a thickness ratio of 15 per cent showing the

[1] John Stack, *Tech. Note* 364, *cit. supra.*

[2] See p. 127, *supra.*

[3] "An Experimental Determination of the Intensity of Friction on the Surface of an Aerofoil," by A. Fage and V. M. Falkner, *R. and M.* 1315.

sudden thickening of the boundary layer that marks the transition point when N_x is from 300,000 to 400,000. These figures are perhaps a little higher than would have been expected for flat plates in the same tunnel.

The variation of the critical value of N_x with form is not impossible. There would be every reason to expect that a three-dimensional convergence or divergence of flow, such as must exist on any solid of revolution, would affect conditions in the boundary layer; every reason to be sure, too, that the type of flow at the boundary would be influenced by the pressure gradient along the surface, which we have already seen[1] as producing stagnation and sudden thickening of the boundary layer and subsequent separation. Thus it is not altogether surprising to find, in several tests of streamline forms or airship models of varied size and at various speeds,[2] that the change to turbulent flow appears at a value of N_x of from 600,000 to 900,000, or from two to three times the characteristic flat-plate figures. In one case the apparent critical value of N_x actually approaches 2,000,000. The highest critical values in general seem to occur on the models with the smoothest change of curvature and with the most gradual change near the nose, and so on those with the maximum diameter farthest back along their length.

As boundary-layer behavior on a three-dimensional object, or on any object with a pressure gradient along its surface, differs from that on a flat plate, so also does skin friction. Even if the coefficients are assumed constant, which cannot be taken for granted, it would appear necessary to integrate over the surface of the object to take account of the variations of intensity of friction with variation of local velocity. In the case of a well-faired form the friction might be increased 25 per cent or more by the difference between the average local velocity in the streamlines adjacent to the surface and the velocity of the undisturbed stream. Ten per cent would be a fair average of increase in such a case, and in prophesying the total of frictional drag on a

[1] P. 129, *supra*.

[2] "Investigation of the Boundary Layers and the Drags of Two Streamline Bodies," by E. Ower and C. T. Hutton, *R. and M.* 1271; "Measurements of Flow in the Bounday Layer of a 1/40 Scale Model of the U. S. Airship Akron," by Hugh B. Freeman, *Rept.* 430. Data bearing indirectly on the same point are available, also, in *Rept.* 342, by H. L. Dryden and A. M. Kuethe, and *R. and M.* 1199, by B. M. Jones, both *cit. supra*.

strut or a streamline body the coefficients as plotted in Fig. 92 may well be raised by that fraction. The point-to-point analysis of friction, abandoning entirely any assumptions regarding the coefficients, yields some interesting results, but they are not of such practical importance, nor do they lead to any physical conclusions of so general an order, as to be pursued here.[1] It is enough for us to note that the formula

$$D_f = C_{D_f}\frac{\rho}{2}S'V_0^2 \qquad (47)$$

where C_{D_f} is the coefficient for the appropriate Reynolds' number as determined by flat-plate experiments and S' is the total exposed surface of the object, gives a close approximation to the friction for all sorts of forms; that it can be made still closer by some such empirical allowance for excess of mean velocity along the surface as was just proposed.

Surface Condition and Friction.—All the data so far offered, except a few of the figures bearing on the relation between artificially produced turbulence and separation, derive from tests of smooth surfaces. As the air in immediate molecular contact with the surface is quiescent in any case, the precise degree of smoothness makes very little difference, but when roughening has progressed to the point of projecting its irregularities outward into the normally moving portion of the boundary layer it does make a difference, and a very pronounced one. The skin friction on a varnished board is substantially equal to that on a carefully cleaned sheet of plate glass, but let the board be covered with sandpaper and it will show a very different result.

It is difficult to define roughness in quantitative terms, and the amount of work done with the direct purpose of discovering the effects of surface condition has been surprisingly small.[2] Froude,

[1] "The Skin Friction on a Circular Cylinder," by A. Fage, *R. and M.* 1231; "An Experimental Determination of the Intensity of Friction on the Surface of an Aerofoil," by A. Fage and V. M. Falkner, *R. and M.* 1315; "Measurements of Flow in the Boundary Layer of a 1/40 Scale Model of the U. S. Airship Akron," by Hugh B. Freeman, *Rept.* 430.

[2] "Skin Friction of Various Surfaces in Air," by Willis A. Gibbons, *Rept.* 6; "Skin Frictional Resistance of Plane Surfaces in Air," by C. Wieselsberger, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. I, 1921, analyzed and abstracted by W. S. Diehl in *Tech. Note* 102; "Wind Tunnel Tests on a Wing Covered with Monel Metal Gauze," by

in the pioneer experiments on skin friction in the latter years of the nineteenth century, found a surface coated with "fine sand" to have a frictional drag 150 per cent higher than a polished one at a Reynolds' number of 3,000,000, well up into the turbulent boundary-layer range. When "coarse sand" was used the percentage of increase was 200. Gibbons found uncoated linen of fine weave to have 25 per cent more friction than plate glass. One coat of varnish on the linen reduced the margin of excess over plate glass to 6 per cent. Two-ply cotton balloon fabrics, without varnish or other coating, ran from 70 to 85 per cent above the plate-glass norm.

An airfoil covered with monel-metal gauze, 100 mesh uncoated, has four times the skin friction of a smooth-surfaced one. Airplane fabric treated in accordance with common commercial practice for wing finishing has about 10 per cent more friction than the same fabric lacquered, waxed, and polished. Schrenk's flight tests show a frictional drag on a surface "roughened with poppy seed" (probably the approximate equivalent of Froude's "coarse sand") about 150 per cent higher than for the same surface polished. All these tests, wherever made, were on substantially full-scale objects, with dimensions parallel to the wind ranging from 4 to 8 ft.

All this leads to no formula, but it spots in enough points to allow of making a fair guess at the frictional coefficients that ought to be used for any particular kind of surface that may confront us.

Gibbons concluded, and the conclusion finds a certain amount of support in the work of other experimenters, that the total friction should increase more rapidly with speed for rough surfaces than for smooth ones, so that, whereas the ordinary formula for determining the frictional coefficient with a turbulent boundary layer shows it varying as the inverse fifth root of the Reynolds' number, the coefficient for a very rough surface should

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F. B. Bradfield, *R. and M.* 1032; "The Aerodynamic Characteristics of Airfoils as Affected by Surface Roughness," by Ray W. Hooker, *Tech. Note* 457; "Effect of the Surface Condition of a Wing on the Aerodynamic Characteristics of an Airplane," by S. J. DeFrance, *Tech. Note* 495; "Measurement of Profile Drag on an Airplane in Flight by the Momentum Method," by Martin Schrenk, *Luftfahrtforschung*, May 18, 1928, translated as *Tech. Memo.* 557.

be entirely independent of N, and the total frictional force in such a case directly proportional to the square of the speed. For moderate amounts of roughness the exponent attached to N in the formula would take values intermediate between 0 and -0.2. For the cases encountered in practice it will hardly be necessary to make any change in the formula as plotted in Fig. 92, though it certainly is true that the difference between a highly polished surface and one of fairly low-grade finish is much more apparent at very high Reynolds' numbers than at those of 1,000,000 or less.

Analogy is always expected between the effects of surface condition of a model and those of the state and degree of turbulence of the air stream. Experiments on the effect of stream turbulence on skin friction have been too infrequent to allow generalization, but the indications are that the frictional coefficients under a given boundary-layer condition are increased by turbulence as by surface roughness. This is quite apart from the increase due to the accelerated transition into the high-friction regime of a turbulent boundary layer. When the boundary-layer flow is turbulent over virtually the entire surface of the object being tested, turbulence in the stream may increase the friction above the level indicated by the upper line in Fig. 92, perhaps by as much as 15 per cent.

Total Drag.—Aerodynamic drag is always a summation of two parts—drag due to tangential components of pressure at the object's surface and that due to normal components, or, alternatively defined, frictional drag and eddy-making or turbulence drag. To adopt still another basis of definition, the first is the component of drag that proceeds from conditions in the boundary layer; the second, the component produced by a breakdown of streamline flow beyond the boundary layer.

In the practical problems of aircraft design both components are present in practically every case, but it frequently happens that one or the other is exceedingly small. On a wing at a small angle of attack the drag may be as much as 93 per cent frictional, and under conditions of high-speed flight the frictional component seldom falls below 75 per cent of the total. On a good streamline form or airship body, too, friction may account for substantially 90 per cent of the total, and a great problem of the aerodynamic design of airships in recent years has been to find forms that

would permit the reduction of fineness ratio, and so of exposed surface per unit of volume, without unduly increasing the eddy-making resistance. On a sphere the skin friction contributes only about 5 per cent of the total drag at low Reynolds' numbers, only about twice that above the critical range, and on flat plates at right angles to the wind its share of the total is virtually negligible.[1]

Scale effects on drag are of two principal kinds—the one due to the variation of frictional coefficient, the other to change in general flow structure and specifically in point of separation.

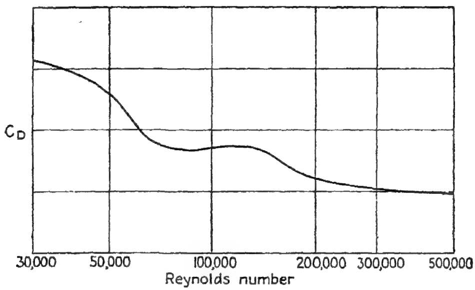

FIG. 93.—Irregular variation of drag coefficient with Reynolds' number.

Both, of course, are connected in their major and most spectacular manifestations with the change of conditions in the boundary layer. The influences of the two components of drag are of opposite sign, the appearance of turbulent flow tending to increase the friction and generally to decrease the eddy-making component through a delay of separation, but it is unusual for the two effects precisely to counterbalance each other. The change of separation point is as a rule more critical, in that it is limited to a smaller range of N, than the change of friction. The reason is obvious, the frictional coefficients continuing to increase from the point where the transition point in the boundary has just appeared at the trailing edge up to a Reynolds' number where it has arrived indefinitely close to the leading edge, while

[1] Yet even on flat plates it has been found that high stream turbulence may increase the drag by as much as 5 per cent. ("The Effect of Turbulence on the Drag of Flat Plates," by G. B. Schubauer, and H. L. Dryden, *Rept.* 546.)

separation is likely to be influenced only by the passage of the transition point across a critical region. The combination of the two components may produce a highly irregular curve from a Reynolds' number (for a solid of revolution) from 50,000 to 1,000,000 or thereabouts, as indicated in Fig. 93, an imaginary but by no means an unlikely case.

CHAPTER VII

AIRFOIL SECTIONS

The number of airfoil sections which have at some time been tested in wind tunnels, and among which the designer is able to choose, is very large. Of the numerous compilations of airfoil data that have been made, the most complete, kept up to date until about 1929 by the periodic issuance of new parts,[1] includes 859 sections. Allowing for the many sections that have been tested since the issuance of the last part, and for the many others for which the results have been adjudged of too little interest to justify inclusion even in the broadest compilation, or which have been the property of individuals or companies that have held them as confidential, the total number that have been designed and, sometime, somewhere, tried must be well over two thousand.

Historical Development.—In view of the existence of this great mass of material, too large to permit of study by a direct section-to-section comparison, some systematic method of procedure is necessary. In addition to showing something of the types of section characteristic of good modern practice and outlining their historical evolution, it is important that the effect of certain particular changes of form be studied in detail in order that the reasons for the course that airfoil development has taken may be understood and that the designer may act intelligently in modifying the form of an existing airfoil to secure specific changes in its characteristics, if that appears desirable.

The history of airfoil development is outlined in Fig. 94, where some typical sections are shown with the approximate dates of their introduction.

It will be observed that in the early days, as represented by the first pair of contours shown, but little attention was given

[1] "Aerodynamic Characteristics of Airfoils," Pt. I, *Rept.* 93; *ibid.*, Pt. II, *Rept.* 124; *ibid.*, Pt. III, *Rept.* 182; *ibid.*, Pt. IV, *Rept.* 244; *ibid.*, Pt. V, *Rept.* 286; *ibid.*, Pt. VI, *Rept.* 315.

to the structural desirability of a thick section. The R. A. F. 6[1] shows the beginning of a tendency to flatten the lower surface for increased spar depth and also for certain aerodynamic

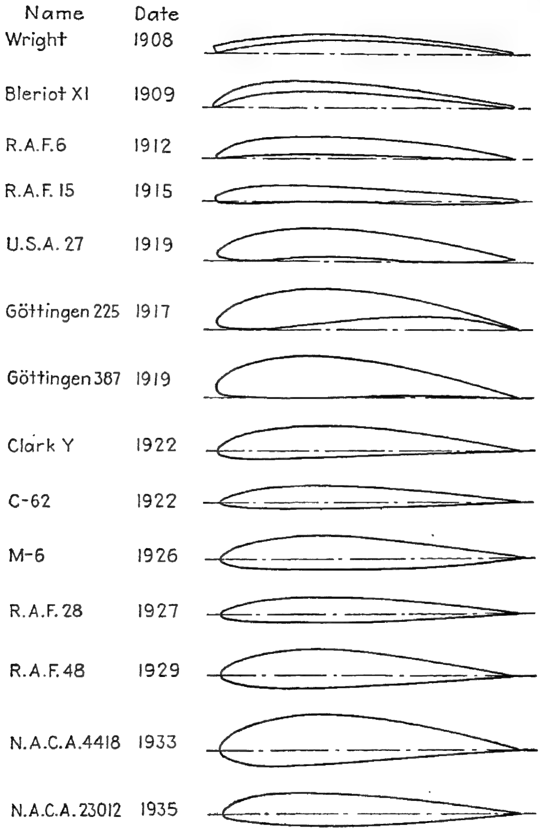

FIG. 94.—The historical development of airfoil sections.

advantages, while in the R. A. F. 15 that same tendency was carried still further, the lower surface being depressed directly under the probable spar locations and left concave between the spars—again with both structural and aerodynamic benefit,

[1] R. A. F. denotes Royal Aircraft Factory, the British experimental plant where the sections originated.

especially in the reduction of the drag at low angles of attack as a result of the reversal of curvature on the lower surface near the leading edge to give a smoother penetration of the air and create less turbulence. The R. A. F. 15, also, displays the first appreciation of that aerodynamic advantage of a rounding of the leading edge for which Theodorsen was later to provide a mathematical explanation.[1]

The R. A. F. 6 marked a transition from the old to the new in airfoil design. Up to that time sections had been laid out by the individual airplane designer; and if wind-tunnel tests were made prior to 1912, it was seldom on the initiative of the actual user of the airfoil. From 1912 on, however, the wind tunnel came to be recognized as the invaluable aid of the aeronautical engineer, and the initiative in airfoil work passed from the designer of airplanes to the laboratory staff. Since that time new sections have generally originated in the laboratory, and no designer has thought of employing a radically new form until after a wind-tunnel test has been made.

The R. A. F. 6 marked the beginning of a new era in another sense, too, for it was the first section to come into use on a number of airplanes of widely different types and to continue in use for a number of years. It was a brilliant improvement over anything that had previously been generally known, and as late as 1918 it was still being employed, especially on heavily loaded machines of moderate speed range such as the flying boats used during the last years of the war.

The R. A. F. 6 has now given way and would no longer be considered for employment in a new design, but its immediate successor in Fig. 94, the R. A. F. 15, has had a still longer and more honorable career. Designed and first tested in 1915, it continued in frequent use until about 1927 and indeed for airplanes having enough external wing bracing so that great thickness of the section is not necessary, and not intended for the very highest speeds, it seems even now to have no very marked superior. The R. A. F. 15, very slightly modified, was used on the DH-4, the backbone of the production program of the U. S. Army Air Service during the World War and of its service equipment for a number of years thereafter. In Great Britain

[1] See p. 73, *supra.*

it was probably employed on a considerable majority of all the types designed and constructed from 1917 to 1925, inclusive.

The next step was toward sections of lower efficiency and higher lift, a shift impelled rather by structural factors than by aerodynamic ones. Experience with the airplane in war had emphasized the desirability of a reduction of the maze of external bracing by struts and wires that had been characteristic of the designs of 1916—a reduction desirable both because of its beneficial effect on performance and for easier assembly, alignment, and maintenance of the machine. The larger the number of wires that have to be adjusted in a wing truss, the longer it will take to put the airplane together and make it ready for service.

Reducing the amount of external bracing implies an increase in the length of the unsupported spans between struts, an increase which may finally be carried to the length of cantilevering the wings from the fuselage and eliminating external bracing members entirely. The increase of unsupported span of course causes, other things being equal, a rise of the bending moment in the wing spars and makes it necessary that the spars be made deeper in order that they may be more efficient as beams. An attempt to cut down the number of wires leads inevitably to a search for thicker airfoil sections. German designers, led by Fokker, moved in that direction during the war; the French very soon after its end; the Americans several years later, the Italians later still; the British, as a whole, not until 1934.

The introduction of the thick sections started off along two somewhat different lines, both illustrated in Fig. 94. The U. S. A. 27 kept the same general form as the R. A. F. 15, but with all the ordinates approximately doubled, increasing the maximum height of the upper surface above the chord from 6.9 to 11.9 per cent of the chord. The Göttingen 225, on the other hand, a German war-time product, was of distinctly new form, with the lower surface convex over the first part of its length and deeply concave thereafter, and with the point of maximum thickness therefore, thrown far forward along the chord. That section, like many others of the long series tested at the Göttingen laboratory, was based on the mathematically derived Joukowski series,[1] and the lower surface form is that characteristic of

[1] See p. 71, *supra.*

Joukowski airfoils. Whatever its aerodynamic advantages may be, it is obviously structurally bad, except for a single-spar wing, as the rear spar in its normal location can be but little deeper than in an R. A. F. 15. It has the additional disadvantage, and this is at its very worst in a single-spar structure with its limited torsional strength, of having an exceptionally rapid center-of-pressure travel that throws a large negative twisting moment into the wing in high-speed flight. This it shares with all other airfoils having a deeply concave lower surface with its maximum camber far back on the chord. Later members of the Göttingen series therefore showed a modification of the lower surface, while keeping the deep leading edge rounded on a large radius. The Göttingen 387 illustrates the tendency.

Following the end of the war and the opportunity that it brought for engineers of the Allied nations to inform themselves on German aerodynamic development and vice versa, the two lines began to draw together again into one. It kept a general form midway between that of the Göttingen 387 and that of the U. S. A. 27, but the lower surface was flattened in the interest of better performance. The Clark Y, developed as the U. S. A. 27 had been by the well-known American designer Virginius E. Clark, best represented the trend. It probably shares with the R. A. F. 6 and the R. A. F. 15 the honors of having been used on more different types of airplanes, at least in the United States and in the British Empire, than any other three sections brought forward since flight began.

A collateral line of development was the search for improved sections of about the thickness of the R. A. F. 15, carried on with special vigor by the designers of racing biplanes. Where maximum-speed performance was virtually the sole interest, the airfoil could be judged entirely on its drag at the particular lift coefficient corresponding to the maximum speed. Flexibility and the preservation of a good efficiency over a wide range could be sacrificed, and the sharp leading edge again became permissible. The C-62 is a good type of the highly specialized high-speed section.

The next phase of evolution was the product of higher speed ranges and cantilever wings and of the demand that they created for reduced center-of-pressure travel, to keep down the torsional loads on the structure. The M-6, convex and with a strongly

reflexed trailing edge, was typical of the efforts to answer that problem. Since the appearance of the M-6 there have been no further changes of a general order. The story of American airfoil development since 1926 has been one of empirical refinement, striving for fractional reductions in drag or slight improvements in maximum lift without sacrifice elsewhere. Probably the most admirable of the products of the search has been the N. A. C. A. 23012, not very different from the Clark Y except in having the point of highest camber of the median line (or of maximum mean camber, as it will hereafter be referred to) considerably farther forward, and in having the mean camber itself somewhat smaller in magnitude. Reproduced with it, as another striking product of the last five years, is the N. A. C. A. 4418, typical of the best that has been done with a section in the extra-thick group.

British practice has followed very much the same course, except that the British have been last of the great aircraft-building nations to look with favor on the cantilever or semi-cantilever monoplane. Clinging to the wire-braced biplane, they clung also to the thin airfoil of which the R. A. F. 15 was an admirable type. In England as elsewhere, however, increased speed range impelled the designer toward the convex form with its superlatively low minimum drag. The R. A. F. 28 is an example, and the R. A. F. 48 is a still more recent development in sequel to a dawning British appreciation of possible virtue in the monoplane for other purposes than breaking speed records.

Present-day Sections.—The natural desire, at this point, is to give comparative data for a few of the sections displayed in Fig. 94. Unfortunately it cannot be done, or if it were the results would be misleading in the extreme. They have been tested at all manner of different Reynolds' numbers and under all manner of different conditions. Even though the Reynolds' number difficulty has been partly eliminated by the testing of a majority of the more popular airfoils in the variable-density tunnel of the N. A. C. A. and at $N = 3{,}000{,}000$ or thereabouts, those tests have been spread over such a length of time, and so many changes have been made in the tunnel installation and in the operating technique in the interim, that hardly any two sets of data can be considered as strictly comparable except where they were actually obtained in the course of a single series of

runs. Designers seeking the best available figures for use in their calculations can turn to the original sources or to one of the handbooks that exists for the express purpose of presenting compiled data.[1] In a text of general principle and general conclusion, the presentation of essentially incomparable data in a form that would be sure to invite the reader to attempt to compare them and so to lead himself into false generalizations has regretfully to be rejected as more potent for harm than for help.

In lieu of figures that cannot be offered because they could not be reliable for their present purpose, we must be content with qualitative approximations. Broadly speaking, the N. A. C. A. 23012 and the R. A. F. 48 are probably the best sections so far developed for all-round performance. The Clark Y is only slightly inferior. Where high speed is the only consideration, the M-6 belongs in the very first category, particular by virtue of the structural advantage of its upturned trailing edge and virtually stationary center of pressure. If more thickness than the R. A. F. 48's 15 per cent is needed (though that happens only near the roots of cantilever wings) the N. A. C. A. 4418 or a uniformly thickened modification of the N. A. C. A. 23012 is probably the best choice. Such sections as the R. A. F. 15 can be discarded on structural grounds, for under any conditions the margin of aerodynamic superiority of the best of the thin sections over the best of those in the medium-thickness group (Fig. 94) is at most so slender that it is completely overbalanced by the structural advantages on the other side. No designer, whatever the type and projected function of his airplane, need any longer hesitate to allow himself the luxury of a wing with a thickness ratio of 12 per cent or better.

To the general rule of incomparability of data there is one exception, and in Figs. 95, 96, and 97 full data are given in several forms for what is perhaps the best section of all and for

[1] "Aviation Handbook," by Edward P. Warner and S. Paul Johnston, New York, 1932; *Repts.* 93, 124, 182, 244, 286, and 315, *cit. supra;* "Collection of Wind-tunnel Data on Commonly Used Wing Sections," by F. A. Louden, *Rept.* 331; "Model Tests with a Systematic Series of 27 Wing Sections at Full Reynolds' Number," by Max M. Munk and Elton W. Miller, *Rept.* 221; "Design Information for Aircraft," *Aeronautics Bull.* 26, U. S. Dept of Commerce. Also *Rept.* 460, *cit. infra.*

one of its principal rivals. Both the N. A. C. A. 23012 and the Clark Y have been tested in the N. A. C. A. full-scale tunnel, at Reynolds' numbers up to over 4,000,000 and under conditions

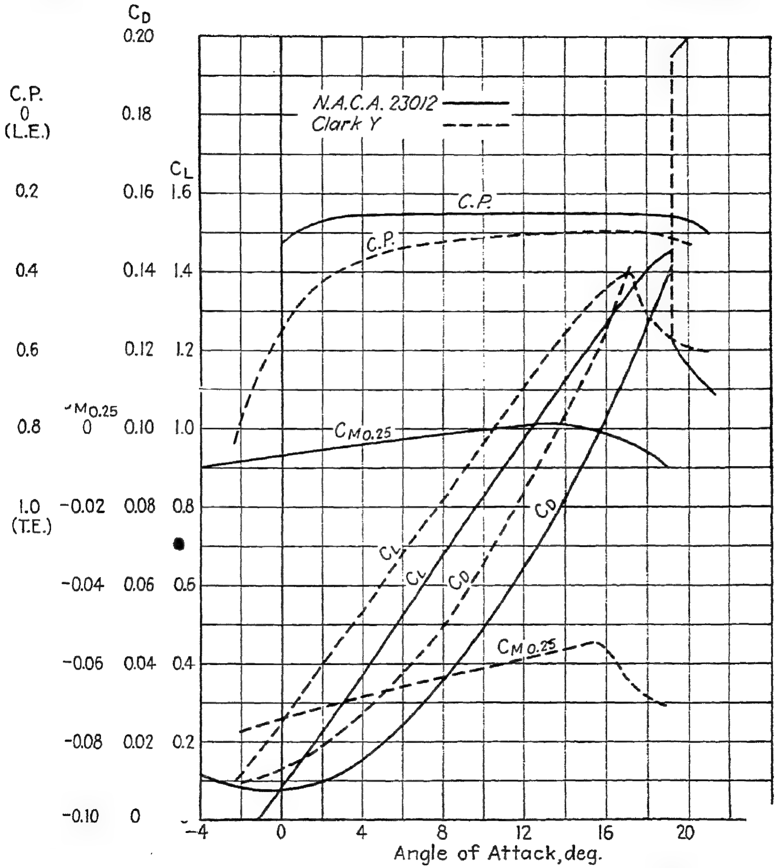

FIG. 95.—The characteristics of two representative airfoil sections.

designed to be strictly identical.[1] In the three illustrations just

[1] "Scale Effect on Clark Y Airfoil Characteristics from N. A. C. A. Full-scale Wind-tunnel Tests," by Abe Silverstein, *Rept.* 502; "Characteristics of the N. A. C. A. 23012 Airfoil from Tests in the Full-scale and Variable-density Tunnels," by Eastman N. Jacobs and William C. Clay, *Rept.* 530.

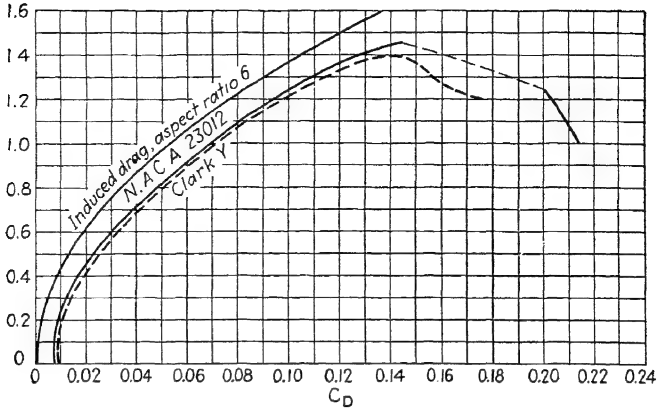

FIG. 96.—Polars of two airfoil sections.

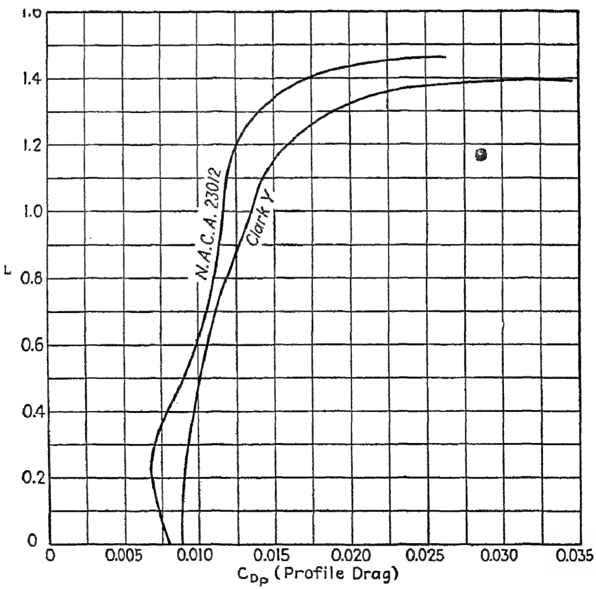

FIG. 97.—Profile-drag polars of two airfoil sections.

mentioned the figures for the two airfoils appear and reappear, plotted against angle of attack, plotted as a polar for an airfoil of aspect ratio 6 with the induced-drag parabola included, and with profile drag alone plotted against lift to permit of a larger scale.[1] In course of time some one of the great laboratories will no doubt put us in a position to superpose just such curves for a dozen of the best known and most used sections, with full confidence in their strict comparability, but they have not done it yet.

Though the aerodynamic characteristics cannot be directly compared, the geometrical ones can. They were shown directly in Fig. 94. In Figs. 98 and 99 they reappear, disguised, plotted in an indirect and somewhat peculiar but highly effective

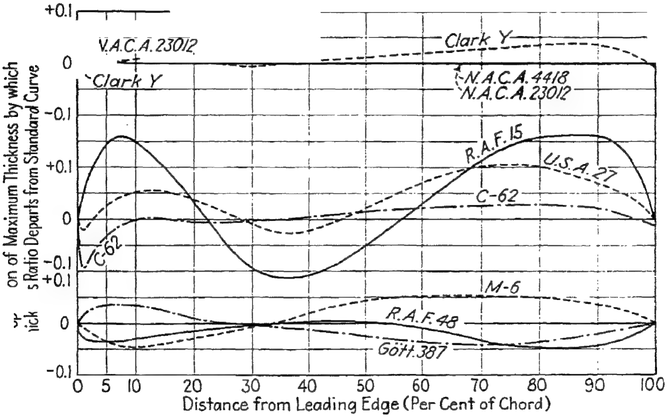

FIG. 98.—Thickness ratios of typical airfoil sections.

fashion. The contour of any section may be completely defined either by a tabulation of the ordinates of its two surfaces at a number of stations along the chord or, more conveniently for direct comparison of sections and analysis of the differences among them, by treating separately (1) the maximum thickness

[1] The induced drag has been calculated with allowance for uniform airfoil chord from tip to tip of the wing. It is thus about 5 per cent larger than for a wing of the same aspect ratio with an elliptical loading curve. All the data given in Figs. 95 to 97 have been corrected for wind-tunnel wall effect, which in a closed throat with a model having a span of half the tunnel diameter amounts to a reduction of the measured drag at each point by about 12 per cent of the induced drag at that same point.

of the section; (2) the maximum camber of its median line, or line midway between the upper and lower surfaces; (3) the form of the curve showing the relative thickness variation along the chord; (4) the form of the curve showing, similarly, the variations

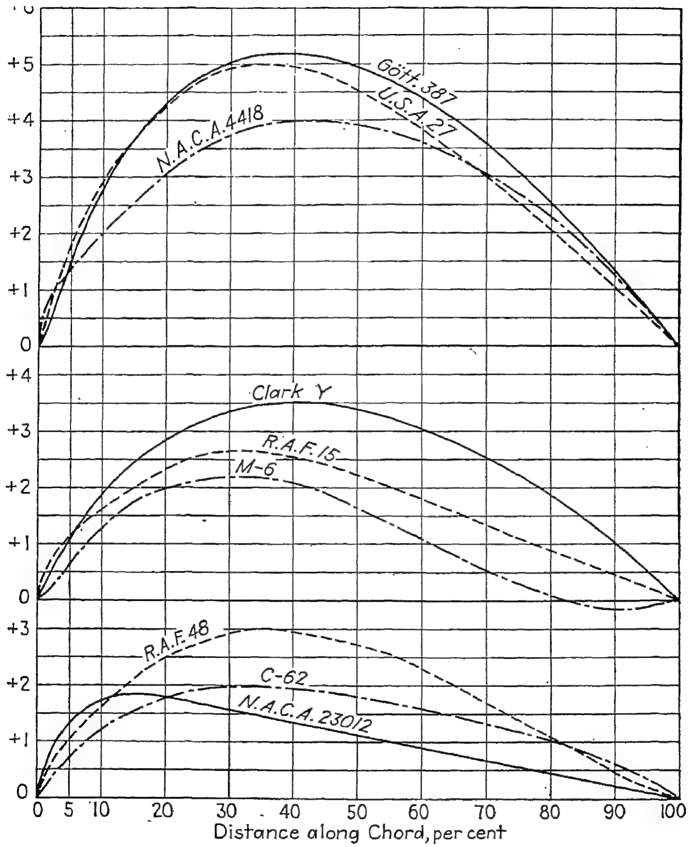

FIG. 99.—Camber ratios of typical airfoil sections.

of camber of the median line. It is convenient also, though not essential to a complete specification, to include (5) the coordinate, along the chord, of the point of maximum thickness; and (6) the similar coordinate for the point of maximum mean camber.

(1), (2), (5), and (6) are tabulated on page 172 for nine of the most interesting sections. (3) and (4) might, of course, be plotted directly, but small differences in form become much more apparent if the curve of mean camber be traced on a fore-shortened scale greatly exaggerating its vertical coordinates (a ratio of exaggeration of 10 to 1 has been used in this case), and if that for thickness be still further magnified by plotting the amounts by which the ratios of the thicknesses at the several points along the chord to the maximum thickness differ from the corresponding ratios at the corresponding points for some mathematically definable standard. In this case the standard selected is one extensively used by the N. A. C. A. in developing

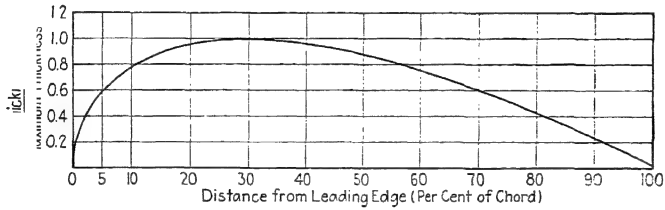

FIG. 100.—Standard airfoil-thickness distribution curve.

a series of airfoils of which we shall hear more later. It is plotted in Fig. 100, its ordinates are tabulated in Table III, and it is defined by the equation[1]

$$\frac{y}{y_{\max}} = 2.968\sqrt{x} - 1.260x - 3.515x^2 + 2.842x^3 - 1.015x^4 \quad (48)$$

where x is the distance from the leading edge and given as a fraction of the chord.

Not only is this the best graphical process for making a minute comparison of the forms of various sections; it is the most effective process for checking the fairness of their contours as well. The fairness of the section defined by the equation and the smoothness of its variation of curvature from nose to trailing edge are, of course, perfect, since the function and all of its derivatives are continuous. The curve of differences between

[1] "The Characteristics of 78 Related Airfoil Sections from Tests in the Variable-density Wind Tunnel," by Eastman N. Jacobs, Kenneth E. Ward, and Robert M. Pinkerton, *Rept.* 460.

one fair curve and another of the same general form should itself be fair. If the enormously exaggerated curve of differences plotted as specified in the previous paragraph turns out to be

TABLE III.[1]—ORDINATES FOR STANDARD CURVE OF THICKNESS DISTRIBUTION ALONG CHORD

Distance from leading edge, percentage of chord	Thickness / Maximum thickness
0	0
1.25	0.3156
2.5	0.4307
5.0	0.5923
7.5	0.6998
10.0	0.7802
15.0	0.8906
20.0	0.9560
25.0	0.9899
30.0	1.0000
40.0	0.9669
50.0	0.8820
60.0	0.7603
70.0	0.6105
80.0	0.4371
90.0	0.2412
95.0	0.1344
100.0	0.0210

[1] Taken from *Rept.* 460.

smooth in form, then the airfoil contour to which it relates must itself be smooth—as it always should be, for best results.[1] The method is somewhat confusing at first sight. To help in establishing it as a familiar tool, the particular case of the R. A. F. 15

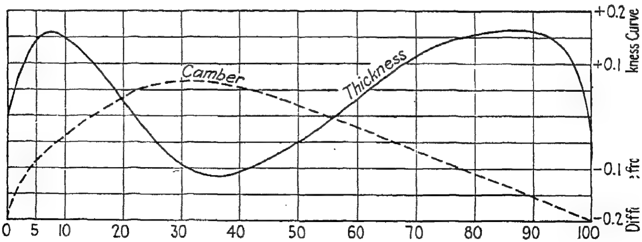

FIG. 101.—Thickness ratio and camber ratio of the R. A. F. 15.

[1] For a more elaborate discussion of this whole subject, see "The Fairing of Airfoil Sections," by Edward P. Warner, *Tech. Note* 146.

has been worked out through a tabulation in Table IV, with the results plotted in Fig. 101.

TABLE IV.—DERIVATION OF MEAN CAMBER CURVE AND DIFFERENTIAL THICKNESS CURVE FOR R. A. F. 15

Distance from leading edge, percentage of chord	Upper camber	Lower camber	Mean camber	Thickness	Thickness / Max. thickness	Difference from standard
0	0	0	0	0	0	0
1.25	1.65	−0.73	0.46	2.38	0.364	+0.048
2.5	2.46	−0.98	0.74	3.44	0.525	+0.094
5.0	3.53	−1.29	1.12	4.82	0.737	+0.145
7.5	4.21	−1.44	1.385	5.65	0.860	+0.160
10.0	4.64	−1.43	1.605	6.07	0.928	+0.148
15.0	5.25	−1.24	2.005	6.49	0.992	+0.101
20.0	5.57	−0.86	2.355	6.43	0.985	+0.029
30.0	5.61	−0.31	2.65	5.92	0.907	−0.093
40.0	5.35	−0.26	2.545	5.61	0.859	−0.108
50.0	4.91	−0.51	2.20	5.42	0.830	−0.052
60.0	4.36	−0.83	1.765	5.19	0.796	+0.036
70.0	3.68	−1.05	1.315	4.73	0.725	+0.115
80.0	2.86	−1.01	0.925	3.87	0.591	+0.154
90.0	1.81	−0.79	0.51	2.60	0.398	+0.157
95.0	1.20	−0.65	0.275	1.85	0.284	+0.154
100.0	0	0	0	0	0	−0.021

Geometrically, the most surprising thing about this group of sections, selected from many sources as they were, is their astonishing uniformity of thickness distribution. With the sole exception of the R. A. F. 15, all have their maximum thickness at between 27 and 34 per cent of the chord. With the exception of the R. A. F. 15 and the U. S. A. 27, for which the curve shows very clearly the arbitrary thickening in the neighborhood of 15 and 65 per cent of the chord in order that the spar depth might be increased, the thickness everywhere lies within 6 per cent of its maximum of the standard curve. Within that range, however, the curves vary widely in detail.

In distribution of camber there is no such uniformity. It scatters infinitely, its maximum ranging from 15 to 70 per cent

of the chord (and from 15 to 40 per cent for sections of the very highest standard of all-round performance). A curious note here is the comparative lack of smoothness of the mean camber curves even for such excellent airfoils as the R. A. F. 15 and the N. A. C. A. 23012, both of which show abrupt changes of radius of curvature and protracted flat spots.

TABLE V.—GEOMETRICAL CHARACTERISTICS OF AIRFOIL SECTIONS

Airfoil	Maximum thickness, percentage of chord	Location of maximum thickness, percentage of chord from leading edge	Maximum mean camber, percentage of chord	Location of maximum mean camber
R. A. F. 15	6.4	17	2.65	30
C-62	8.0	32	2.0	34
U. S. A. 27	11.1	26	5.0	36
Clark Y	11.7	30	3.5	40
N. A. C. A. 23012	12.0	30	1.85	15
M-6	12.2	33	2.2	31
R. A. F. 48	15.0	32	3.0	36
Göttingen 387	15.2	28	5.2	38
N. A. C. A. 4418	18.0	30	4.0	40

Scale Effect.—Most of the airfoil data now used are taken from variable-density or full-scale wind tunnels at Reynolds' numbers of between 3,000,000 and 6,000,000. We have already made note of the effect of time of test and detail of testing method on results. Even the latest of variable-density-tunnel figures, however, cannot be treated as perfectly comparable with those of free flight at the same Reynolds' number. The abnormally high turbulence inseparable from the enclosure of the complete testing system in a tank, with the consequent rapid recirculation of the air through the return passages, increases the drag appreciably. It has the further effect of increasing the effective Reynolds' number in respect of its influence on maximum lift, as explained in the previous chapter.[1] The standard test in

[1] Pp. 138, 145, *supra*.

the N. A. C. A. variable-density tunnel, run at a Reynolds' number of 3,000,000, gives a minimum drag coefficient from 0.0005 to 0.0010 higher than can be found in the less turbulent full-scale tunnel at the same Reynolds' number, a maximum lift such as would be obtained in a free stream only at a Reynolds' number of about 8,000,000. The actual working range of N on present-day aircraft is from 1,000,000 to 7,000,000 in landing conditions, from 2,500,000 to 25,000,000 at maximum speed.

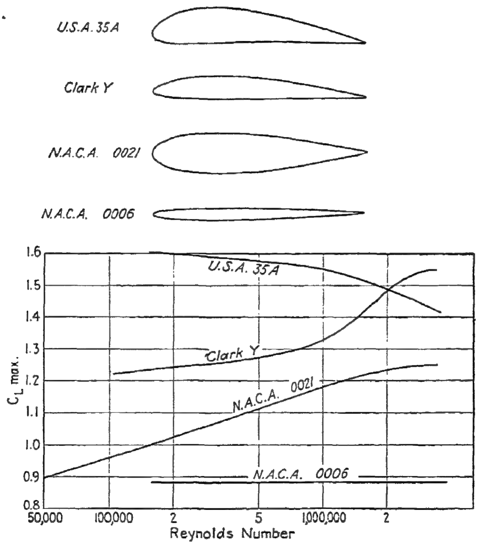

FIG. 102.—Typical modes of variation of maximum lift with Reynolds' number.

The maximum-lift measurements from the variable-density tunnel are then literally "full Reynolds' number," and for small airplanes or those with a low landing speed they are considerably more than that and some care must be exercised in using them. For an airfoil of which the $C_{L_{max}}$ increases rapidly with N, an appropriate downward revision (seldom exceeding 6 per cent) ought to be made in such a case. Unfortunately there is no general rule to cover the matter, and the theoretical discussion in the previous chapter provides hardly even qualitative guidance.

The groups of curves brought together in Figs. 102 and 103 serve to suggest some possible modes of variation of $C_{L_{max}}$, and also to show how great a variety of form of lift curve near the peak is likely to be encountered—a factor which has an important relation to scale effect on lift. The best generalizations that can be offered here are (1) that maximum lift tends to increase very slowly, or actually to diminish, with increase of Reynolds' number for airfoils having a large mean camber and a high maximum lift at low values of N; (2) that for airfoils

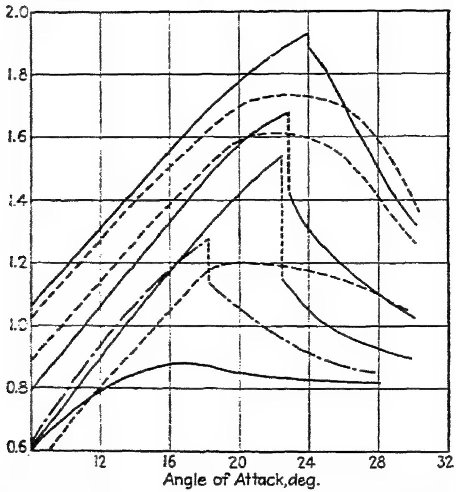

FIG. 103.—Typical modes of variation of lift with angle of attack in the neighborhood of the maximum.

ordinarily considered as of low maximum-lift type, on the other hand, the increase with Reynolds' number is rapid; (3) that the spread of maximum lift, from highest to lowest, is therefore much less under full-scale conditions than in model tests at low Reynolds' numbers; (4) that in more general terms it is the tendency for maximum lift first to decrease slowly with increasing Reynolds' number, then to increase, considerably more rapidly, and finally to level off at a substantially constant value;[1] (5) that the region of positive slope of the curve of $C_{L_{max}}$ against N apparently may start anywhere between $N = 50{,}000$ and

[1] See p. 145, *supra*.

$N = 8{,}000{,}000$ and may stop at as low a Reynolds' number as 3,000,000 or may continue up to Reynolds' numbers so high as so far to have been beyond the reach of direct measurement; (6) that maximum lift is generally encountered at a somewhat larger angle of attack at large values of N than at low,[1] though exceptions to this are frequent; (7) that a lift curve that has a smoothly rounded peak at low Reynolds' numbers commonly tends to sharpen that peak at high ones, the drop of lift beyond the maximum becoming increasingly abrupt; (8) that, on the other hand, absolute discontinuities in the lift curve, due to an instantaneous change in the type of flow and point of separation, are commoner at the lower values of N, though by no means unknown even when N is 3,000,000 or more.

For drag, at least a crude approximation can be made in a more generalized form. N. A. C. A. variable-density-tunnel data can be used directly as plotted for Reynolds' numbers up to 3,000,000. Beyond that point the drag coefficients may be gradually reduced, by 0.0008 when $N = 10{,}000{,}000$ and 0.0013 when it is 20,000,000. Data taken at a Reynolds' number of around 5,000,000 in the N. A. C. A. full-scale tunnel can be made subject to correction downward by 0.0004 at 10,000,000 and 0.0009 at 20,000,000. For the British compressed-air tunnel, which apparently has a less turbulent flow than the American one, the same corrections can be used as for the N. A. C. A. full-scale installation.

A factor of scale effect that makes little trouble if N is kept always above 1,500,000, but that becomes very important if it is necessary to apply small-scale data, is the change of shape of the profile drag curve. At low Reynolds' numbers separation takes place at a much smaller lift coefficient than at large ones. Also, there is separation on the lower surface at a small negative angle of attack, often above that of zero lift. Separation means high profile drag, and Fig. 104 shows the effect in a rather extreme case. In minimum drag, which approximates the skin-friction rule, there is but little difference between the two curves. At lift coefficients above 1.0 or below 0.3 the difference is enormous. Even the U. S. A. 27, however, flattens its profile drag curve out very nearly to its ultimate form before N reaches 1,000,000. Generally speaking, although the profile drag at very low lift

[1] As explained on p. 145, *supra*.

coefficients or at high ones may vary with N a little more rapidly than the correction factors recommended on the previous page would indicate, the departure is not great enough or consistent enough to justify any attempt at a more elaborate scheme of adjustment or any worry over the errors that may result from using the simple standard subtractive corrections just proposed—so long, that is, as N stays safely in the millions. As verification,

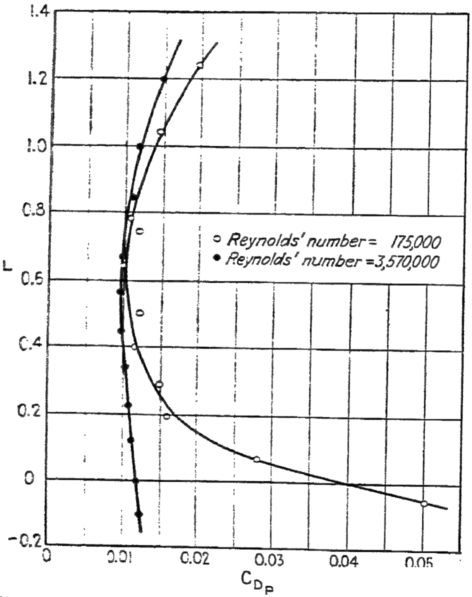

FIG. 104.—Profile-drag curves for the U. S. A. 27 at two different Reynolds' numbers.

Fig. 105 gives the full-scale-tunnel profile-drag curves for the Clark Y at Reynolds' numbers of 1,100,000 and 5,800,000. At negative lift coefficients, the ordinary relationship seems in that case to be reversed, but from $C_L = 0.1$ to $C_L = 0.9$ the curves are parallel within the range of accuracy that one may hope for in scale-correction work. The wide divergence at very high lift coefficients corresponds to a difference in maximum lift in the two cases, but it has already become plain that in the immediate neighborhood of $C_{L_{max}}$ no standard rules can apply.

More on Airfoil Criteria.—The general similarity of form of the profile-drag curves for good airfoil sections, combined with the known analytical form of the induced drag, suggests at once the possibility of fitting an analytical approximation

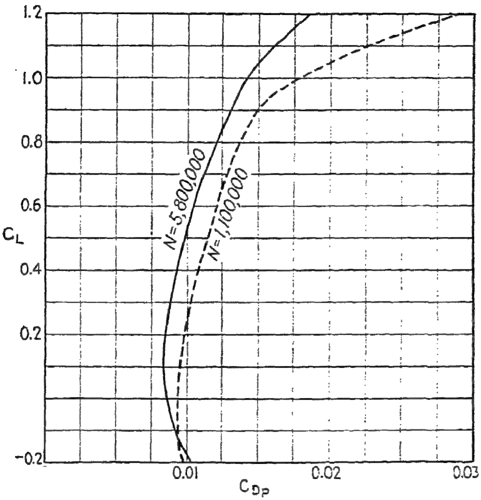

FIG. 105.—Profile drag of the Clark Y at two different Reynolds' numbers.

to the experimental data. The simple curve best adapted to the purpose is a cubic[1] of the form

$$C_{D_p} = C_{D_{p_0}} \quad 1 + \frac{C_L}{2} \qquad (49)$$

$C_{D_{p_0}}$ being the minimum coefficient of profile drag.

Then, adding in the induced component, the total drag is

$$C_D = \frac{C_L^2}{\pi R} + C_{D_{p_0}}\left(1 + \frac{C_L^3}{2}\right) \qquad (50)$$

an empirical expression that proves very closely adherent to the results of most tests taken at high Reynolds' numbers.

[1] A variety of more complex forms, using fractional exponents or transferring the origin elsewhere than to the point of zero lift, have been suggested. But simplicity is a transcendent virtue at this point, and it justifies a certain imperfection of approximation.

Both for the N. A. C. A. 23012 and for the Clark Y, for example, it fits the experimental curve to within 0.0012 for the drag coefficient at all lift coefficients up to 1.0.

On this basis, the ratio of drag to lift is

$$\frac{D}{L} = \frac{C_L}{\pi R} + \frac{C_{D_{P_0}}}{C_L} + \frac{1}{2} C_{D_{P_0}} C_L^2 \qquad (51)$$

Differentiating with respect to lift, and setting the derivative equal to zero to find the point where the function is a maximum, C_L for best D/L is found to depend on the equation

$$\frac{1}{\pi R} = \frac{C_{D_{P_0}}}{C_L^2} - C_{D_{P_0}} C_L \qquad (52)$$

Applying the same process to the criterion of minimum power requirement, $C_D/C_L^{3/2}$, we get

$$\frac{C_D}{C_L^{3/2}} = \frac{C_L^{1/2}}{\pi R} + \frac{C_{D_{P_0}}}{C_L^{3/2}} + \frac{1}{2} C_{D_{P_0}} C_L \qquad (53)$$

which attains its maximum when

$$\frac{1}{2\pi R C_L^{1/2}} + \frac{3}{4} C_{D_{P_0}} C_L^{1/2} = \frac{3}{2} \frac{C_{D_{P_0}}}{C_L^{5/2}} \qquad (54)$$

or

$$\frac{2}{\pi R} + 3 C_{D_{P_0}} C_L = 6 \frac{C_{D_{P_0}}}{C_L^2} \qquad (55)$$

The implications that these formulas can carry will be further examined in the next chapter. For the moment it is of more interest to consider not merely the minimum ratio of drag to lift, but the ratio that would exist at every value of the lift coefficient; first, if the empirical formula for drag were rigorously followed, and, second, if there were various departures from that formula. That corresponds to the general practical problem. The speed of flight, together with the wing loading and the air density, determines the lift coefficient at which the wings of an airplane must operate. The ratio of drag to lift at that coefficient then determines the drag, and the best wing at any speed is the one that gives the lowest values of D/L at the particular lift coefficients that are of most interest in the performance of that particular type of airplane.

To avoid the necessity of making a sheaf of curves or a long tabulation for various values of $C_{D_{p_0}}$, the single standard value of 0.0080 has been chosen. The curve that corresponds to the standard can be constructed from (49), and if great refinement in the comparison of a group of sections is sought their profile drags may be plotted as differences from the standard, precisely as departure of geometrical thickness distribution from an analytical standard have been plotted. Nothing else is so effective in making particular points of strength or weakness stand out. The difference curves have been plotted in Fig. 106 for the N. A. C. A. 23012 and the Clark Y, and their relative virtues, and especially the remarkable efficiency of the 23012 around $C_L = 0.25$, become even more conspicuous than before.

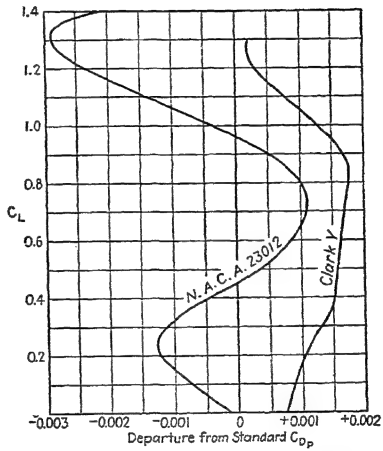

FIG. 106.—Variations from "standard" profile drag for two airfoil sections.

D/L and $C_D/C_L^{3/2}$ need only to be multiplied by the weight of the airplane to give the total drag and the power required (an additional constant having to be multiplied in in the latter case), respectively. Being linear functions of C_D, the change of their magnitude with a given change of C_D is, at any particular value of C_L, invariable. The standards of profile drag for $C_{D_{p_0}} = 0.0080$, of induced drag for aspect ratio 6, of the corresponding values of D/L and of $C_D/C_L^{3/2}$, and of the change in each that results from changing C_D by 0.001 have been tabulated on page 181 and curves for most of the tabulations are plotted in Fig. 107. The use is best shown by example. Take, for instance,

[1] The induced drag in this case has been computed for elliptic loading, on the presumption that in practice the wing tips will always be rounded or raked enough to make the elliptic assumption at least as valid as the rectangular one. In no case, with a wing tip designed with even reasonable care, ought the induced drag to exceed the values here tabulated by more than 3 per cent. In many cases the excess would be negligible.

the conditions at $C_L = 0.4$. The "standard" values for D/L and $C_D/C_L^{3/2}$ are 0.0420 and 0.0664 respectively. The N. A. C. A. 23012 has a profile drag at the selected lift coefficient that is 0.0005 below the standard value. D/L is then reduced by 0.5 of the figure standing opposite the lift coefficient 0.4 in the

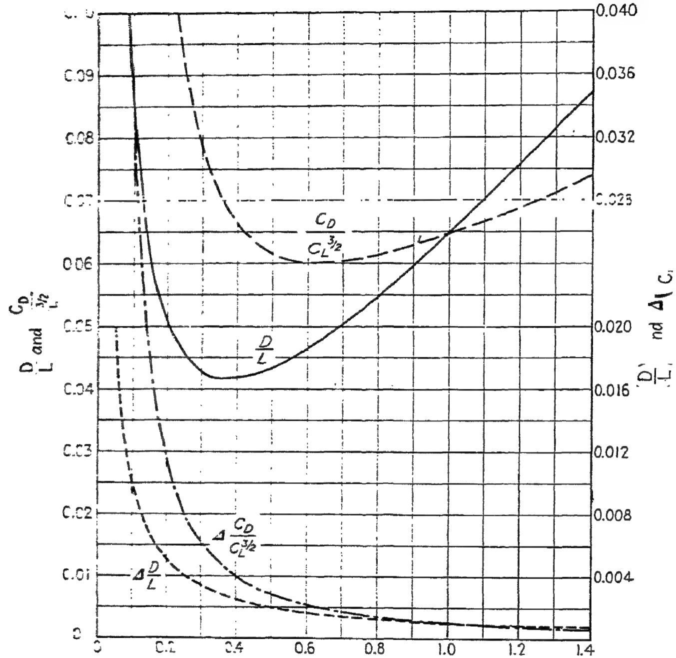

FIG. 107.—Relations between C_L, C_{Dp}, D/L, and $C_D/C_L^{3/2}$, for airfoil of aspect ratio 6.

column $\Delta(D/L)$ or by 0.5×0.0025, bringing it down to 0.0408. $C_D/C_L^{3/2}$, in precisely the same fashion, is found to be 0.0644. The profile drag for the Clark Y at the same lift being 0.0015 above standard, the corresponding values of D/L and of the power coefficient for that section are 0.0458 and 0.0724.

Systematic Study of Variations in Airfoil Form.—One might wander almost eternally in the great and swiftly propagating forest of airfoil data without perceiving form or pattern there. If the area is to be mapped and furnished with a guidebook; if

its segments of greatest fertility are to be discovered; if the study of airfoils is to resolve itself into anything more than a virtual infinity of statements that section A is good and section B is not so good—random wandering must give way to carefully planned

TABLE VI.—WING CHARACTERISTICS WHEN PROFILE DRAG = $0.0080\left(1+\frac{C_L{}^3}{2}\right)$

C_L	C_{D_p} (standard)	C_{D_i} (for aspect ratio 6)	$C_{D_{total}}$	$\frac{D}{L}$ (standard)	$\left(\frac{C_D}{C_L{}^{3/2}}\right)$ (standard)	Changes when C_{D_i} is changed by 0.001 from standard: $\Delta\left(\frac{D}{L}\right)$	$\Delta\left(\frac{C_D}{C_L{}^{3/2}}\right)$
0	0.0080	0	0.0080				
0.05	0.0080	0.0001	0.0081	0.1620	0.724	0.0200	0.0895
0.10	0.0080	0.0005	0.0085	0 0850	0.269	0.0100	0.0316
0.15	0.0080	0.0011	0.0091	0.0607	0.1566	0.0067	0.0173
0.20	0.0080	0.0021	0.0101	0.0505	0.1128	0.0050	0.0112
0.25	0.0081	0.0033	0.0114	0.0456	0.0912	0.0040	0.0080
0.30	0.0081	0.0047	0.0128	0.0427	0.0779	0.0033	0.0060
0.35	0.0082	0.0064	0.0146	0.0417	0.0705	0.0029	0.0049
0.40	0.0083	0.0085	0.0168	0.0420	0.0664	0.0025	0.0040
0.45	0.0084	0.0107	0.0191	0.0424	0.0632	0.0022	0.0033
0.5	0.0085	0.0132	0.0217	0.0434	0.0613	0.0020	0.0028
0.6	0.0089	0.0190	0.0279	0.0465	0.0600	0.0017	0.0022
0.7	0.0094	0.0259	0.0353	0.0504	0.0603	0.0014	0.0017
0.8	0.0100	0.0338	0.0438	0.0548	0.0613	0.0012	0.0014
0.9	0.0109	0.0427	0.0536	0.0596	0.0629	0.0011	0.0012
1.0	0.0120	0.0528	0.0648	0.0648	0.0648	0.0010	0.0010
1.1	0.0133	0.0639	0.0772	0.0702	0.0668	0.0009	0.0009
1.2	0.0149	0.0760	0.0909	0.0757	0.0690	0.0008	0.0008
1.3	0.0168	0.0892	0.1060	0.0815	0.0712	0.0008	0.0007
1.4	0.0190	0.1035	0.1225	0.0875	0.0739	0.0007	0.0006

survey. And the obvious basis of survey is a controlled analysis of the consequences of varying particular geometrical characteristics of an airfoil, one at a time.

To report the fact that over a period of almost twenty years there was no serious attempt to make such a survey is to record an extraordinary lacuna in aerodynamic effort, a curious blind spot in those responsible for its direction. The official British wind-tunnel laboratory had made a start on it in 1911, laid it

aside within a couple of years thereafter for other work that promised quicker returns and seemed to have greater urgency of practical application to design. During the succeeding decades there were many studies of so-called series of airfoils, the members of each series differing among themselves in respect of some single geometrical characteristic, but each series was as a rule so brief and the various series were so totally unrelated to each other in every respect (including that of the Reynolds' number at which the tests were run) that to put them all together and extract even the feeblest of general conclusions was a first-class problem in deduction. In 1924 Ackeret undertook at Göttingen[1] a program on a much more extended front, though one still far from complete. Not until 1931 was a really comprehensive investigation, under standardized conditions, started. Not until 1933 were its results made available to the public.[2] From the seventy-eight airfoils covered in that study, together with half a dozen more essentially of the same family that were included in a supplementary investigation,[3] it is at last possible to draw generalization. Though we may make occasional reference to particular series of tests previously undertaken, the N. A. C. A. researches of 1931 and the years since make everything else in the field essentially obsolete. Unique both by their scope and by the close approach to full-scale conditions that the use of the variable-density tunnel allowed, they lay the foundation of all future airfoil design.

The obvious geometrical characteristics of an airfoil, if no thought be given to the variables that have proven fundamental in developing the theory of airfoil action, are the form of the upper

[1] J. Ackeret, *Proc.* Wissenschaftliche Gesellschaft für Luftfahrt, 1924, translated in *Tech. Memo.* 323. The results were more completely reported, and analyzed and discussed, in "Systematic Investigation of Joukowsky Wing Sections," by O. Schrenk, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, May 28, 1927, translated in *Tech. Memo* 422; and in "Contribution to the Systematic Investigation of Joukowsky Profiles," by Gottfried Loew, *Zeitschrft für Flugtechnik und Motorluftschiffahrt*, Nov. 28, 1927, translated in *Tech. Memo.* 461.

[2] "The Characteristics of 78 Related Airfoil Sections from Tests in the Variable-density Wind Tunnel," by Eastman N. Jacobs, Kenneth E. Ward, and Robert M. Pinkerton, *Rept.* 460.

[3] "Tests in the Variable-density Wind Tunnel of Related Airfoils Having the Maximum Camber Unusually Far Forward," by Eastman N. Jacobs and Robert M. Pinkerton, *Rept.* 537.

surface and the form of the lower surface. Very naturally the original disposition was to treat the two separately, and most of the systematically developed series available up to the end of 1932, excepting only a few German and Polish experiments, followed the line of holding the form of one surface constant and varying the other. Particularly prevalent was the practice of varying the camber of the upper surface while keeping the lower one flat, and of series of that sort there have been a considerable number.[1]

Such tests have their value, particularly in connection with propeller design, but for general purposes it now appears far more rational to classify the basic geometrical characteristics of an airfoil in the fashion suggested on page 167. Of the six controlling characteristics there described, four of them numerical and the other two having to be defined by curves, the effects of (1), (2), and (6), maximum thickness, maximum mean camber, and location of point of maximum mean camber, have been exhaustively covered by the National Advisory Committee tests. A little, in somewhat scattered fashion, has been done on (3) and (4), the form of the curves of thickness distribution and of mean camber. Of (5), the effect of location of point of maximum thickness, there has rather curiously been virtually no direct treatment up to the present time. That gap ought to be filled in the near future. That it is a gap, and that we cannot too casually assume an inherent superiority for the standard Advisory Committee practice of locating the maximum thickness at 30 per cent of the chord from the leading edge, is suggested by the fact that as excellent an airfoil as the R. A. F. 15 has its maximum thickness barely half that far back.

Effect of Airfoil Thickness.—The relation between thickness and lift characteristics is summarized in Fig. 108, where the lift

[1] As, for example, "Determination of the Lift and Drift of Airfoils Having a Plane Lower Surface and Variable Camber, the Upper Surfaces Being Obtained by Varying the Ordinates in a Constant Ratio," *R. and M.* 60; "Airfoils for Airscrew Design," by W. L. Cowley and H. Levy, *R. and M.* 362; "Experiments at Two Speeds on Lift, Drag, and Center of Pressure in Aerofoils Suitable for Airscrew Design," by A. Landells, R. Jones, and F. J. E. China, *R. and M.* 152; "Experiments on Thick Wing Sections," *Bull.*, p. 184, Expt. Dept., Airplane Eng. Div., U. S. Army Air Service, December, 1918; "The Aerodynamic Properties of Thick Airfoils," Pt. II, by F. N. Norton and D. L. Bacon, *Rept.* 152. See also p. 201, *infra*.

curves have been plotted against angle of attack for airfoils of three different thicknesses and otherwise identical. Five conclusions are immediately apparent. (1) The maximum lift increases with thickness up to a certain point, then decreases,

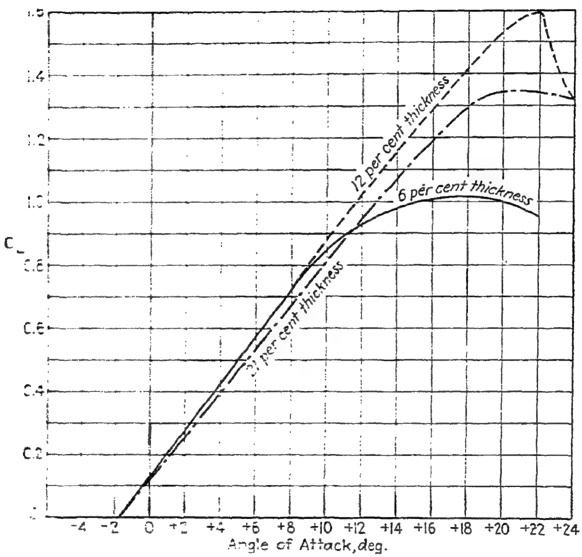

Fig. 168.—Typical variation of lift-curve form with airfoil thickness.

reaching a maximum at some intermediate thickness. (2) The form of the peak of the lift curve changes, from a very smoothly rounded shape and a very gradual drop of lift on the thinnest section to an absolutely discontinuous form on that of medium thickness and then back to a smooth curve again with still thicker sections (though never again to as large a radius of curvature at the peak of the curve as in the case of the thinnest forms). (3) The slope of the lift curve is generally diminished with increasing thickness, in at least qualitative accord with Theodorsen's conclusions,[1] though the diminution is slight. (4) The angle of attack for maximum lift tends to increase with thickness up to a

[1] See p. 74, *supra*.

thickness of about 12 per cent, thereafter to stabilize or to decrease slightly with further thickening. (5) The angle of zero lift, faithful to the theory that allows of its computation as a function of mean camber line alone,[1] is virtually independent of thickness.

All this is typical and can be taken as established without further remark except for some quantitative note on the first point. If maximum lift is plotted against thickness it proves to reach its absolute peak value for a thickness of from 9 per cent (for a series with 6 per cent mean camber) to 13 per cent (for a series uncambered and symmetrical). The progression from one value to the other is substantially uniform, the optimum thickness from the single point of view of maximum lift falling off steadily as the mean camber is increased. Moving the point of maximum camber farther back from the leading edge has the effect of increasing the optimum thickness slightly after the 40 per cent point is passed, a series with maximum camber at mid-chord showing its highest maximum lift with a thickness from 1 to 2 per cent greater than an otherwise identical series having the deepest camber at 30 per cent or at 40 per cent of the chord. The drop in maximum lift is rapid as the thickness is reduced to below its optimum value, much more gradual in response to an increase above that level. Quantitatively, a reduction of the thickness ratio to 4 per cent below the optimum value reduces the maximum lift by from 12 to 19 per cent (the larger effects with the least deeply cambered series), while an increase of similar degree above the optimum effects a reduction of maximum lift of from 3 to 6 per cent (in the reverse order of magnitude as among the various series). Much of all this is suggested by Fig. 109, where maximum-lift-thickness curves have been plotted for two extreme series.

The effect of thickness on minimum profile drag proves, as would be anticipated, unfavorable in every instance. Figure 110 shows just how unfavorable for three particular series, and it is a little surprising to find the influence of thickness so uniform. It might have been anticipated that for the most deeply cambered forms, where increase of thickness has the effect of diminishing the concavity of the lower surface, the improved smoothness of flow below the airfoil that would result would make up at least

[1] See p. 107, *supra*.

in part for the greater disturbance of the flow over the upper surface, and that thickness would accordingly have less adverse effect in such a case than in the case of a symmetrical series. Nothing of the sort appears, however, from the results of the

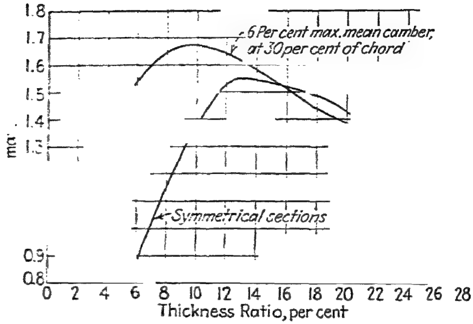

FIG. 109.—Relation of maximum lift to airfoil thickness.

tests, which indicate as a nearly enough uniform rule that every increase of 1 per cent in thickness increases the minimum profile drag by $4\frac{1}{2}$ per cent of the value that it had at the original thickness. Compounding, an increase of 6 per cent in thickness increases the minimum drag by 30 per cent.

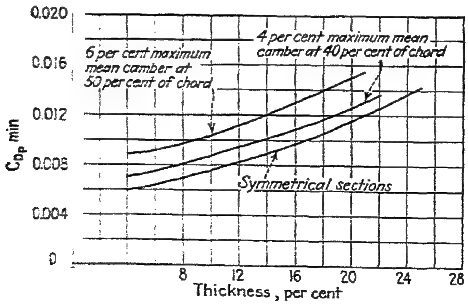

FIG. 110.—Relation of minimum profile drag to airfoil thickness.

There is, as Figs. 111 and 112 testify, a little more to it than that. The minimum profile drag prevails only over a limited range of lift coefficients, and the range varies from section to section. The curves of "departure from standard profile drag,"[1]

[1] See p. 179, *supra*.

plotted for two series in the figures just referred to, show how much narrower the range of maximum relative efficiency is likely to become for a very thin section than for one of moderate thickness. If an average is taken over the whole range of operating conditions the thin section has much less advantage than it would appear to have if the comparison were made on a basis of minimum profile drag alone. It still has something of a lead to its credit, however, a lead most particularly apparent in the 4 per

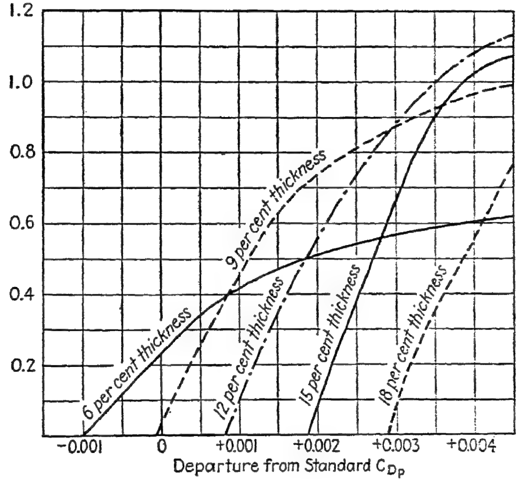

FIG. 111.—Variation of profile-drag characteristics with thickness; symmetrical airfoils.

cent camber series covered by Fig. 112, and the necessity of balancing its aerodynamic virtues against its structural and practical weaknesses still exists.

Pitching-moment coefficients sustain theoretical prediction[1] by showing themselves virtually independent of thickness. What little effect there is may be put aside for later discussion as a by-product of the very definite and primary dependence of moment on camber.

There remain three criteria of general efficiency, the variation of which with thickness calls for fleeting notice. Figure 113 records for two widely different series the relation between thick-

[1] See p. 109, *supra*.

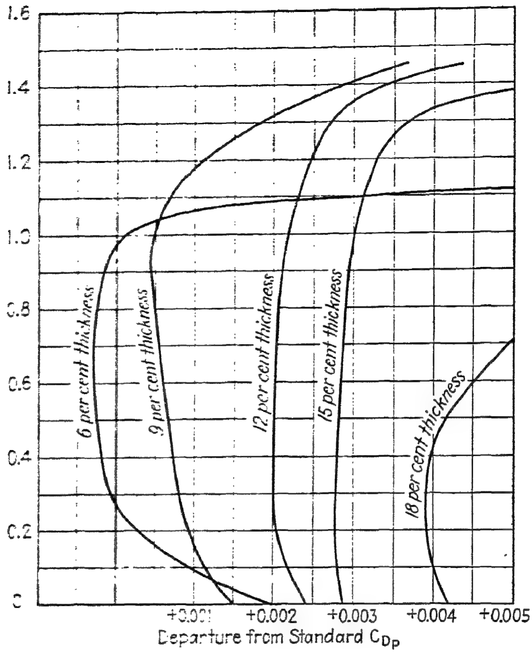

FIG. 112.—Variation of profile-drag characteristics with thickness; airfoils of the 4 per cent camber series.

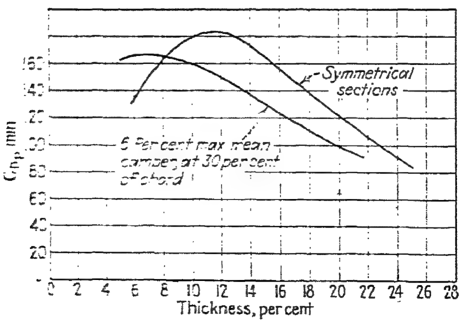

FIG. 113.—Relation of $C_{L_{max}}/C_{D_{p\,min}}$ to airfoil thickness.

ness and the ratio $C_{L_{max}}/C_{D_{p min}}$, once accepted as the best of indications of an airfoil's usefulness at high speeds, less significant now since variable-lift wings[1] have become a common feature of design, but still a useful index. Figure 114 does the same for D/L and (for a single series, the symmetrical) for $C_D/C_L^{3/2}$. For the first criterion, best results are to be had with a thickness from 7 to 12 per cent, the higher optimum values corresponding to the lower cambers. For the other two bases of comparison, indices of minimum thrust required to overcome total wing drag and of minimum power needed to push the wings

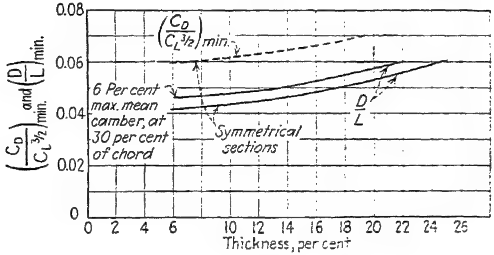

FIG. 114.—Relation of $(D/L)_{min}$ and $(C_D/C_L^{3/2})_{min}$ to airfoil thickness.

through the air, respectively, the thinner the section the better. In both cases, for both thrust and power, a 6 per cent thickness is approximately 6 per cent better than one of 12 per cent, while an increase to 18 per cent thickness increases both thrust and power to about 10 per cent above the 12 per cent thickness level.

Effect of Changes in Mean Camber.—Structurally, thickness is, of course, an airfoil's predominant characteristic. Aerodynamically, the performance is much more sensitive to changes in mean camber. It is camber that determines the angle of zero lift. It is camber that determines the moment characteristics and the center-of-pressure travel. So much is evident from theory. It might be foreseen as a matter of common sense, and it proves to be true in practice, that camber is the predominant element in fixing maximum lift, minimum drag, and point at which minimum drag occurs as well.

With the same general course pursued as when thickness was put under the lens, Fig. 115 has been plotted for the lift characteristics of four sections, all with a 12 per cent thickness ratio

[1] Treated in Chap. X, *infra*.

and differing only in camber. The four curves lie substantially parallel, the angle of zero lift changing approximately 2 deg. for every change of 2 per cent in camber. The maximum lift increases steadily as camber increases. The peak of the lift curve becomes progressively less sharp as increments are added to the camber, and with 6 per cent camber its curvature has become as smooth as that of the curve for a symmetrical section

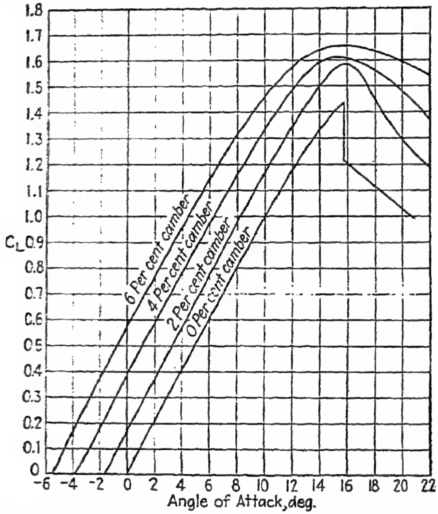

FIG. 115.—Typical variation of lift-curve form with change of maximum mean camber.

of half the thickness.[1] The first two points run in perfect harmony with anticipation, except that the rate of change of the zero-lift angle is about 5 per cent less than would be expected from the theory. The third seems at first sight like more of a surprise, for it would be expected that increasing camber would produce an increasing disturbance of the air flow and an increasing likelihood of discontinuities in the curves. Increasing camber brings with it, however, an increasing height of the upper surface and decreased radii of curvature of the contours there. With as smooth a basic form as was used in these tests and with the point

[1] See Fig. 108, *supra*.

of maximum camber as far back as it was in this particular series, changes of that order in upper-surface form seem likely to promote a gradual forward progress of the separation point as the angle increases, rather than an abrupt appearance of burbling and a sudden discontinuous movement of the separation point from a location near the trailing edge to one far advanced on the chord. The validity of that line of reasoning to account for the observed phenomena will become more apparent in examining the effects on the lift curve of a shift in the location of the point of maximum camber.[1]

The relation of maximum lift to mean camber appears in Fig. 116, where curves have been plotted for a number of series. To look at the curves is to feel some discouragement over the prospects of generalization, for they seem to have little in common. For sections of great thickness, and especially for those with the point of maximum camber forward on the chord, the maximum lift is virtually independent of the camber. As the thickness diminishes, or as the point of maximum camber on a fairly thick section moves back more than 40 per cent of the chord from the leading edge (variations of position within the first 40 per cent of the chord making but little difference), the camber becomes more and more important. With a 6 per cent thickness ratio, the section with 6 per cent camber has 65 per cent more maximum lift than the symmetrical one, and both for

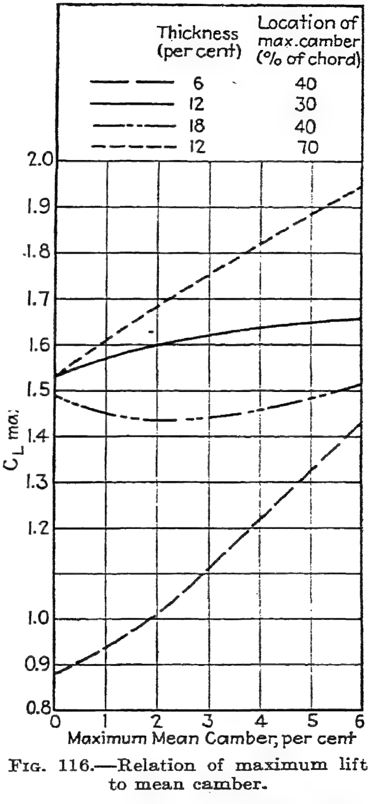

FIG. 116.—Relation of maximum lift to mean camber.

[1] See p. 205, *infra*.

that series and for the 12 per cent thickness series with the maximum camber located 70 per cent of the way back the maximum lift appears to be rising without limit.

That it cannot, in fact, do so is easy to see. There must be some restrictions on the sharpness of curvature possible on the upper surface without causing separation even at small angles, and the greater the camber the sharper the curvature becomes. It is unfortunate that the National Advisory Committee tests were not carried far enough to provide specific data on the conditions that produce an absolute maximum, but Ackeret's tests[1] went considerably further. They went, in fact, to the fantastic length of including an airfoil of 15 per cent thickness and 40 per cent mean camber (more like a turbine blade than a wing section), as shown in Fig. 117. For the particular series used by Ackeret the maximum lift was still increasing even up to that point, as per Fig. 118, but the curve is finally rounding over and showing signs of reaching a maximum at around 45 to 50 per cent camber.

Fig. 117.—Airfoil of 40 per cent mean camber, tested by Ackeret.

Figure 118 is smooth enough in form, but the individual curves that supply its points are not so tractable. Characteristic of abnormally deep cambers is an extreme irregularity of flow and a correspondingly irregular variation of lift and drag. The lift curves in Fig. 119, drawn from Ackeret's tests for one normal camber (5 per cent) and two abnormally deep ones (30 and 40 per cent), speak for themselves. Such eccentricities, which commonly begin to make their appearance at a camber of from

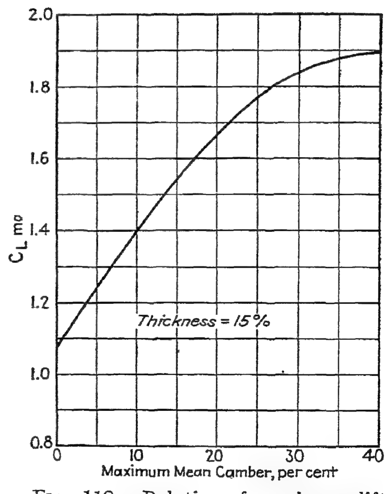

Fig. 118.—Relation of maximum lift to mean camber, up to 40 per cent camber.

[1] *Tech. Memo.* 422, *cit. supra.*

15 to 25 per cent, would be paralleled but probably not duplicated in a succeeding test of the same model. They express, in other words, not so much a real characteristic of the airfoil as a more or less complete indeterminancy of aerodynamic behavior and the possibility of finding a number of different lift curves on as many different trials. All would presumably be highly irregular, but very likely no two in exactly the same fashion, the precise nature of the flow and magnitude of the forces existing at any instant being dependent in part upon chance, in part upon the recent past history of the relation between the air stream and the model. If the angle of attack is increased slowly up to a certain value one result will be obtained; if it is increased rapidly to the same angle there will be another result; if, yet again, the same angle is reached by decreasing from a larger angle still another lift is likely to be found. These aberrations are of limited practical importance in aircraft design, but the knowledge of their existence helps to instill respect for the complexity of the subject and for the infinite variety of behavior of which a separating and turbulent air flow is capable.

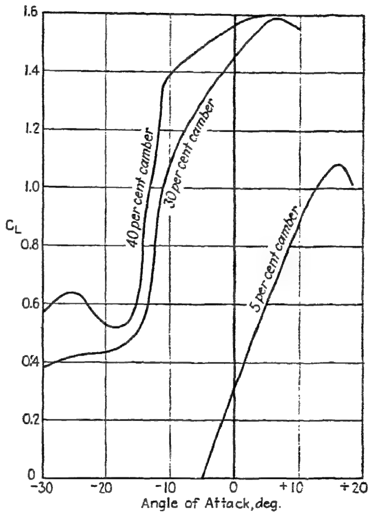

FIG. 119.—Lift curves for airfoils of abnormally high camber.

To revert for a moment to the question of the angle of zero lift, the rough rule can be laid down from these tests that the angle in degrees, referred to a chord connecting the extreme leading and trailing edges, is numerically equal to the deepest mean camber in percentage of the chord. Munk's graphical method for finding the angle makes it vary considerably with the location of the point of maximum camber, but the rougher rule just suggested seems almost as accurate and certainly close enough to the truth for all practical purposes. It gives the zero-lift angle correctly to within 1.0 deg. for every one of the seventy-eight

sections included in the National Advisory Committee's airfoil research, and within 0.6 deg. in every case but eight. All of the eight are abnormal either in having an exceptionally deep camber or in having its deepest point extraordinarily far back on the chord. For such sections as are normally used in design the one-for-one degree-for-percentage rule can be expected to tell the truth within half a degree in practically every instance.

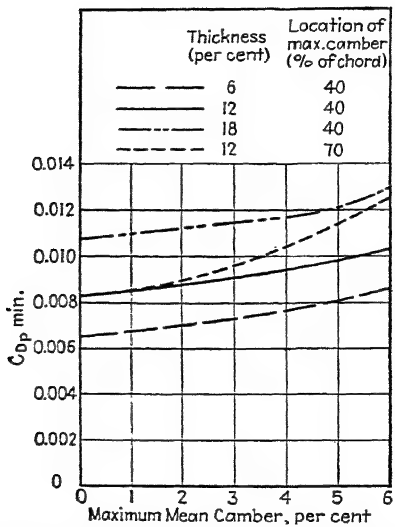

FIG. 120.—Relation of minimum profile drag to mean camber.

The effect of camber on drag is as pronounced as its effect on lift, and again it is true that the effect is one more on phase relationships than on magnitudes. The major influence on lift is a lateral shifting of the lift curve as a whole, with corresponding change of the zero-lift angle. The major influence on drag proves to be a vertical translation of the profile-drag polar, with a corresponding change of the value of the lift coefficient at which the minimum profile drag is found. Figure 120 shows the dependence of minimum profile drag on camber up to 6 per cent, for several series as tested by the National Advisory Committee, and Fig. 121 is drawn on a different scale to carry the same data out to much higher cambers from Ackeret's tests.

Up to a 10 per cent camber the increase of minimum drag with camber is comparatively slow, averaging about 30 per cent higher for an airfoil with 6 per cent camber than for a symmetrical section (except when the point of deepest camber is at mid-chord or farther back, in which case the sensitiveness of drag to camber is more marked). Beyond 10 per cent the increase is much more

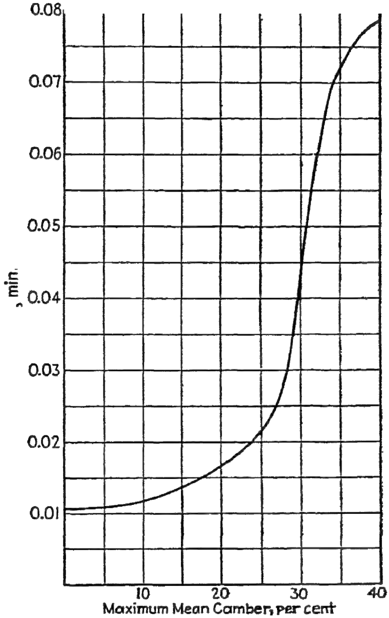

Fig. 121.—Relation of minimum profile drag to mean camber up to 40 per cent camber.

rapid. Beyond 20 per cent it is almost vertical, and if there were nothing else to indicate the hopelessness of such deep cambers Fig. 121 would do it.

Figures 122, 123, and 124 are more interesting, for they have a wider scope. They represent, for three series differing in thickness, "differences from standard profile drag" for several cambers, and they make it obvious that any possible camber over a wide range has its merits if we look in the right place for them. However many cambers might be run to cover the spread between 0

and 6 per cent, whether four as in the present case or forty or more, they will form a sheaf of which each element will have its own brief moment in the sun, projecting at the extreme left of the group. Here, for instance, in the 12 per cent thickness series, the symmetrical section does best for lift coefficients from 0 to 0.14, a 2 per cent camber gains the ascendency from 0.14 to 0.55, and so on. If a section of 1 per cent camber had been added to the series it would have forced itself in between the 0 and 2 per

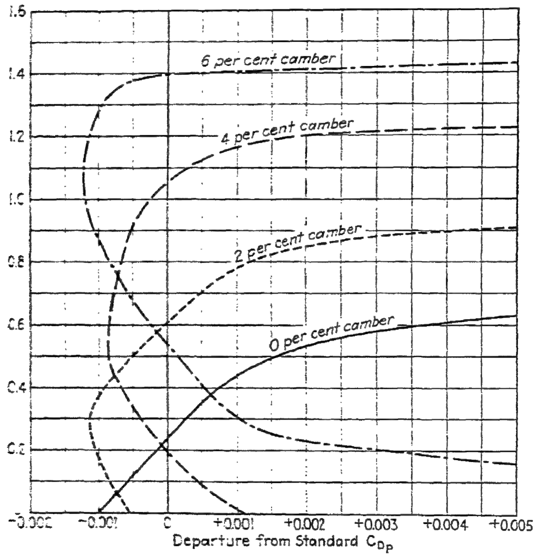

FIG. 122.—Variations of profile-drag characteristics with camber; airfoils of the 6 per cent thickness series.

cent curves and claimed for itself a part of the zone of present superiority of each of them, leaving each of its predecessors still supreme in a restricted range. Figure 125 is a plot of cambers against the lift coefficients at which the supremacy of each will be, so to speak, unchallengeable. It is, in other words, a plot giving the camber that should be chosen for an airfoil for which a low drag at one particular lift coefficient is the paramount, and in fact the sole, consideration. The form of the curve is independent of location of maximum camber to within the experi-

mental error, dependent on thickness in only a small degree, and can be taken as reasonably valid for all cases except that of the airfoil with a reflexed trailing edge. To that it does not apply at all, nor do most of the other rules that can be developed from a study of airfoils with a mean-camber curve continuously concave downward. The influence of reflexing is separately treated

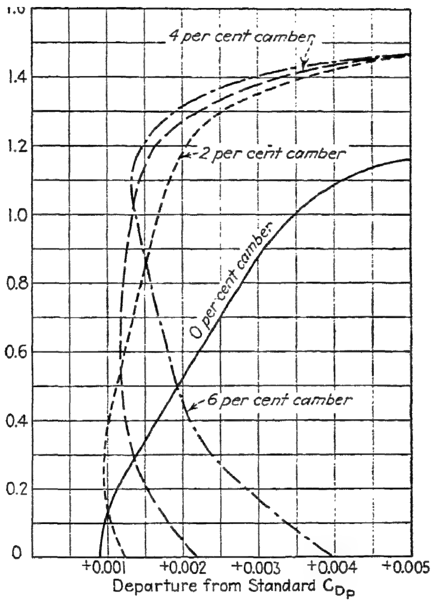

FIG. 123.—Variations of profile-drag characteristics with camber; airfoils of the 12 per cent thickness series.

on a later page.[1] One other exception or stipulation is that it never seems to be advisable, under any conditions, to combine a camber of more than 5 per cent with a thickness of more than 12.

Figure 125 is most important in connection with thin airfoils, the thicker ones, as indicated by a comparison of Figs. 122 and 123, being less abruptly sensitive to the influence of variations in camber. Roughly, for an airfoil of 6 per cent thickness the lift coefficient can be changed an average of 0.3 either way from

[1] P. 212, *infra*.

its ideal value, as shown by Fig. 125, without increasing the profile drag to more than 0.0005 above the value that it would

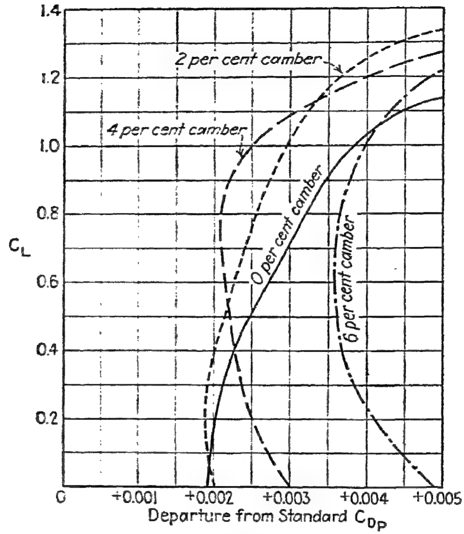

FIG. 124.—Variations of profile-drag characteristics with camber; airfoils of the 15 per cent thickness series.

have at the same lift coefficient if the optimum camber for that particular lift coefficient were being used. To put it another way, there is an average range of C_L of 0.6 over which a rigid wing would be inferior by 0.0005 or less to a wing of which the camber could be universally varied so as to be ideal at every instant for the lift coefficient at which it was then operating. For airfoils of 12 per cent thickness, the corresponding range is increased from 0.6 to 1.1. In neither instance does the conclusion apply to symmetrical airfoils, which lose their efficiency very rapidly as the lift is increased. In the 6 per cent thickness group,

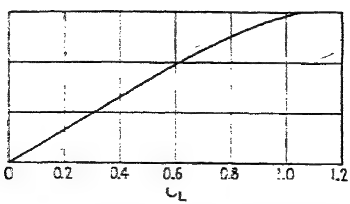

FIG. 125.—Variation of optimum camber, for lowest possible profile drag, with lift coefficient at which airfoil is to be rated.

even at a lift coefficient as small as 0.15 (as low a value as is generally attained at maximum speed at sea level), a symmetrical section has 10 per cent more profile drag and 8 per cent more total drag than one with 1 per cent of mean camber.

Figures 126 to 128 show the variation of $C_{L\max}/C_{D_{p\min}}$ and, for a wing of aspect ratio 6, of D/L and $C_D/C_L^{3/2}$. The first is entirely dependent on thickness, showing its maximum for very large cambers in thin wings and for very small cambers in thick ones. The other ratios find minima at around $2\frac{1}{2}$ and 4 per cent camber, respectively.

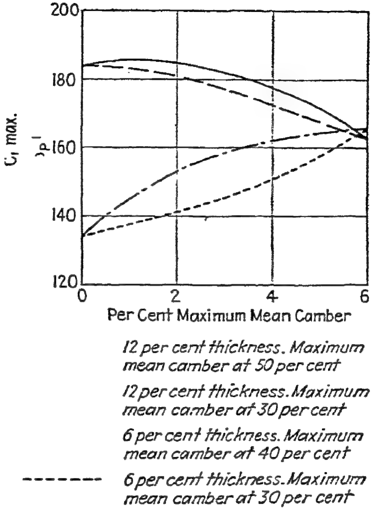

FIG. 126.—Relation of $C_{L_r}/C_{D_{p\min}}$ to mean camber.

Finally there are moments to be considered. Here again, as for zero-lift angle, theory shows camber to be the controlling factor and theory is justified. The negative coefficients (diving moments) at zero lift prove in most cases to be from 10 to 15 per cent less

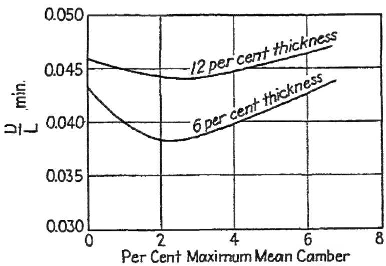

FIG. 127.—Relation of $(D/L)_{\min}$ to mean camber.

than the values predicted by Munk's method.[1] They have a slight tendency to increase numerically with increasing

[1] See p. 110, *supra*.

thickness up to about 10 per cent and to decrease thereafter, but the effect of thickness is virtually negligible in comparison with that of camber. Munk's method is simple enough to be applied to any airfoil for which experimental data are lacking, but as a still rougher rule it may be said that the negative moment at zero lift for any airfoil of normal camber form and without a reflexed trailing edge is approximately 0.02 for every percent of maximum mean camber. If the point of deepest camber is moved forward to 20 per cent of the chord from the leading edge the constant of proportionality decreases to 0.015. With the

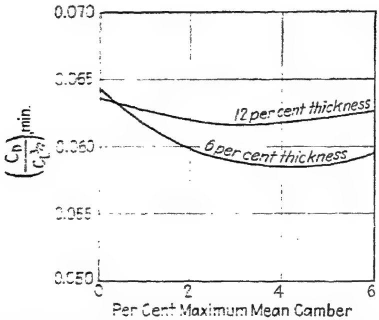

FIG. 128.—Relation of $(C_D/C_L^{3/2})_{min}$ to mean camber.

maximum at 50 per cent the diving moment per percentage of camber increases to 0.025, with another 0.005 of increase for every additional 10 per cent of movement of the maximum camber toward the trailing edge. Thickness has practically no effect up to 15 per cent, but beyond that point the diving moment for a given mean camber falls off gradually as the thickness increases. With a thickness ratio of 18 per cent the diving moments are about 10 per cent smaller than with one of 12, other things being equal.

The general conclusion, so far as it is feasible to draw any, is that the symmetrical airfoil is ruled out for general service by its bad drag characteristics at lift coefficients above 0.2, and that mean cambers above 4 per cent go out of court on low general efficiency, bad high-speed performance, and bad structural characteristics due to excessive negative moments and consequent torsion in the structure. Except for certain very

special purposes, the designer's consideration can be limited to mean cambers between 1 and 3.5 per cent.

Variation of Upper Camber with Flat Lower Surface.—Of data on the effect of a combined variation of thickness and mean camber by changing the upper surface alone there is, as has already been noted,[1] no dearth. Supplementary to the early researches already cited, most of them carried out at very low Reynolds' numbers, there has been run within the past decade a single series of tests from the National Advisory Committee's variable-density tunnel.[2] Conclusions ought henceforth to discount everything that preceded that series, for thick sections are peculiarly subject to scale effects and tests on small models or at low speeds at atmospheric density are peculiarly liable to be misleading. Especially is that true of maximum-lift figures, as the scale effect on maximum lift absolutely reverses as the thickness and camber increase. For sections with a flat lower surface and a thickness ratio of 0.15 or more, the maximum lift increases rapidly with N up to about $N = 100{,}000$ and then begins to drop off as N is still further increased. In the series of tests to which this discussion particularly relates, the maximum lift for a section of 20 per cent thickness ratio was 20 per cent lower for $N = 3{,}550{,}000$ than for $N = 172{,}000$. Further discussion will be confined to the high Reynolds' number characteristics.

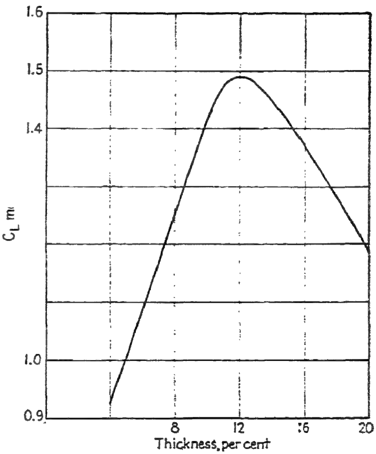

FIG. 129.—Relation of maximum lift to upper camber, flat lower surface.

The sections composing the series varied in upper camber from 4 to 20 per cent. They all had the same form of thickness curve, all the vertical dimensions being modified in the same ratio to pass from one section to another. The maximum thick-

[1] See p. 182 for bibliography.

[2] "Characteristics of Propeller Sections Tested in the Variable-density Wind Tunnel," by Eastman N. Jacobs, *Rept.* 259.

ness, and necessarily the maximum mean camber as well, was uniformly at 30 per cent of the chord from the leading edge. The leading-edge radius was uniformly 10 per cent of the thickness.

The variation of maximum lift and minimum profile drag, plotted against thickness in Figs. 129 and 130, is what might be expected from the separate consideration already given to the influence of thickness and mean camber. The absolute maximum of lift perhaps comes a little sooner than might have been expected. We might well have guessed that the influence of increasing camber would overcome the slightly unfavorable effect of increasing thickness after passing the optimum-thickness point at about 12 per cent, and that the highest possible lift would be attained, under the conditions of this series of tests, with a thickness of 16 per cent or even more. Just that happens at low Reynolds' numbers, but in the tests at high values of N the lift curve reaches its ultimate peak, as shown by Fig. 129, at a 12 per cent thickness. The best thickness for maximum lift is then only about 2 per cent smaller when the mean camber is held constant, the upper and lower surfaces being bulged out equally and oppositely to accumulate thickness, than when the lower surface is kept static and the mean camber is increased half as rapidly as the thickness itself.

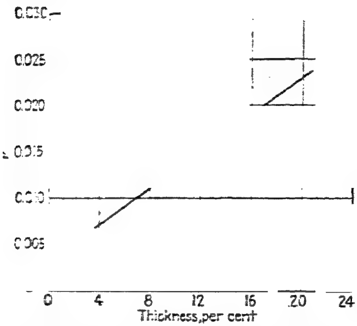

FIG. 130.—Relation of minimum profile drag to upper camber, flat lower surface.

Other effects of thickness on the lift curve are unimportant. The peak has a slight tendency to become rounder and smoother with increasing thickness, contrary to the conclusion, often drawn from tests at low values of N, that thick sections are especially liable to discontinuities of lift. The zero-lift angle, in conformity with the rough rule suggested on page 193, is (in degrees) approximately half the percentage of upper-surface camber.

Minimum drag of course increases steadily with thickness, and the almost perfectly uniform rate of increase is a little surprising. It would have seemed more reasonable to expect

an accelerating rate of increase as the thickness mounted. The exact values on this curve must not be taken too seriously, however, as the operating technique in the variable-density tunnel was still unperfected at the time when the work was done. The curve must be regarded rather as indicative of a general tendency.

Profile drags have been plotted in Fig. 131 for four of the sections. Their most interesting revelation is that the tendency

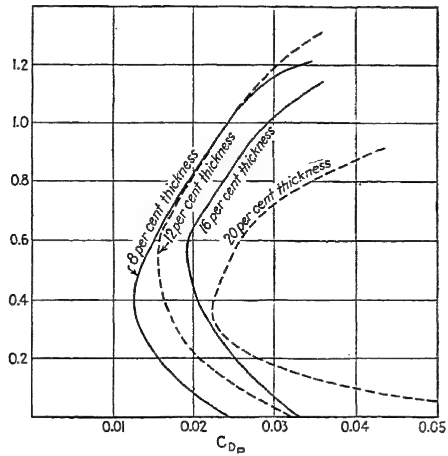

FIG. 131.—Variation of profile-drag characteristics with upper camber; lower surface flat in all cases.

to increasing flatness of the profile-drag curve and an increased range of efficient operation with increasing thickness does not extend, for the flat-bottomed sections, beyond a thickness of about 15 per cent. The thickest section shows the abrupt increase of profile drag both ways from the minimum that ordinarily characterizes sections of 8 per cent thickness or less.

Effect of Position of Maximum Camber.—The most remarkable thing about this variable is its unimportance. Attention has already been called to the fact that two of the best airfoils now known, the N. A. C. A. 23012 and the Clark Y, have points of deepest mean camber separated by 25 per cent of the chord. To the tentative conclusion there indicated, that it makes very

little difference anyway, systematic testing now proceeds to give support.

For minimum profile drag the best location is at 40 per cent of the chord for thin airfoils, moving forward gradually to 30 per cent with 15 per cent thickness and 6 per cent camber. On the average, a shift of maximum camber to 10 per cent from its optimum point increases the minimum drag coefficient by 0.0001 for every percent of camber. Location of maximum camber has virtually no effect on the form of the drag curve or upon the point where the airfoil attains its highest relative efficiency.

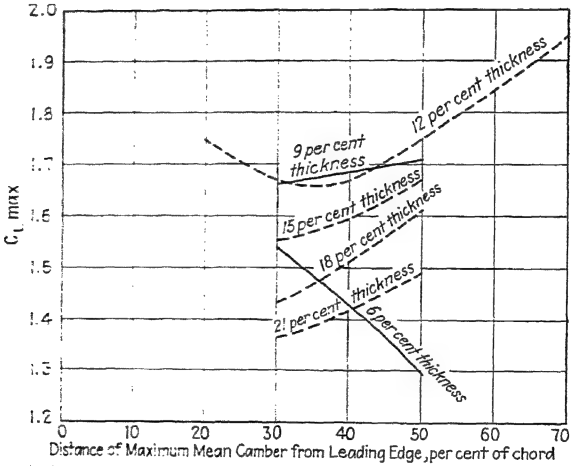

FIG. 132.—Effect of location of point of maximum mean camber on maximum lift.

The lift is more responsive, though even here it is only for the largest cambers that the effects so exceed the experimental errors that one can be sure of their reality. In Fig. 132 curves of maximum lift against location of maximum mean camber have been drawn for 6 per cent mean camber and for several different thicknesses. Unfortunately, the tests extended over a wide range of location for only one thickness, and the other curves contain too few points to provide a wholly satisfactory definition of their form.

The maximum-lift data point exactly in the opposite direction from those for minimum drag. Now it appears that the maxi-

mum camber should be far forward on thin sections, far back on thick ones. A compromise value of 35 per cent would be not far off the mark in any case, unless for some reason special stress were to be laid on a high maximum lift. In that event, which the general adoption of variable-lift devices[1] has made a rare one nowadays, a thick section with a large mean camber centered very far back on the chord is the indicated recipe. One very curious point in the curves is the apparent reversal of slope at the left-hand end of the one for 12 per cent thickness. Apparently the maximum lift increases indefinitely with movement of the maximum-camber point toward either extreme—a conclusion so surprising as to be wholly unacceptable on the basis of a single test.

Moving the maximum-camber point back increases the numerical value of the zero-lift angle, by about 0.4 deg. for every 10 per cent of movement when the camber is 6 per cent. It also proves, as might have been anticipated from what we know of air flow, that the lift curve peaks most smoothly and with least likelihood of a sudden break when the maximum camber is near mid-chord and that a shift toward any extreme position, most particularly one toward the leading edge, has a damaging effect on its smoothness. That effect, however, is almost too small to be traced except in extreme cases.

Most important of all is the effect of camber location on moment. As has already been pointed out,[2] moving the maximum-camber point forward 10 per cent of the chord reduces the moment at zero lift about as much as reducing the camber by a quarter. The resultant structural benefit justifies pushing the point of deepest camber forward from the 35 per cent previously determined as an optimum compromise for lift and drag to 30 per cent, or possibly even to 20. The N. A. C. A. 23012 has its maximum at 15 per cent, but in that case the form of the camber line differs somewhat from the general standard. The nature and the effect of the difference will be discussed later.[3]

Shape of Leading Edge.—One of the few elements of thickness distribution to have been the subject of direct investigation is the leading-edge radius. Many of the very early airfoil

[1] See Chap. X, *infra*.

[2] P. 200, *supra*.

[3] P. 215, *infra*.

sections had sharp leading edges, although even at that time it was being realized, by analogy between the airfoil section and the streamline body and by observation of the form of a bird's wing, that there might be advantages in a rounded rather than a sharp point. Some sections, such as the Eiffel 9, were even then designed with the leading edge thickened to an exaggerated degree. In the course of airfoil evolution it came to be the universal practice to round the leading edge over to a radius, if for no other reason than that it made the construction of the wing simpler. Subsequently it came to be realized that the rounding brought with it a positive aerodynamic improvement, and especially a broadening of the range of angles within which the airfoil remained efficient. In some cases that simple finishing of the leading edge in a fillet rather than in a sharp point has been extended to the point of making the minimum radius of curvature at the leading edge something like 25 per cent of the maximum thickness of the wing, and in numerous instances it exceeds 15 per cent of the thickness. This is especially characteristic of the Göttingen sections and others derived from forms based on the Joukowski transformation. Thus, while the leading-edge radius of the Göttingen 387 is 19 per cent of the maximum thickness, that of the U. S. A. 27 is only about 7 per cent. The N. A. C. A. 23012's is 13 per cent, which is fairly representative of good modern practice for wings of that thickness. The ratio of leading-edge radius to maximum thickness tends to increase, for obvious geometrical reasons, as the thickness increases.

It is not easy to experiment directly on the effect of changing leading-edge thickness, as an attempt to modify that feature of a section without altering anything else almost necessarily results in a modification of the whole appearance and in a marked sacrifice of fairness of form. The only systematic series of tests available on cambered sections dates back more than twenty years[1] and was run at such low Reynolds' numbers as to be of little present use. The far-reaching airfoil researches of the National Advisory Committee have extended over a few studies on the effect of leading-edge radius, but they were applied only to symmetrical sections. The change of nose form was accom-

[1] "Experiments upon the Effect of Thickening the Leading Edge of an Airfoil," *R. and M.* 72.

panied by a slight change of thickness aft of the mid-point of the chord, in the interest of fairness of contour, but the effect of the change there was probably negligible compared with that of the changes at the nose. The three forms tested, standard,

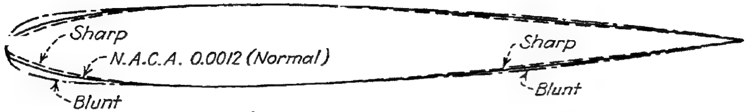

FIG. 133.—Changes in leading-edge form.

extra thin, and extra blunt, are shown for 12 per cent thickness in Fig. 133.

The lift curves for the three forms, again for 12 per cent thickness, are reproduced in Fig. 134. Apparently there is a

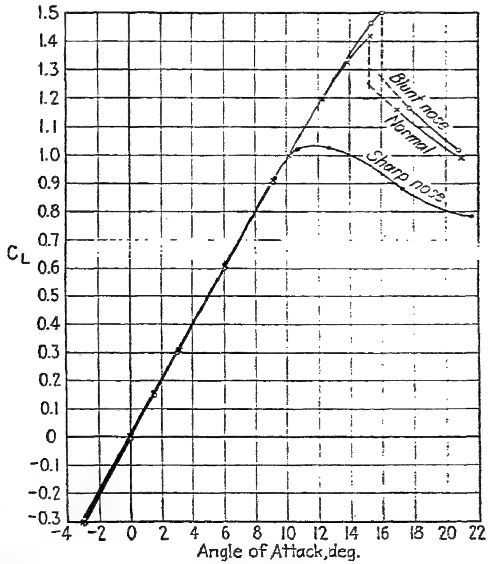

FIG. 134.—Effect of sharpness of leading edge on form of lift curve; symmetrical sections.

critical minimum value of leading-edge radius, below which the maximum lift suffers acutely. Comparison of the results shown in Fig. 134 with those obtained in similar tests on sections of 6 and 18 per cent thickness suggests the definite inadvisability,

however thick or thin an airfoil may be, of using a leading-edge radius of less than 0.75 per cent or more than 5 per cent of the chord. A continuation of the study into cambered forms, limited to the single experiment of fitting several sections with perfectly sharp leading edges, confirms this conclusion so far as maximum lift is concerned. The effect continues to be almost disastrous. There is, however, a possibility that the change of leading-edge form may have so pronounced a beneficial effect on an airplane's spinning characteristics[1] as to justify the sacrifice.

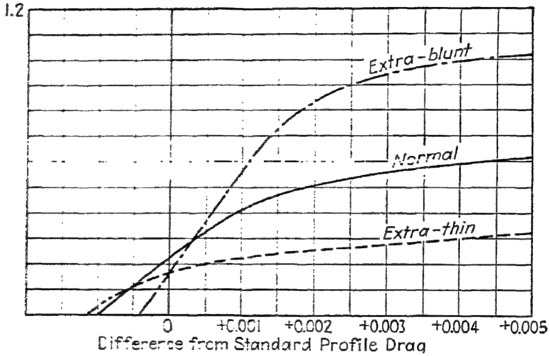

FIG. 135.—Variation of profile-drag characteristics with sharpness of leading edge; symmetrical sections.

In accordance with the common rule that a very high maximum lift is likely to mean a sharp peak on the lift curve at high Reynolds' numbers, the lift curves are smoothest at the peak for the sharp-nosed sections.

Drag is but little affected except at high lift coefficients. Difference-from-standard-profile-drag curves have been drawn for the 6 per cent thickness series in Fig. 135, for the two Clark Y sections in Fig. 136. As might be expected, decreasing the leading-edge radius decreases the minimum profile drag somewhat, though hardly enough to compensate for the loss of maximum lift. Rounding of the nose has the effect, as might be expected from analogies with what has gone before, of extending

[1] "Characteristics of Two Sharp-nosed Airfoils Having Reduced Spinning Tendencies," by Eastman N. Jacobs, *Tech. Note* 416; "The Effect on Lift, Drag, and Spinning Characteristics of Sharp Leading Edges on Airplane Wings," by Fred E. Weick and Nathan F. Scudder, *Tech. Note* 447.

the range over which the profile drag remains reasonably close to its minimum value. Thus at lift coefficients above 0.2 the sharp-nosed section is definitely inferior to the normal one of the symmetrical series, while above 0.4 the blunt-nosed one is as clearly superior to normal. The 12 per cent thickness series shows much less effect than the 6 per cent one, again as might be expected. The sharp nose on the Clark Y is not so bad from a practical point of view as that on the symmetrical sections, for the 3 per cent mean camber of the Clark section gives it a range of superior efficiency well displaced from zero lift.[1] The

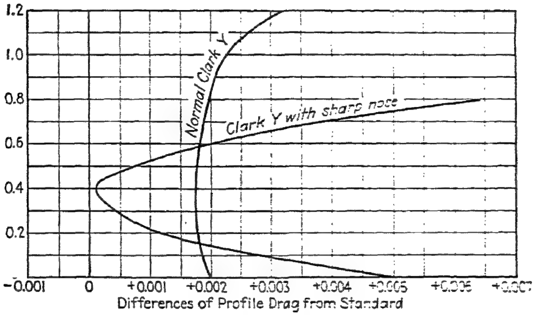

FIG. 136.—Variation of profile-drag characteristics with sharpness of leading edge; Clark Y section.

sharp-nosed modification keeps a good efficiency over a range of C_L from 0.1 to 0.6, a spread within which 90 per cent of the demands on an airplane's performance are concentrated. The possible advantages of a section with a sharp nose, and a camber of from 1 to 2 per cent, depending on the wing loading and the speed expected to be attained, for cases where maximum speed is the paramount consideration, will be obvious. Racing airplanes afford the major case of that sort.

Thickness Distribution near Trailing Edge.—Though systematic studies of the effects of variation in thickness distribution are virtually lacking, certain early studies involving independent modification of the upper and lower surfaces bear on the point.

In certain sections (as in the R. A. F. 15 and U. S. A. 27) the lower surface undergoes a reversal of curvature and shows a

[1] The discrepancy between the "normal Clark Y" curve plotted here and that in Fig. 179 is due to different conditions of testing, and especially to a difference of Reynolds' number.

convexity in the last 30 or 40 per cent of the chord. The reason is primarily structural. The rear spar is usually shallower than the front one in any case, and special efforts are therefore made to keep the section as deep as possible at the rear-spar location. Such comparisons as can be made indicate that the lift and drag curves are but very little changed, the tendency being to decrease the maximum lift slightly and to decrease the minimum drag in an even smaller proportion, while the drag at moderate lift coefficients remains substantially unaffected unless the thickening is very marked. None of the effects on lift and drag is likely to exceed 2 or 3 per cent in magnitude.

FIG. 137.—Thickening of rear part of airfoil.

Raising the upper surface as shown in Fig. 137, again with the object of deepening the rear spar, has a somewhat more pronounced effect. The best series of tests[1] on that particular change shows an increase of 7 per cent in the minimum D/L ratio as the result of a 40 per cent increase in the depth of an R. A. F. 6 section at 80 per cent of the chord from the leading edge. A 20 per cent decrease in depth of upper chamber at the same point lowered the minimum ratio about 2 per cent. Thickening the section had no appreciable effect on the maximum lift, but the 20 per cent reduction in thickness cut the maximum down by 5 per cent. The most important effect was on minimum drag, which was 6 per cent higher for the thickest section than for the standard form and was lowered a trifle more than 6 per cent when the upper camber was trimmed down by one-fifth. These results probably would not change very much with changing thickness of the basic section, nor is there reason to suppose that these particular figures would be very different at larger values of Reynolds' number. Obviously, however, there is a point of diminishing returns from cutting down the upper surface. Such thinning of the section would not continue to bring improvement indefinitely, as the curvature of the upper surface would ultimately become distorted and unfair.

[1] "Investigation into the Effect of Thickening an Aerofoil towards the Trailing Edge," *R. and M.* 72.

The combined effect of raising the upper surface and reversing the curvature of the lower one is to increase the thickness of the airfoil near the trailing edge without changing either the maximum thickness or the form of the mean-camber line. The combined result is a very small increase in minimum drag, a relatively somewhat larger increase in drag in the cruising range (around $C_L = 0.4$), practically no change in maximum lift. The data are too antiquated and too incomplete to allow of more quantitative conclusions.

Amputation of the Trailing Edge.—Occasionally the necessity arises, to improve the pilot's field of view or from some other consideration of practical design, of cutting away a part of a wing's trailing edge. Such cutouts, though they usually extend over only a small part of the span, may run locally to a depth of a third or even a half the chord, and tests have been made at Göttingen[1] to approximate their effect under the worst conditions possible by assuming the deformation of the trailing edge prolonged from tip to tip of the wing.

FIG. 138.—Extreme case of cutting away of trailing edge.

As separation takes place, at high angles of attack, far forward of the trailing edge it is not surprising to find that the rearmost part of an airfoil has but little effect on its maximum lift. If coefficients are based always on the true area of the airfoil as tested, the maximum-lift coefficient actually increases with each successive removal of segments from the trailing edge until 60 per cent of the original chord is gone and the airfoil looks like Fig. 138. In other words, while removal of bits of the trailing edge reduces the gross lift in pounds, it reduces it less rapidly than the area itself is reduced by the removal. The coefficient based on true area during the test therefore rises, approximately as a linear function of the amount removed up to the point where the chord has been reduced by a half and the maximum-lift coefficient increased by 25 per cent. To put it in unit terms, $C_{L_{max}}$ goes up 1 per cent for every 2 per cent of chord that is trimmed squarely off the trailing edge.

[1] "Experiments on Airfoils with the Trailing Edge Cut Away," by J. Ackeret, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. III, translated in *Tech. Memo.* 431.

As might be expected, the effect on drag is quite different. The price of geometrical discontinuity is written in terms of turbulence in the wake, breakdown of streamline flow along the contour, and increased drag. Taking 10 per cent of the chord off the trailing edge increases the profile-drag coefficient by 20. With 20 per cent off, the profile-drag coefficient is doubled, with 30 per cent it is more than tripled, with half the chord gone the coefficient is multiplied by eight. Such distortions are, in short, good to keep away from. Though there is no evidence that the performance of an airfoil suffers appreciably by terminating abruptly at a trailing edge having a thickness of ½ per cent of the chord or less, rather than being carried out to a knife-edge, the effect of a trailing-edge thickness of 2 per cent or more is manifestly bad in the extreme.

Reflexed Trailing Edges.—The upper and lower surfaces of an airfoil near the trailing edge are sometimes modified together,

Fig. 139.—Reflexing of trailing edge.

without changing the thickness, by bending the trailing edge upward as a whole, as shown in Fig. 139. Commonest of all irregularities encountered in the form of the mean-camber curve, it is a feature most often introduced to reduce the diving moments and to secure an improved center-of-pressure travel, reducing the rapidity with which the center of pressure moves toward the trailing edge as the angle of attack is lowered.

Restricting the center-of-pressure travel has advantages both structural and aerodynamic, the former in the shape of a more uniform distribution of load between the spars and a reduced torsion, the latter in that of a reduced tail size and tail load needed to insure longitudinal stability.

Manifestly it is impossible to draw any universally valid conclusions regarding the effects of a reflexed trailing edge on lift, drag, and moment, for much must depend on the initial form of the airfoil. From the best series of tests offering an absolutely direct comparison between the behavior of given airfoils with and without the reflexing,[1] the rough general rule can however

[1] "A Comparison of the Aerodynamic Characteristics of Three Normal and Three Reflexed Airfoils in the Variable-density Wind Tunnel," by George L. Defoe, *Tech. Note* 388.

be drawn that in its effect on angle of zero lift and on moment 1 per cent of reflex is equal and opposite to 1 per cent of camber (percentages, as always, being taken in terms of the chord). Given a reflexed airfoil like that shown in Fig. 139, then, redraw it with the reflex eliminated as there indicated, measure the camber from the new chord so established, measure also the amount of vertical movement of the trailing edge in passing from the original to the reconstructed section (which is defined as the amount of reflex), put both measurements in terms of percentages of the chord, and take the difference between them. Nearly enough for most preliminary purposes, the angle of zero lift in degrees is numerically equal to the difference so determined, the sign of the angle being negative when the camber exceeds the reflex, and vice versa. Nearly enough for estimating purposes pending the availability of full data for a particular section, too, the moment coefficient at zero lift can be taken as 0.02 times the difference between camber and reflex. Given a particular section as a starting point, then, the amount of reflex needing to be given to reduce the diving moment to any desired level can be computed, with an error that should seldom reach 1 per cent.

The maximum lift, also, shows itself susceptible to the application of a general rule. Quite uniformly for all the sections that have been tested, including some run in England many years ago,[1] the maximum lift falls off by 4 per cent for every per cent of reflex. The form of the lift curve is otherwise unaffected.

Concerning drag, only one generalization is possible—that reflexing brings the minimum profile drag to a lower lift coefficient. Whether the minimum value will be decreased or the opposite depends on the form of the basic section. If its camber was large in proportion to its thickness, or if the deepest camber was far back on the chord, then reflexing will certainly be helpful, but only at the cost of sacrificing the high maximum lift which alone would have justified the adoption of such a form in the first place. With a mean camber of less than one-third the thickness, the effect of moderate amounts of reflex on minimum

[1] "Experiments on an Aerofoil with Reverse Curvature towards the Trailing Edge," *R. and M.* 72; "Further Experiments with Aerofoils Having Reversed Curvature towards the Trailing Edge," by F. H. Bramwell, *R. and M.* 110.

drag is inappreciable. Figure 140 gives the profile-drag difference curves for a section of 13 per cent thickness and 3.4 per cent camber, with the maximum camber at 35 per cent of the chord, and also for the same section with 3 per cent of trailing-edge reflex.[1] They speak for themselves. The reflexing is helpful at high speeds, provided that its effect on maximum lift can be overlooked. In the region from $C_L = 0.5$ to $C_L = 0.8$, normally corresponding to take-off and climb, its effect is bad.[2]

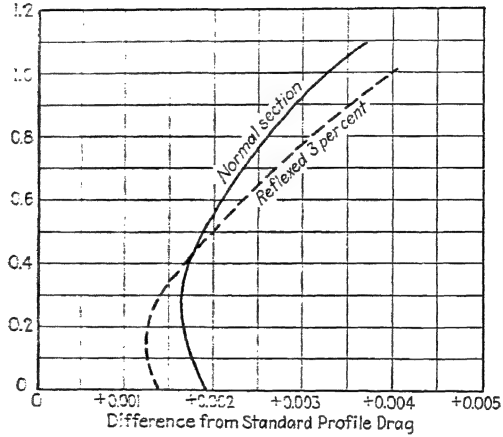

FIG. 140.—Effect of reflexing of trailing edge on profile-drag characteristics.

Distribution of Camber.—Aside from the work on trailing-edge reflex, very little has been done on the effect of changes in the form of the mean-camber line with its maximum point kept undisturbed. There exists in fact but a single series rigorously confined to that single variable—the series that contains the N. A. C. A. 23012.[3] The difference between the two camber

[1] Based on *Tech. Note* 388, *cit. supra.*

[2] Anyone wishing to pursue this subject further will find test results for a great variety of reflexed airfoils, though without any corresponding results for the corresponding unreflexed forms, in "Model Tests with a Systematic Series of 27 Wing Sections at Full Reynolds' Number," by Max M. Munk and Elton W. Miller, *Rept.* 221.

[3] "Tests in the Variable-density Wind Tunnel of Related Airfoils Having the Maximum Camber Unusually Far Forward," by Eastman N. Jacobs and Robert M. Pinkerton, *Rept.* 537.

forms set up for comparison is shown in Fig. 141, where they are superposed with the customary ten-to-one exaggeration of the vertical dimensions. The camber for the 2212, a smooth curve from leading to trailing edge, follows the standard form for the N. A. C. A. group upon which most of the discussion in this chapter has been based. That for the 20000 series, a straight line all the way back from just behind the maximum, appears much less smooth.

In lift and drag characteristics, the change proves to make no measurable difference whatever except that it diminishes the negative value of the angle of zero lift by 0.3 deg. The principal

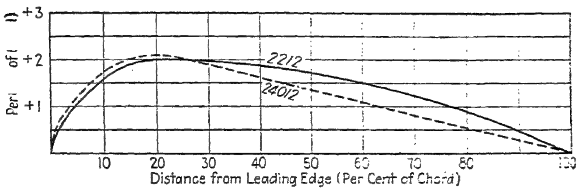

FIG. 141.—Difference of form of camber line between N. A. C. A. 2000 and N. A. C. A. 20000 airfoil series.

influence is shown in the moment, C_M at zero lift being changed from -0.029 to -0.013 by increasing the camber forward of the maximum and cutting it down aft of that point—a reduction of coefficient that may make a difference of 3,000 lb. in the load carried on the rear spar of a bombing airplane in a vertical dive.

One other thing the change of form of the mean-camber line does. It makes it possible, with apparent advantage, to push the point of maximum mean camber forward to locations unprecedentedly near to the leading edge. The N. A. C. A. 23012 has its maximum at 15 per cent, and it can even be pushed forward to 5 per cent without appreciable loss. The farther forward it goes, also, in accordance with what we have already seen to be the general rule, the lower the diving moments fall. With the maximum camber of 2 per cent at 6 per cent from the leading edge the moment reaches zero, precisely as in a symmetrical section or in one with the percentage of reflex equal to the percentage of camber.

Upon this whole subject of mean-camber-line form, as upon that of thickness distribution, there is a vast amount of work

yet to be done in the few wind tunnels equipped to undertake it. Taken together, they provide airfoil research with its greatest opportunity to make within the course of the next few years still further gains in efficiency, minute in magnitude but looming large when translated into the aggregate power consumption of an airplane or the aggregate cost of operating it over its useful life.

Discontinuities of Form.—Numerous attempts have been made to develop freak sections designed to turn the inertia of the air to profit by causing the streamlines at some point to take a form more or less independent of the actual contour of the section in that region. One of the earliest of such efforts was represented by the Constantin leading edge, of the form shown in Fig. 142. The theory was that the air would be directed

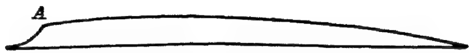

FIG. 142.—Airfoil with "Constantin entry."

upward from the point marked A by the concave portion scooped out over the leading edge and would therefore leave a discontinuity of flow behind the point of discontinuity of section, with resultant high suction over the part of the wing immediately behind that point. Numerous tests were made,[1] and the general conclusion was that the Constantin leading edge might improve a poor airfoil, but always harmed a good one, in respect of both lowered lift and increased drag. It is now known, and it should be amply apparent from what has already been said in this chapter and the three preceding ones, that the highest lift coefficients are secured with a substantially smooth streamline flow of air over the upper surface of the airfoils, rather than by deliberately introducing discontinuities and turbulence.

Another section of somewhat eccentric form, originally offered for test by Howard Wright and illustrated in Fig. 143, has been tried both in France and in England.[2] The object of such sections, with double or triple camber on the upper surface, is

[1] "Nouvelles recherches sur la résistance de l'air et l'aviation," by G. Eiffel, p. 118, Paris, 1914; "Experiments on a Number of Aerofoils of Various Sections," by F. H. Bramwell, *R. and M.* 110; "Experiments on Thick Wing Sections," *Bull.* Expt. Dept., Airplane Eng. Div., U. S. Army Air Service, December, 1918 (U. S. A. *T. S.* 15).

[2] G. Eiffel, *op. cit.*, p. 123 (Eiffel 47–51); *R. and M.* 110, *op. cit.*

primarily to secure a more uniform distribution of pressure along the chord and a reduced movement of the center of pressure with changing angle of attack. Incidentally, the designers hoped for a possible increase in maximum lift by the introduction of two or three peaks of suction on the upper surface in place of one. It would, of course, be anticipated that the drag would be increased as compared with that of an ordinary airfoil because of the increase of turbulence of flow which must manifestly result from the introduction of so many abrupt changes of curvature.

FIG. 143.—Multiple-camber airfoil.

The results of the tests on multicambered sections showed nothing remarkable. The expectation of a high maximum lift was justified in some degree (about a 12 per cent increase in the British tests), but at the expense of a 30 per cent increase in minimum drag, a 9 per cent rise in minimum drag/lift ratio, and some reduction in the maximum thickness of the wing, to say nothing of the structural difficulties which might attend on building ribs to so odd a form. Furthermore, the highest maximum lift ever found with such a section is inferior to what can be had with a good airfoil of ordinary shape. The center-of-pressure travel certainly is not appreciably improved, and in fact the rate of movement of the center of pressure with change of angle actually increased in one case as compared with the rate for a standard section. The matter is mentioned here solely for historical purposes and to confute those who may be so naive as to rediscover the same idea. If a stationary center of pressure is wanted, it is far easier to get it by reflexing the trailing edge or by reducing the camber than by such tricks as these.

Airfoil Performance at Abnormal Angles.—Most of an airplane's activities are carried on at angles of attack between 0 and 15 deg. Occasionally, however, either in deliberate acrobatics or in consequence of unintentionally falling into an abnormal attitude, angles far outside the normal range have to be encountered. Efficiency under such conditions is usually of little moment, but the form of the lift and drag curves may have a controlling influence over stability and especially over the ability to recover from a spin. Discussion under those headings

can be reserved for a later volume, but it ought to be extended somewhat beyond the range so far treated before leaving the general presentation of airfoil data.

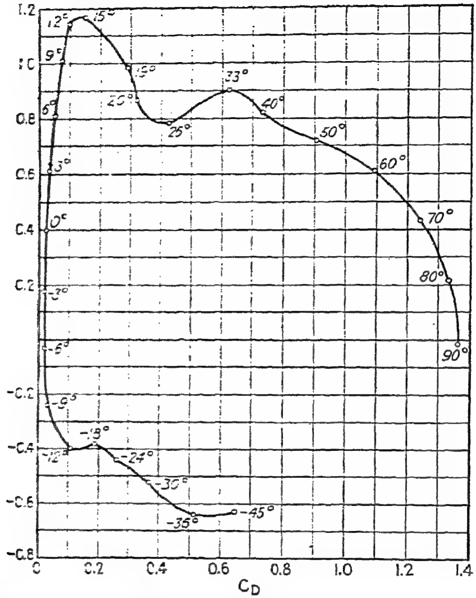

FIG. 144.—Polar curve for Clark Y over 135 deg. range of angle of attack.

In Fig. 144[1] polars have been plotted from −45 to 90 deg. Contrary to common practice, the lift and drag scales in this

[1] From "Wind Tunnel Tests on a Series of Wing Models through a Large Angle of Attack Range," by Montgomery Knight and Carl J. Wenzinger, *Rept.* 317. Additional data of the same order, with interpretation of their characteristics, are in "The Aerodynamic Characteristics of Six Commonly Used Airfoils over a Large Range of Positive and Negative Angles of Attack," by Raymond F. Anderson, *Tech. Note* 397; "The Aerodynamic Characteristics of Airfoils at Negative Angles of Attack," by Raymond F. Anderson, *Tech. Note* 412; "Air Forces and Air Moments at Large Angles of Attack and How They Are Affected by the Shape of the Wing," by Richard Fuchs and Wilhelm Schmidt, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Jan. 14, 1930, translated in *Tech. Memo.* 573; "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. III; "Lift and Drag of Two Aerofoils Measured over 360 Deg. Range of Incidence," by C. N. H. Lock and H. C. H. Townend, *R. and M.* 958.

case are the same, so making room for the inflated values of drag at high angles and making it possible also to measure off the resultant force at any angle directly as the length of the radius vector from the origin to the appropriate point on the polar. The variation of resultant force (which is virtually coincident with component normal to the chord) at high angles is of greater practical significance than the variation of lift.

The double maximum of lift and normal force, with the second peak at between 25 and 45 deg., is characteristic. So is the negative burbling (for the cambered section) at an angle smaller than that of maximum positive lift and a lift coefficient less than half the maximum on the positive side. The center of pressure retreats steadily with increasing angle after passing the first

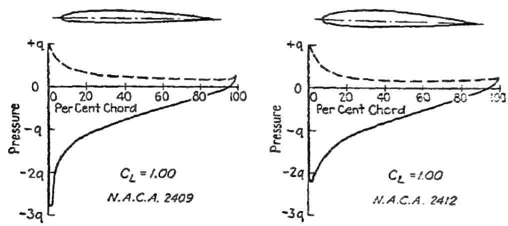

FIG. 145.—Pressure-distribution diagrams at $C_L = 1.0$, showing effect of thickness of airfoil.

maximum of lift, and lies between 45 and 52 per cent of the chord for all angles between 45 and 90 deg.

Effect of Form on Pressure Distribution.—In Figs. 145 to 148, inclusive, comparative pressure-distribution diagrams[1] are shown for a number of groups of sections. The diagrams show comparison not at a common angle of attack but at a common lift coefficient, the representation being in every case for $C_L = 1.0$.

Figure 145 indicates the effect of increase in thickness from 9 to 12 per cent, the form of the mean camber line remaining unchanged. The geometrical consequence of the increase is to decrease the relative rapidity of change of curvature over the upper surface, and so to diminish the suddenness with which the magnitude of the pressure drop falls off after passing the leading edge. The very high value of the pressure drop shown in the

[1] From "Determination of the Theoretical Pressure Distribution for Twenty Airfoils," by I. E. Garrick, *Rept.* 465.

left-hand diagram is characteristic of a sharp curvature of the streamlines and so of impending separation. The pressure along the lower surface is slightly higher with the thinner section than with the thicker of the two, since the latter has the larger convexity of lower surface form.

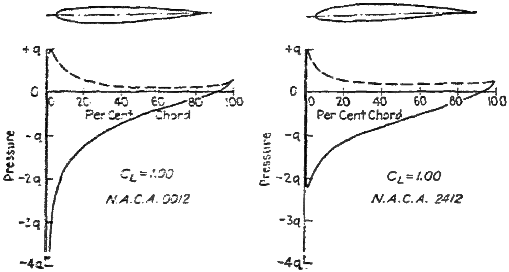

FIG. 146.—Pressure-distribution diagrams at C_L = 1.0, showing effect of camber

Figures 146 and 147 both relate to the effect of camber. As the camber is increased from zero towards 6 per cent, the sharpness of the curvature of the section contour immediately behind the leading edge is reduced, while curvature farther back along the upper surface is increased. It would then be anticipated

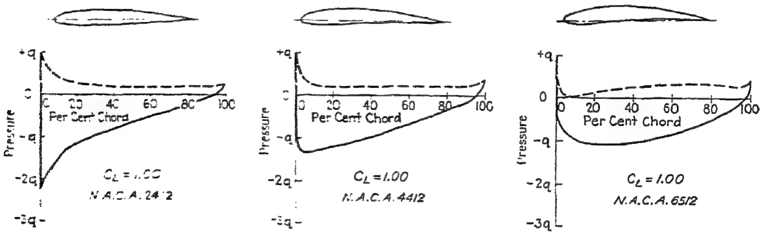

FIG. 147.—Pressure-distribution diagrams at C_L = 1.0, showing effect of camber.

that the effect of increasing camber would be a reduction in the pressure drop immediately adjacent to the leading edge and an increase in its magnitude farther back along the surface. The anticipation is perfectly borne out by the curves. Equally clear from the geometry of the sections are the reasons for the gradual decrease of positive pressure on the forward part of the lower surface and its gradual increase on the after part, as increasing

camber diminishes the suddenness of change of direction of the streamline encountering the lower surface, on the one hand and introduces an actual concavity of lower surface to maintain a continually increasing downward inclination of the streamlines as they approach the trailing edge, on the other.

The last figure of the group (Fig. 148) indicates the consequences of the reflexing of the trailing edge. The pressure distribution on the first half of the chord is relatively little affected, except that to attain the lift coefficient for which the comparison is made the section with the reflex form must be set at a larger angle of attack and consequently displays a higher

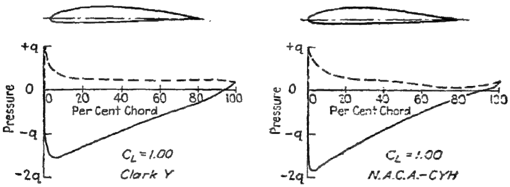

FIG. 148.—Pressure-distribution diagrams at $C_L = 1.0$, showing effect of reflexing trailing edge.

maximum of pressure drop. The major changes, however, are near the trailing edge, where the reflexing produces a reversal of the curvature of the streamlines near the trailing edge of the upper surface and an increased degree of convexity of the lower surface, operating to reduce the pressure drop above the airfoil and to reduce the pressure below, with a pronounced reduction of the amount of lift contributed by the rearmost quarter of the section.

Airfoil Performance in the Compressibility Range.—The theory of airfoil action at superlatively high speeds has already been discussed, and the results of wind-tunnel experience summarized in general terms.[1] It will be recalled that the principal characteristic was a very rapid increase of drag over a small range of speed, usually starting at from 70 to 80 per cent of the speed of sound, and that the increase started at progressively lower speeds as the lift coefficient at which the section was operating was increased. With that much known, it becomes the problem of the aeronautical engineer, and especially of the propeller designer, to find airfoil forms that will postpone the

[1] See p. 111, *supra*.

compressibility effect to higher speeds or that will minimize it when it appears.

It cannot be said that they have had any great measure of success. Twenty-two airfoils have so far been tested in the high-speed wind tunnel of the National Advisory Committee and reported upon.[1] They cover a wide range of form, but while they show certain differences of behavior the differences are much smaller than might have been hoped for. Better results can be obtained in propeller designing with a good high-speed section than with a poor one, but the essential problem remains unsolved. With the best that is now known, compressibility effects seem to set up an insuperable barrier to flight at over 600 m. p. h. with anything like the present type of airplane. Above that speed, the power required to drive the wings through the air swiftly increases to impossible values.

A convenient general index of high-speed efficiency is the value of the ratio V/v (v being the speed of sound in air, 770 m. p. h.) at which the profile-drag coefficient for zero lift equals 0.03, or from three to four times the low-speed value of the same coefficient for good airfoils. For the twenty-two sections tested by Stack the ratio ranges from 0.70 to 0.89. It is primarily a function of thickness, and the most important lesson on compressibility is that the thickness ratio should be kept to the lowest possible figure. At 80 to 85 per cent of the speed of sound, the drag of a section of 6 per cent thickness may be barely half that of one of 9 per cent. Hence the virtues of thin metal blades, as against the heavier sections of a wooden blade, for high-speed propellers on high-powered engines.

When attention is narrowed to the sections of 9 per cent thickness, of which there were fourteen, the critical ratio as just defined is found always, except for one section with a flat lower surface, to lie between 0.81 and 0.87. Even that difference, small as it seems, may make an enormous difference in performance for a propeller that has to run with the widest part of its blade in the critical region.

The sections that have the smallest profile drags at low speeds prove generally to have the longest delayed appearance of the

[1] "The N. A. C. A. High-speed Wind Tunnel and Tests of Six Propeller Sections," by John Stack, *Rept.* 463; "Tests of Sixteen Related Airfoils at High Speeds," by John Stack and Albert E. von Doenhoff, *Rept.* 492.

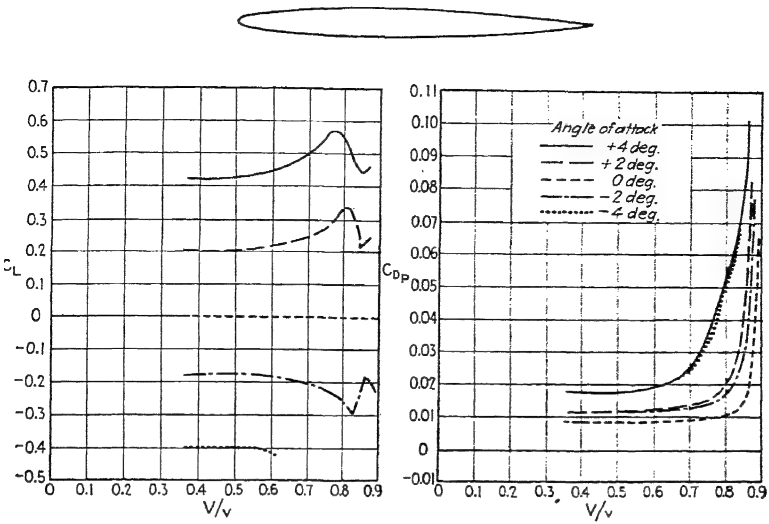

FIG. 149.—Symmetrical section for ultra-high speeds, with curves showing compressibility effect on lift and drag.

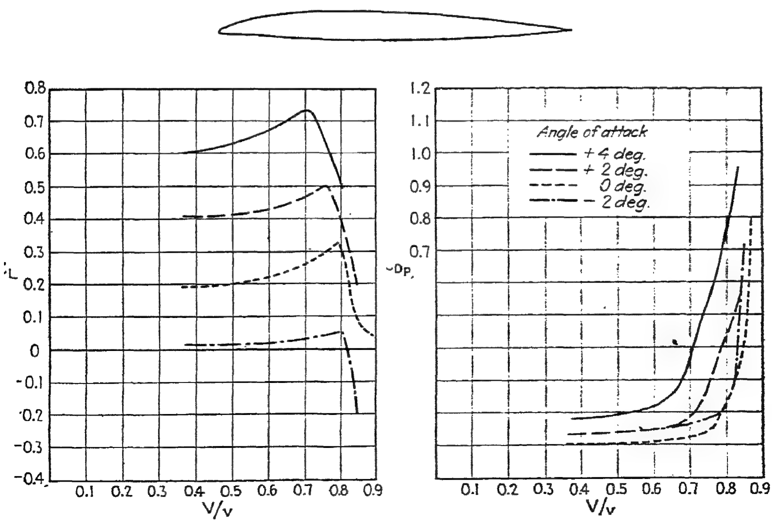

FIG. 150.—Cambered airfoil section found best suited for ultra-high speeds, with curves showing compressibility effect on lift and profile drag.

compressibility burble as well. There is a slight predisposition toward a sharp nose and a displacement of the point of maximum thickness back to nearer the trailing edge than usual, but the best sections show less emphasis on those points than would be expected from experience with projectiles. A symmetrical airfoil with its maximum thickness at 60 per cent of the chord proves to be distinctly inferior in the avoidance of compressibility effects to one with the thickest point at 40 per cent. Symmetrical airfoils prove to display high-speed characteristics virtually independent of the leading-edge radius, provided only that it does not exceed 1 per cent of the chord. Whether or not the same is true for cambered sections cannot, from evidence so far accumulated, be affirmed. There have been no comparative tests of forms other than the symmetrical ones differing only in leading-edge radius. The forms of the best symmetrical section so far discovered and of the cambered section found to give the greatest retardation of the compressibility burble at a lift coefficient of 0.4 are displayed in Figs. 149 and 150, together with curves of lift and of profile drag against V/v at several different angles of attack for each section.

CHAPTER VIII

EFFECT OF PLAN FORM ON AIRFOIL PERFORMANCE

The lift and drag of an airfoil are not dependent on sectional form alone. They are controlled to some extent by other geometrical characteristics of the wing assembly. Especially, as we have already seen,[1] are they controlled by the plan form of the individual wing.

There are several ways in which a wing can be varied in plan. A rectangular shape may be maintained while the ratio between the long and short sides of the rectangle changes, or it may be tapered in respect of chord length to have the shape of a double trapezoid instead of that of a rectangle, or the tips may be raked or rounded in all sorts of fashions. Taper in section, too, or the decrease of thickness of the airfoil section from the mid-point of the span out to the tip, although it is not strictly a matter of plan form, may properly be coupled with it for discussion. As taper in section is seldom entirely separate from taper in chord length, and as both relate to changes along the span, they may well be considered together. The same is true of "washout" or "washin" of angle, or the twisting of the wing so that the angle of attack varies along the span.

Aspect-ratio Effects.—By far the most important of possible plan-form modifications is change of aspect ratio, defined as ratio of span to chord. When the wing is rectangular in form the application of that definition is obvious, but when the plan form is tapered or the tips much beveled for rake it is customary, as previously noted, to take the aspect ratio as given by the formula

$$R = \frac{b^2}{S}$$

where R is the equivalent aspect ratio, b the extreme span, and S the area.

[1] P. 87 *et seq.*, *supra.*

A great number of aspect-ratio tests have been made in wind tunnels, some of the work having been published as early as 1910. They are still useful in studying the effect of aspect ratio on maximum lift, but as a basis for the determination of correction factors to be applied to drag, the principal reason for which they were undertaken, they have long since been superseded by the results of induced-drag computations.[1]

Computation has now taken the field over so completely that most of the aspect-ratio work done in the wind tunnel[2] dates back a dozen years or more. The best of it indicates a steady but slow increase of maximum lift with increasing aspect ratio. Such an increase could not be quantitatively predicted

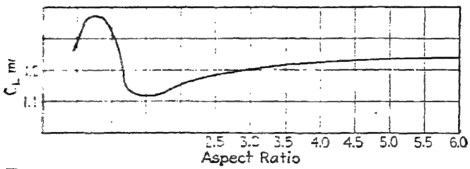

FIG. 151.—Effect of aspect ratio on maximum lift.

by any existing theory, but it might reasonably have been foreseen, as extending the leading edge and so reducing the flow of air parallel to the span and cutting down the tip losses would be expected to raise the lift per unit of area, especially on the parts of the wing near the tips. The effect is so small as to be of but little significance to the designer in his selection of a plan form, as the maximum lift is decreased only about 5 per cent by cutting the aspect ratio from 6 to 3 and increased a similar amount by increasing from 6 to 9. The change within the range of aspect ratios ordinarily used is approximately 1½ per cent for each unit by which the ratio is changed.

When the aspect ratio is allowed to drop below 2, as it never does for wings but may on occasion for tail surfaces, the slope of the curve of maximum lift against aspect ratio reverses. At an aspect ratio of approximately unity the maximum lift suddenly

[1] See pp. 87–95, *supra*.

[2] "Effects of Varying Aspect Ratio," *Bull.*, Exp. Dept., Airplane Eng. Div., U. S. Army Air Service, August, 1918; "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. I, by L. von Prandtl, C. Weiselsberger, and A. Betz, p. 50, Berlin, 1921.

rises to about 20 per cent above the value for a wing of the same section and of an aspect ratio in the ordinary range of design practise for wings[1]—a curious phenomenon first observed on flat plates by Eiffel in 1909, responsible at least in part for the original inspiration of the slotted wing,[2] and still hard to explain. With rounded tips the peak is still more evident, an "airfoil" of circular plan form having an aspect ratio of 1.27 and a maximum lift just 50 per cent above that of a wing of the same section and of aspect ratio 6. Figure 151 shows the relationship over a considerable range of aspect ratios.

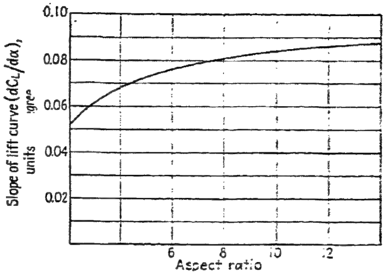

FIG. 152.—Variations of slope of lift curve with aspect ratio, for elliptic loading.

The relation between aspect ratio and lift-curve slope has already been treated in the light of the results of mathematical analysis,[3] and in Fig. 152 it is plotted on the assumption of an elliptical curve of load distribution along the span and on the further assumption, a fair approximation for the average airfoil of

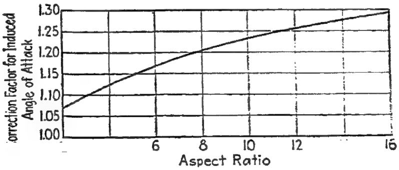

FIG. 153.—Effect of rectangular form of wing on induced angle of attack.

from 8 to 12 per cent thickness, that the slope at infinite aspect ratio is 0.1 per degree, 10 per cent below the theoretical slope for a thin airfoil in a perfect fluid. The exact form of the span-load curve has a pronounced influence on the lift-curve slope, and Fig. 153 gives a curve of the coefficient by which the aspect ratio of a wing with square tips and a chord constant throughout the span

[1] "Characteristics of Clark Y Airfoils of Small Aspect Ratios," by C. H. Zimmermann, *Rept.* 431.

[2] P. 297, *infra*.

[3] P. 92, *supra*.

must be divided to get an equivalent elliptic-loading aspect ratio for use in determining the slope from Fig. 152. Thus a rectangular wing of aspect ratio 7 is equivalent in lift-curve slope to one with an ideal loading curve and an aspect ratio of 7/1.19 or 5.9, and from Fig. 152 the anticipated slope is read off for such a wing as .076.

The N. A. C. A. tests on wings of superlatively low aspect ratio show the theoretical indications on slope well sustained

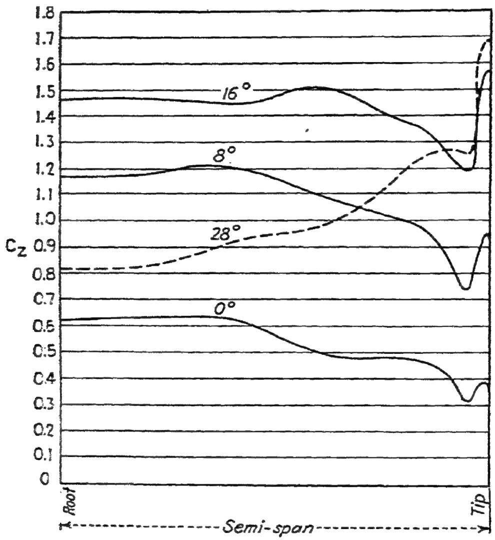

FIG. 154.—Span-load grading curves for rectangular wing at several angles of attack.

by experiment even down to aspect ratios as low as 1.5, where the slope is less than half that for aspect ratio 6.

A rectangular form of wing of course does not imply a constant load from center to tip. The change of downwash along the span[1] and the induction of the tip vortex changes the true angle of attack and the direction in which the air flows across the wing, with resultant change in loading intensity. All this has been discussed qualitatively in Chap. V,[2] and it is quantitatively

[1] See p. 82, *supra*.
[2] P. 84, *supra*.

shown in Figs. 154 and 155. It will be observed that burbling appears much later at the wing tip than near the center line, the explanation lying in the increase of downwash in that region. At

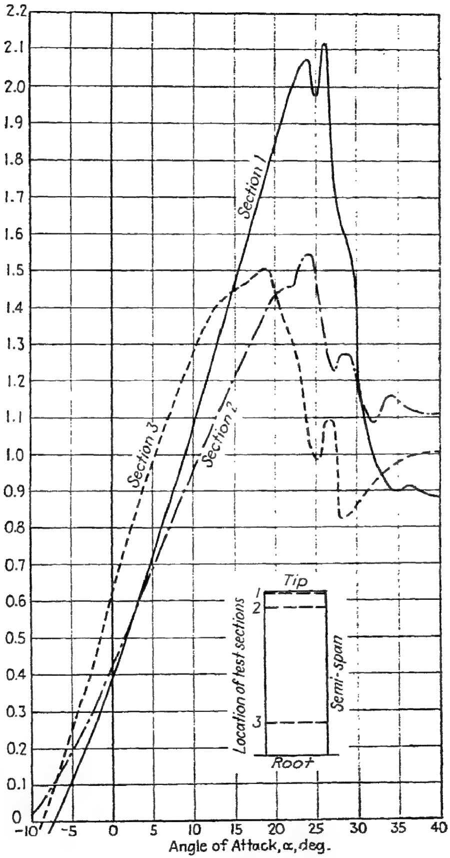

FIG. 155.—Variation of lift at angle of attack at various points along the span of a rectangular wing.

angles beyond that of normal stall, the lift intensity may increase steadily, from center line to tip (as in Fig. 154, at 28 deg.).

One more aspect-ratio effect on the shape of the lift curve near its peak is shown in Fig. 156, along with the other lift-

curve phenomena already mentioned. With increasing aspect ratio, beyond a ratio of 3 or thereabouts, the peak of the lift curve becomes progressively sharper, the curve running up substantially as a straight line more and more nearly to the maximum point and then dropping off with increasing abruptness after passing the maximum. It is probably a matter of secondary importance, but it may have some influence on stability and

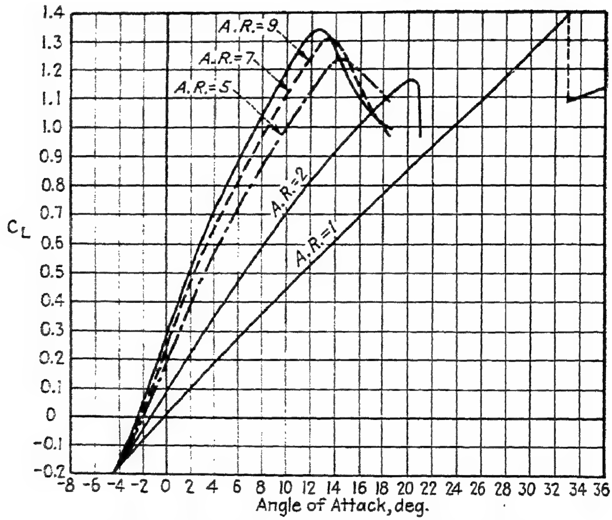

FIG. 156.—Relation of lift-curve form to aspect ratio.

control at low speeds. When the aspect ratio is reduced to below 3 the trend reverses, and by the time it reaches unity the peak of the curve becomes, as will be noted in Fig. 156, not only sharp but discontinuous.

Of greater importance in its effect on stability, and especially on spinning tendencies, is the form of the lift curve beyond the peak. That will be discussed in more detail elsewhere, but for the moment it may be noted without illustration that the second maximum of lift, to which allusion has previously been made,[1] becomes more and more accentuated as the aspect ratio increases.

[1] *Rept.* 431, *cit. supra.*

At an aspect ratio of 8, in the best series of tests available,[1] the second maximum comes at an angle of 35 deg. and falls only 20 per cent below the first in value. With the ratio reduced to 4, on the other hand, the second peak has disappeared entirely as a maximum, reducing itself to the status of a little bump in the curve and a momentary hesitation in the steady decline of lift. At negative lifts, on the other hand, burbling as shown by a kink in the lift curve becomes progressively more pronounced as the aspect ratio is reduced—but that may have been an eccentricity of a particular section in the single set of tests that exist.

On moment coefficient, as on zero-lift angle, aspect ratio is substantially without effect—again in accordance with theoretical prediction.

Aspect Ratio and Wing Efficiency.—Here the wind tunnel is abandoned in favor of the slide rule. Since the effect of aspect ratio is felt only in induced drag, that effect will manifestly be most important when the induced drag itself is largest. The minimum drag, which comes near to the angle of zero lift, is scarcely affected at all by changes of plan form, but the minimum D/L and the minimum value of $C_D/C_L^{3/2}$ show the influence of any change very strongly. With structural requirements neglected for the moment, then, the tendency would be to equip airplanes intended for climbing to great altitudes or for maximum economy of flight over long distances with wings of exceedingly high aspect ratio, while for racing machines, flying at their maximum speed at very low lift coefficients, the ratio would be of little importance except as it might affect stability and control. The relation between aspect ratio and particular type of service desired has never been shown more clearly than in fifteen years' development of soaring gliders. The conditions of maximum efficiency of a glider are much the same as those of maximum efficiency in climb, and structural weight is of relatively little importance, so that some of the best of the gliding craft have been fitted with wings with aspect ratios as high as 16 or even 20. In powered airplanes the normal range is from 5 to 8 in monoplanes and from 6 to 9 in biplanes.

Turning now from the qualitative to the quantitative, a first step is to tabulate and plot the induced drag for various

[1] *Rept.* 317, *cit. supra.*

lift coefficients and aspect ratios. The table follows. A selection of the curves, parabolas all, is in Fig. 155.

C_L	C_{D_i} (induced-drag coefficient) for aspect ratios						
	4	6	8	10	12	16	20
0.1	0.0008	0.0005	0.0004	0.0003	0.0003	0.0002	0.0002
0.2	0.0032	0.0021	0.0016	0.0013	0.0011	0.0008	0.0006
0.3	0.0071	0.0048	0.0036	0.0029	0.0024	0.0018	0.0015
0.4	0.0127	0.0085	0.0064	0.0051	0.0042	0.0032	0.0025
0.5	0.0199	0.0133	0.0100	0.0080	0.0066	0.0050	0.0040
0.6	0.0286	0.0191	0.0143	0.0115	0.0095	0.0072	0.0057
0.7	0.0390	0.0260	0.0195	0.0156	0.0130	0.0098	0.0078
0.8	0.0509	0.0340	0.0255	0.0204	0.0170	0.0127	0.0102
0.9	0.0645	0.0430	0.0322	0.0258	0.0215	0.0161	0.0129
1.0	0.0796	0.0530	0.0398	0.0318	0.0265	0.0199	0.0159
1.2	0.1146	0.0764	0.0573	0.0458	0.0382	0.0286	0.0229
1.4	0.1560	0.1040	0.0780	0.0624	0.0520	0.0390	0.0312

All the foregoing is for the elliptic-loading case. The influence of span-load distribution upon induced drag is much less important than its effect on lift-curve slope, but it is too large to be neglected. In Fig. 158 is plotted the factor by which the induced drag as tabulated above or as calculated from (20) must be multiplied to get the proper value for a square-tipped wing. Alternatively, and for many practical problems more conveniently, the method already employed on page 228 can be employed again, and the true aspect ratio of a square-tipped wing can be divided by the factor read off from Fig. 158 to get the equivalent aspect ratio for an elliptically loaded wing of the same induced-drag characteristics as the actual square-tipped one.

The next step requires a reversion to the concept of "standard profile drag" as introduced in the previous chapter,[1] and a return to the analytical expressions there developed for D/L and $C_D/C_L^{3/2}$. It is, of course, obvious, since the numerical increase of induced drag with lift becomes progressively slower as the aspect ratio increases, that the minimum values of D/L and of $C_D/C_L^{3/2}$

[1] P. 179.

will be attained at higher lift coefficients with large aspect ratios than with small ones. By taking the expressions (52) and (55), and substituting 0.0080 for $C_{D_{p\min}}$ and solving for various values of R, the data are obtained for plotting Fig. 159. By calculating

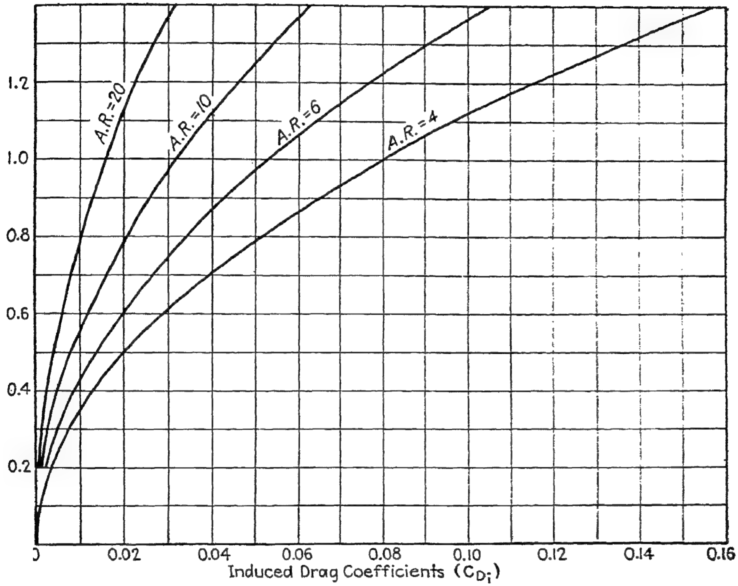

FIG. 157.—Variation of induced drag with aspect ratio.

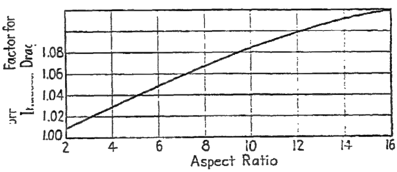

FIG. 158.—Effect of rectangular form of wing on induced drag.

the minimum values at the values of C_L so determined, Fig. 160 results. This, again, is based on the assumption of elliptic loading. In dealing with a wing with square or nearly square tips its aspect ratio should be divided by the factor read from Fig. 158, and the equivalent aspect ratio so obtained should be used as the

abscissa above which the appropriate figures will be read off in Figs. 159 and 160.

The virtues of high aspect ratio as a promoter of aerodynamic efficiency are plain, but it is plain too that they are subject to a law of diminishing returns. The higher the aspect ratio is to start with, the less the advantage that is to be derived from each succeeding increment of further increase. The higher, incidentally, the structural price that must be paid for each increment, and the greater the danger of wing flutter and other structural troubles. To get the best possible aerodynamic performance, furthermore, operation must be at a progressively higher lift coefficient and so at a progressively lower speed of flight as the ratio increases, and the reduction of economical cruising speed and climbing speed are often so objectionable on practical operating grounds as to inhibit the actual use of the full limits of efficiency on a high-aspect-ratio machine. As a practical problem of design, the gain in efficiency to be had by increasing aspect ratio beyond about 8 (in a monoplane) seldom appears worth the capturing.

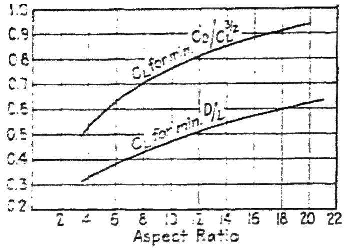

FIG. 159.—Variation of C_L for maximum efficiency with aspect ratio.

The curves give the effect of aspect ratio on minimum wing drag and minimum power coefficient, for an airfoil having "standard" profile-drag characteristics, with perfect rigor. As a rougher approximation, for use in preliminary estimating of the effects of a change, it may be borne in mind that the minimum of D/L decreases 3 per cent for every 10 per cent of increase in aspect ratio, while the minimum of $C_D/C_L^{3/2}$ changes 7 per cent for every 10. Further discussion may await the section of the book that deals expressly with airplane performance and

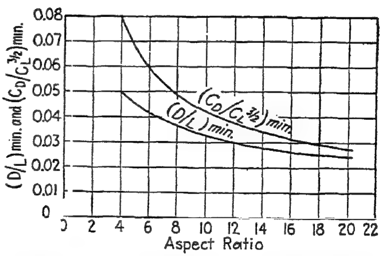

FIG. 160.—Variation of minimum thrust and minimum power coefficients with aspect ratio.

its dependence on the various major design variables, of which aspect ratio is one.

As a final aid to computing the effects of aspect-ratio changes, the amounts by which D/L and $C_D/C_L^{3/2}$ are changed from the

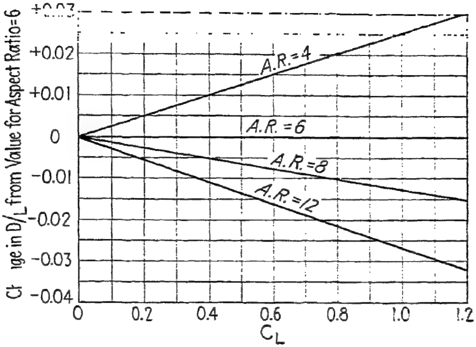

FIG. 161.—Effect of aspect ratio on D/L with standard profile drag.

values tabulated for aspect ratio 6 on page 180 are plotted for several aspect ratios in Figs. 161 and 162. A tabulation like that already made for aspect ratio 6 can be built up at will from the curves, by adding to or subtracting from the original tabu-

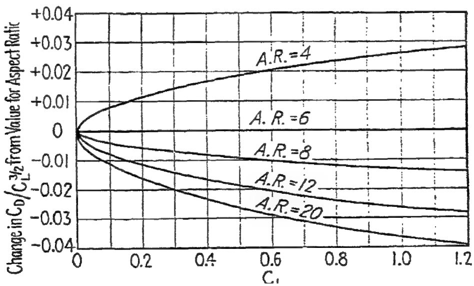

FIG. 162.—Effect of aspect ratio on $C_D/C_L^{3/2}$ with standard profile drag.

lated values such corrections as the curves indicate. The values of $\Delta(D/L)$ and $\Delta(C_D/C_L^{3/2})$, the effects of unit departures from standard profile drag, are the same for all aspect ratios. It may be stated, without taking the space for an arithmetical demonstration, that the general form of the curves of Figs. 159 and

160 would be much the same for all reasonable airfoil sections, whether their profile-drag characteristics adhere closely to the selected standard or lie a considerable distance above it. The absolute values of the ordinates on the curves would of course be changed, but it would be by a shifting of the curves approximately parallel to themselves, preserving essentially their present slope and curvature.

Effect of Slits through Wings.—The general nature of the influence of a continuous opening through a wing parallel to the chord, such as may be left at the point of juncture of two panels or of the attachment of a wing to the fuselage, was described in

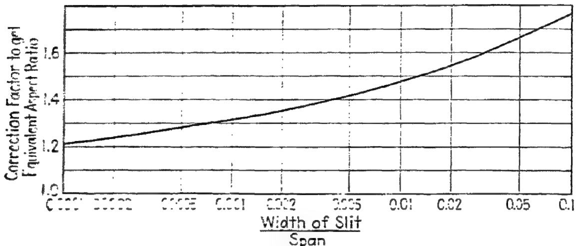

Fig. 163.—Change of induced drag or of apparent aspect ratio, due to slit through wing parallel to the chord.

an earlier chapter.[1] It remains only to give the figures, and Fig. 163 does that. The quantity plotted is the correction factor for aspect ratio, or factor by which the actual aspect ratio must be divided to give the equivalent aspect ratio with due account for the slit. The same factor gives the ratio of actual induced drag to the induced drag that would exist if there were no slit. Anyone who may prefer (as some engineers do, though it is not recommended) to keep the induced-drag calculations entirely in terms of span and span loading can divide the actual span by the square root of the figure taken from Fig. 163 to get the equivalent span after the slit is allowed for.

The curve has been plotted in logarithmic units to permit of close reading for very narrow slits. Unfortunately there is no adequate wind-tunnel check on this particular correction factor. Without it, it is a little difficult to believe that a slit with a width of 1/10,000 of the span, or barely 1/16 in. on the ordinary airplane,

[1] P. 101, *supra*.

can reduce the effective aspect ratio by 17 per cent. Boundary-layer effects of which the theory takes no account seem likely to intervene and reduce the flow through the slit. However that may be, and judgment must be suspended until there is more direct evidence, there is no doubt that the effect of any slit of appreciable width is serious and that great care ought to be given to covering all openings. Where the width exceeds 0.002 of the span the corrections plotted in Fig. 163 can be used with some confidence, though they probably tend toward overestimation of the effect in all practical cases.

End Plates on Wings.—Again to revert to a subject briefly touched in Chap. V, the vertical plate screening the tip of a wing against lateral flow has to be considered from the point of view of practical design. It comes out of the examination so very badly that it seems unnecessary to give exact figures. The frictional drag on the plates themselves overweighs their effect in reducing induced drag over the range of lift coefficients where efficiency is most important, and only at values of C_L above 0.5 are they likely to produce a net decrease in drag. The only cases in which they seem likely to be of any practical value are those of a plane designed exclusively to break the duration record (not distance; the plates would be disadvantageous for that) or one intended to make a record for getting into the air with an absolute minimum of power. On an airplane designed to be flown by manual power, for example, the plates would probably be worth while. Circular plates with a diameter of twice the chord reduce the total drag at $C_L = 0.8$ by about 15 per cent on a wing of aspect ratio 6. Their effect, of course, becomes proportionately more important as the aspect ratio is reduced. Anyone who wants more data than that can turn to the original sources.[1]

Cutouts in Wings.—It often appears necessary, in the interest of vision or of easier accessibility of a cockpit, to deform either the leading or the trailing edge of a wing. Though research on the effects of such a deformation has been scattered,[2] and much

[1] "Flügel mit seitlichen Scheiben," by F. Nagel, *Vorlaufige Mitteilungen der aerodynamischen Versuchsanstalt zu Göttingen,* No. 2, 1924; "The Effect of Shielding the Tips of Airfoils," by E. G. Reid, *Rept.* 201; "Drag of Wings with End Plates," by Paul E. Hemke, *Rept.* 267.

[2] "Messungen an Profilen mit abgeschnitte Hinterkante," "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. III, pp. 82, 94,

of it has been concerned with forms quite different from those generally employed in practice, its indications of startlingly bad effect on wing performance are clear. The best test for practical inference, covering a cutout extending over a maximum of one-quarter of the span and to a maximum depth of 53 per cent of the chord at the trailing edge and 7 per cent at the leading edge (illustrated in Fig. 164), shows a loss of maximum lift of

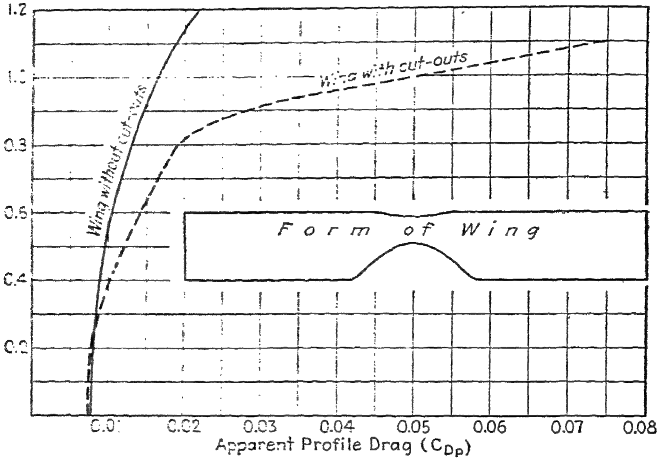

FIG. 164.—Effect on the aerodynamic characteristics of a wing of cutting away the trailing edge.

23 per cent, a negligible effect on drag up to $C_L = 0.4$, and drag increases beyond that as plotted in Fig. 164. The sudden break in drag curve at $C_L = 0.8$ is due to the burbling of the cutout portion of the wing at that point and the gradual outward extension of the broken-down flow after the collapse has once started at the center section. That in turn is due to the distorted span-lift curve, which abruptly decreases the downwash at the center section; to the consequent existence of a larger "true" angle of attack at the center section than elsewhere, since it will be recalled that the true angle of attack at any point is equal

1927; "Neuere Messungen an Flügeln mit Ausschnitten," *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Apr. 15, 1929; "Air Force and Moment for N-20 Wing with Certain Cut-outs," by R. H. Smith, *Rept.* 266; "The Aerodynamic Effects of Wing Cut-outs," by Albert Sherman, *Rept.* 480.

to the apparent, or geometrical, angle of attack minus half the downwash; and to the correspondingly early attainment at the center section of the true angle of attack at which early separation and consequent burbling take place. Sherman[1] has explained the theory in some detail and calculated the effect for a particular case. Obviously cutouts, like slits in the wing, are to be avoided at any reasonable cost of weight or inconvenience.

Tapering Wings in Plan.—Since a definition has been given for the mean aspect ratio of tapered wings, it might be supposed that the effect of taper in chord could be allowed for by treating the wing as equivalent to a rectangular one of corresponding aspect ratio. There is no mathematical proof that that would be correct, however, as a change in plan form of the wing changes the lift grading curve farther away from or brings it closer to the semielliptical form on which the induced-drag theory is based and therefore changes the degree of validity of the basic assumption. Experiments are necessary, and some have been made,[2] though only a few that apply strictly to this particular problem. The results of the wind-tunnel work indicate that a small amount of taper, the chord, of course, being a maximum at the center and reduced toward the tips, is of some aerodynamic benefit. The best results seem to be obtained when the chord at the tip is about one-half that at the plane of symmetry of the airfoil, although the ratio may be anything between one-fourth and two-thirds without very appreciable change of effect. If it is remembered that an elliptical gradient along the span is the theoretical ideal, Fig. 165, where a trapezoid of one-half taper has been superposed on an ellipse, shows that the experimental results are just about what ought to have been expected. Glauert has treated the effect of taper on induced drag analytically[3] and found that with a tip chord anywhere between a quarter and a half that at the center of the wing the induced drag should exceed that for an elliptical airfoil by less than 2 per cent. The

[1] *Rept.* 480, *cit. supra.*

[2] "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. I, by L. von Prandtl, C. Wieselsberger, and A. Betz, Berlin, 1921, p. 63 *et seq.* "The Aerodynamic Properties of Thick Airfoils," Pt. II, by F. H. Norton and D. L. Bacon, *Rept.* 152.

[3] "The Elements of Aerofoil and Airscrew Theory," by H. Glauert, p. 152, Cambridge, 1926.

actual values resulting from the computation, both for induced drag and for induced angle of attack (and so, indirectly, for lift-curve slope) are plotted in Fig. 164. The computations are made for an aspect ratio of approximately 6. The experimental results give evidence that something more than a reduction of induced drag is involved, for they show a reduction even at zero lift. As compared with the rectangular wing of aspect ratio 6, the performance of the best tapered wing of the same aspect ratio can be approximated by subtracting from the basic drag at each point 3 per cent of the induced drag plus 10 per cent of the minimum profile drag. As compared with a wing basically rectangular but

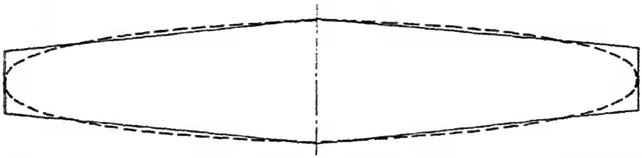

FIG. 165.—Comparison of form of an ellipse and of a one-half straight taper.

with the tips well rounded,[1] the proportion of induced drag to come off is reduced to about 1½ per cent. There is, of course, a structural benefit from tapering in chord, giving as it does the maximum depth of the spars at the point where the moments and shearing forces are largest in a cantilever or semicantilever machine, which applies equally at high and low speeds.

If the tapering is carried to the point of reducing the chord at the tip to zero, running the wing out to a point, there is not only a loss of efficiency as compared with that for the best taper, but the results are actually worse than for the rectangular wing, the drag coefficients being increased above those for the rectangular form of equal equivalent aspect ratio by from one to two times as much as they are reduced by the adoption of that moderate taper which gives best results. The extremely bad effects of tapering to a tip chord of less than one-fifth the root chord are apparent from Fig. 166. There is much experience, also, to indicate that an excessive taper has a very bad effect on lateral control by causing a premature burbling at the tips of the wings due to a reduced downwash there, analogous to the premature burbling found at a cutout center section.[2] To supplement

[1] See p. 242, *infra*.

[2] See p. 238, *supra*.

direct flight experience and theory, pressure-distribution tests[1] provide a certain amount of support for that presumption.

Tapering in chord has some effect on maximum lift, as well as on drag. The best ratio of taper for maximum lift seems to be about the same as for drag, and the effect of changing the rectangular wing to that optimum form was to increase the maximum-lift coefficient by 2 per cent in the German series of tests and by 10 per cent in the American (the two series, of course, related to different basic airfoil sections). The pointed wing

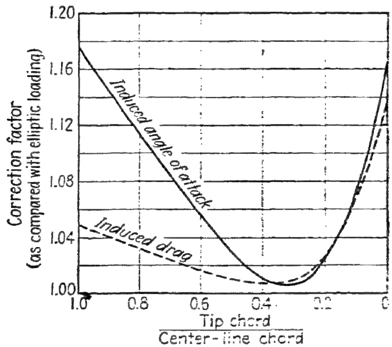

FIG. 166.—Computed variations of induction-correction factors with amount of taper, for aspect ratio 6.3.

gives substantially the same maximum-lift coefficient as the rectangular one.

One of the models tested in Germany was made elliptical in form instead of with the straight taper, with the object of approximating as closely as possible to the mathematically ideal semi-elliptical load-grading curve. The results in all respects were very nearly identical with those for the best straight taper.

Shaping of Wing Tips.—From the wing of elliptical plan form to one with the leading and trailing edges parallel throughout the greater part of their length, but with the tips raked or rounded, is an easy and logical step. On this subject there is experimental evidence from many sources,[2] though surprisingly

[1] "Pressure Distribution over Thick Airfoils—Model Tests," by F. H. Norton and D. L. Bacon, *Rept.* 150.

[2] "The Effect of Shape of Wing Tips on the Lift, Drag, and Center of Pressure of an Aerofoil," by E. F. Relf, *R. and M.* 152; "The Effect of

little of it is modern. All of the data are in agreement in showing that changing the tip form from the original rectangle slightly increases both the lift coefficient and the lift/drag ratio. In general, provided the effect of the changing tip form does not extend inward more than one chord length from the tip, the maximum-lift coefficient increases so little with change of tip form that it can hardly be allowed for. The effect seldom exceeds 1½ per cent. In so far as there is an increase of lift, it is more apparent with positive rake than with negative (the rake of the wing tip is said to be positive when the trailing edge is longer than the leading one), and more apparent with a raked tip than with a rounded one.

The effect on drag is more apparent. The results are somewhat conflicting, and much appears to depend on the exact way in which the tips are shaped and on how the edges that are oblique to the wind are rounded over, but as a general rule it can be said that a well-shaped tip reduces the drag at any point by

$$\Delta C_D = 0.015 C_{D_i} + 0.05 C_{D_p} \tag{56}$$

where C_{D_i} and $C_{D_{p\min}}$ have their customary significance of induced-drag coefficient and minimum coefficient of profile drag. The benefits are thus just one-half those promised on page 240 from the best form of taper, and in the specific case of minimum drag a reduction of about 5 per cent can be expected, while the minimum of D/L should be reduced by about 3 per cent. There is, of course, a further gain from the fact that shaping of the tip reduces the area and so increases the true aspect ratio b^2/S that

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Aspect Ratio and Shape of Wing Tip on Aerofoil Characteristics," by H. Glauert, *R. and M.* 575; "Comparative Tests of a Biplane of R. A. F. 15 Wing Section When Fitted with Wing Tips of Four Different Types," by H. B. Irving and E. Ower, *R. and M.* 557; "A Comparison between the Aerodynamic Properties of Two Aerofoils of the Same Section but with Square and Rounded Wing Tips, Respectively," by W. L. Cowley and C. N. H. Lock, *R. and M.* 816; "An Investigation on the Effect of Raked Wing Tips," by F. H. Norton, *Tech. Note* 69; "Lift and Drag Effects of Wing-tip Rake," by A. F. Zahm, R. M. Bear, and G. C. Hill, Rept. 140; "The Effect of Wing-tip Outline and Washout of Camber on the Aerodynamic Efficiency," by J. H. Parkin, H. C. Crane, and S. L. Galbraith, *Aero. Research Paper* 4, *Bull.* 2, School of Eng. Research, Univ. Toronto. Reference should also be made to the German work previously cited in connection with the tapering of wings.

is to be used in computing the induced drag. The effect of tip form on profile drag seems to depend less on plan form than upon a careful smoothing over of the edges of the tip and keeping a good sectional form all the way out to the extreme end.

The best tip forms for drag reduction, if it is assumed that the shaping is to extend in by not more than half a chord length from the tip, seem to be the semicircular and that with a positive rake of about 30 deg. Negative rake is again inferior to positive, as it was in its effect on maximum lift. Structurally, however, negative rake is better than positive, as it keeps the centers of pressure of all elements of the wing more nearly in a straight line parallel to the span. Better yet, and for the same reason, is a combination of two elliptical quadrants, their eccentricities proportioned as shown in Fig. 167 to bring the extreme span at a point about one-third of the way back on the chord.

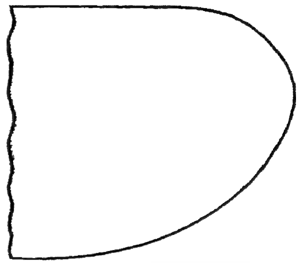

Fig. 167.—Recommended form of wing tip (double elliptical).

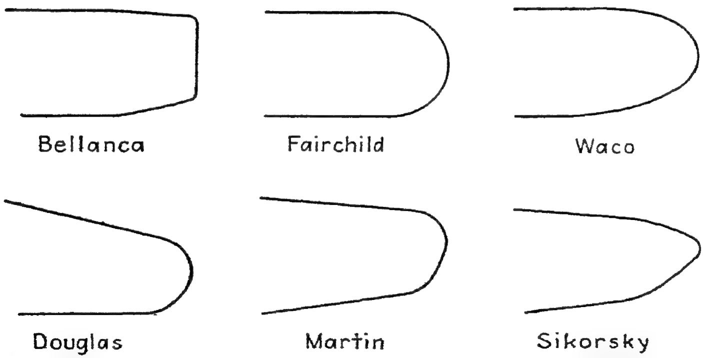

Fig. 168.—Representative wing-tip forms from recent designs.

That is the most typical of present forms, as the combination of wing-tip outlines from half a dozen representative planes of recent design, shown in Fig. 168, serves to show.

The actual span-load-distribution curve can be obtained for any tip form by taking the pressure distribution. Distribution

curves[1] are given for several forms in Fig. 169, and it will be observed that the distorted elliptical tip already highly commended for its structural virtues also approximates very closely indeed to the ideal span-load distribution.

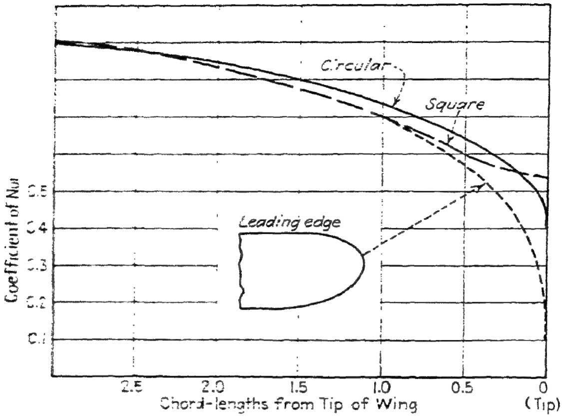

FIG. 169.—Span-load-distribution curves for various wing tips.

Center-of-pressure Movement on Tapered Wings.—Tapering the wing in chord or rounding the tip of course has an effect on the mean position of the center of pressure. It has been shown by Glauert[2] that satisfactory agreement with experimental results can be obtained if the airfoil is assumed divided into a large number of strips by lines drawn parallel to the plane of symmetry and if the center of pressure on each strip is assumed to lie at the same fraction of the distance from the leading edge to the trailing edge of that particular elementary section. While that assumption does not strictly correspond to the facts, satisfactory final results nevertheless appear to be obtained by its use. The mean position of the center of pressure over the whole wing can then be found by integration. The process need not be reproduced here. The corrections to be applied to the center-of-pressure coefficient measured on a rectangular airfoil are generally small, and as is readily apparent from the geometry of the problem

[1] "The Influence of Tip Shape on the Wing Load Distribution as Determined by Flight Tests," by Richard V. Rhode, *Rept.* 500.

[2] *R. and M.* 575, *cit. supra.*

they are likely to be negative when the center of pressure is far back, positive when it is far forward. The range of center-of-pressure travel is reduced by shaping the tips. With a tip form such as was illustrated in Fig. 167, and an aspect ratio of 6, the center-of-pressure coefficient ought to be corrected by approximately one-twentieth of the amount by which it varies from 0.333. If the center-of-pressure coefficient for a given lift coefficient with a rectangular wing is 0.5, then, the coefficient with shaped tips would be reduced by (0.500 − 0.333)/20, or 0.008, leaving a corrected coefficient of 0.492. The same rule, in fact, can be used as an approximation for any tip form, substituting for 0.333 the proportion of the chord from the leading

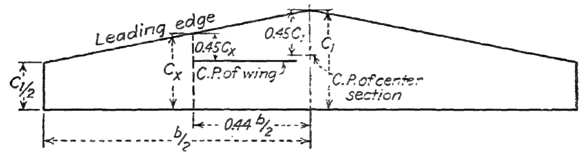

FIG. 170.—Determination of center of pressure for tapered wing.

edge at which the particular tip form used has its extreme span; substituting also, in case of a form of which the shaping extends in to appreciably more or less than one-half chord length from the extreme tip, a divisor changed from the basic value of 20 in inverse proportionality to the length of the shaped portion along the span (giving a divisor of 40 in case of a form of which the shaping is limited to one-quarter chord length from the extreme tip, and so on).

The mean center of pressure of tapered wings can be found in much the same way. Assuming the intensity of loading per unit of span at each point proportional to the chord at that point, the center-of-pressure location can be determined with a maximum probable error of about 1.5 per cent by the geometrical method of laying off a chord line (in the case of a wing of which the tip chord is half the root chord) at 44 per cent of the semispan from center to tip, as illustrated in Fig. 170. The longitudinal position of the center of pressure for the whole wing is then fixed by laying off the appropriate coefficient on the chord so constructed, again as the drawing illustrates. If the airfoil is tapered to a point at the tip the fraction of the semispan at

which the layoff chord is to be constructed is reduced from 44 per cent to 33.[1] If the tip is rounded or raked an additional correction can be made for that fact in accordance with the method explained in the previous paragraph, but with a further adjustment to the divisor in the correction formula in inverse proportion to the ratio of tip chord to mean chord (bringing the divisor, in the case of a typical elliptical tip form, to 30).

It is most convenient, after the center-of-pressure position has been fixed upon by this or any other device, to refer it to the length of the mean chord of the wing, and to the location of a chord midway between the center line and tip projected in upon the plane of symmetry of the airfoil, in computing its coefficient. Coefficients of pitching moment, too, are best computed in terms of the mean chord and referred to an axis at a specified proportion (usually 25 per cent) of the chord located as just described, and that is the customary practice.

Influence of Sweepback.—Wings of constant chord may on occasion be inclined bodily backward or (much more rarely) forward with respect to a line in the plane of the wing and perpendicular to the plane of symmetry. The characteristic is one introduced, as pointed out in the first chapter, most commonly for improvement of the airplane's longitudinal balance, somewhat commonly from considerations of stability, practically never in the interests of efficiency. Nevertheless, it is well to see what happens if sweepback is for some reason employed.

Again there is a scarcity of data. The best of what exists[2] indicates scarcely measurable effects. With 10 deg. of sweep-

[1] These approximations are adequate for most purposes, and in fact they afford an excellent degree of accuracy when the taper is straight and the airfoil section and angle of attack are uniform throughout. For a much more generalized method, which we shall not take the space to discuss at length, see "Charts for Determining the Pitching Moment of Tapered Wings with Sweepback and Twist," by Raymond F. Anderson, *Tech. Note* 483. Another general discussion of the whole subject of taper, but with primary reference to stability characteristics rather than to efficiency, is found in "Investigation of Certain Wing Shapes with Sections Varying Progressively along the Span," by L. Arsandaux, *L'Aéronautique*, March, April, and May, 1928, translated in *Tech. Memo.* 617.

[2] "Experiments on Swept-back and Swept-forward Aerofoils," by D. H. Williams and A. S. Halliday, *R. and M.* 1491; "Span Load Distribution on Two Monoplane Wing Models as Affected by Twist and Sweepback," by Montgomery Knight and Richard W. Noyes, *Tech. Note*, 356.

back, about as much as is ever used in practice except on tailless airplanes, the maximum lift is decreased by some 5 per cent. The angle of zero lift is increased somewhat (in algebraic value), as the changed downwash characteristics make it possible for an appreciable positive lift at the center of the wing to be balanced by a negative lift near the tips. The change of angle is about half a degree for 10 deg. of sweepback.

More important than the change in the maximum amount of lift is the change in the shape of the curve. Sweepback in some

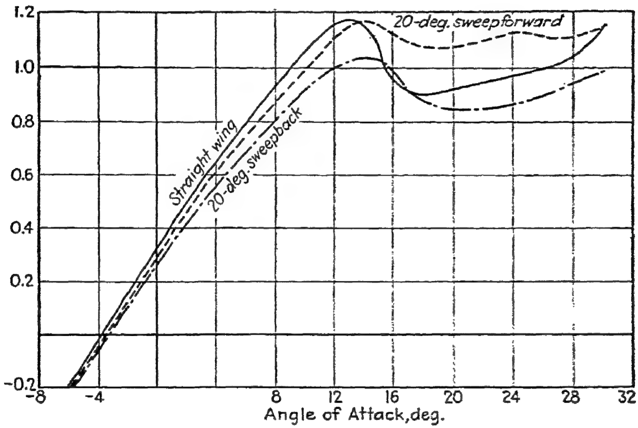

FIG. 171.—Effects of sweepback upon lift characteristics.

cases, and sweep-forward in all, produces a very marked flattening of the peak. An extreme case is shown in Fig. 171, and other tests in general show the same sort of effect, though often in a much less exaggerated form. Tapered wings seem to show it more strongly than straight ones, though there are not enough tests bearing on that point to encourage one to become dogmatic about it. It would need a very impressive demonstration of an extraordinary aerodynamic virtue in the swept-forward wing to make designers reconcile themselves to the use of so odd-looking a shape.

On the effects of sweepback on drag, data obtained under modern conditions are even more completely lacking. The presumption is, from what little is to be found, that changes in drag with any reasonable amount of sweepback would be almost

imperceptible; that so far as they existed at all they would be unfavorable.

Tapering in Sectional Thickness.—The effect of tapering a wing in section is not so easy to discuss, as there is no simple way of expressing the nature of the variation of the form of section statistically, and there has been relatively little direct study of the subject in the wind tunnel.[1] The comparative dearth of information from that source is due in part to the high cost of tapered models for trial in the tunnel. Unfortunately, to explain the deficiency is not to overcome it. The problem of combined taper of sectional thickness and chord is one that arises in a very large proportion of modern design, and one with which we are ill prepared to cope.

The best standard of comparison for a tapered wing is the average of the characteristics of the root and tip sections, assuming that neither of them shows anything very eccentric and that the taper is straight. Unfortunately, out of the few tapered-wing tests that exist there are still fewer in which full data for both the root and tip sections, tested as rectangular airfoils, are to be had. The best of the tests affording opportunity for such comparison show a performance of the tapered wing in general just about equal to the weighted mean (weighted in proportion to the chord lengths at their respective locations) of the separate performances of the sections of which the wing is composed, each section being judged on its characteristics when tested in a plan form identical with that of the model under examination. The first step, in other words, is to take the standard data for the several component sections, each drawn from the customary test of a rectangular model of aspect ratio 6, and to construct a weighted mean set of curves for an imaginary airfoil. The second step is to apply to the curves so drawn the taper correc-

[1] Experiments on the effect of tapering in section are included in *Rept.* 152, already mentioned in other connections, and also in "Tests of U. S. A. 27*A*, 27*B*, and 27*C* Airfoils," *Information Circ.* 224, Airplane Eng. Div., U. S. Army Air Service; "The Aerodynamic Characteristics of Three Tapered Airfoils Tested in the Variable Density Wind Tunnel," by Raymond F. Anderson, *Tech. Note* 367; "Tests of Three Tapered Airfoils Based on the N. A. C. A. 2200, the N. A. C. A. M-6, and the Clark Y Sections," by Raymond F. Anderson, *Tech. Note* 437; Full-scale Force and Pressure-distribution Tests on a Tapered U. S. A. 45 Airfoil," by John F. Parsons, *Tech. Note* 521.

tions already described on page 240. The only error that can be forecast in that method of treatment is that the maximum lift of a wing thick in the center and thin at the tips commonly proves to be a little higher than the weighted mean of the maxima for the component sections. A further arbitrary addition of about 3 per cent on $C_{L_{max}}$ is a safe average. Where the tips of the tapered wing are carefully shaped, instead of being left square, further reduction of drag by about $0.05C_{D_{p_{min}}}$ may be hoped for.

There is, of course, no reason to suppose that the few tests so far made have plumbed the subject to its limit, nor to doubt that a sufficient amount of study will reveal particular combinations of sections or relations of the several sections to each other that will be at least a little better than anything so far tried. Notwithstanding the expense of the models, further wind-tunnel work is badly needed.

One particular advantage of the tapered wing is that, since all the sections along the span do not have the same shape of profile-drag curve or the same angle of minimum profile drag or reach their maximum lift at the same time, the peaks of the curves often tend to be somewhat flatter than for the individual sections. It would, however, be unsafe to generalize. Some of the tapered wings tested at high Reynolds' numbers in the variable-density tunnel show all the indications of sudden burbling, with a discontinuity of the lift curve at its maximum.

Taper either of section or of chord, and certainly the combination of the two, serves to smooth out the sudden changes of pressure distribution toward the tip that characterize the rectangular wing. There still persists, at least at high angles, a local peak of upper-surface pressure drop near the trailing edge at the tip, but its intensity is somewhat diminished. All of this is quite as might be expected from the uniformity of downwash and the decreased role of the tip vortices on a tapered plan form. In Fig. 172 are plotted the normal force contours for a tapered wing at two angles of attack. Of basic section akin to the Göttingen 387,[1] and of thickness tapering from 18 per cent at the center line to 12 per cent at the tip, it showed an almost perfectly regular force distribution at an angle of 3 deg. and a lift coefficient of 0.65. At 21 deg., just beyond the angle of maximum lift but still showing a C_L within 2 per cent of the maximum, the

[1] See p. 159.

highly localized peak of load intensity at the trailing edge rises to about half the leading-edge peak.

Effect of Warp of an Airfoil.—Washout or washin of angle, the changing of the inclination of the chord lines near the wing

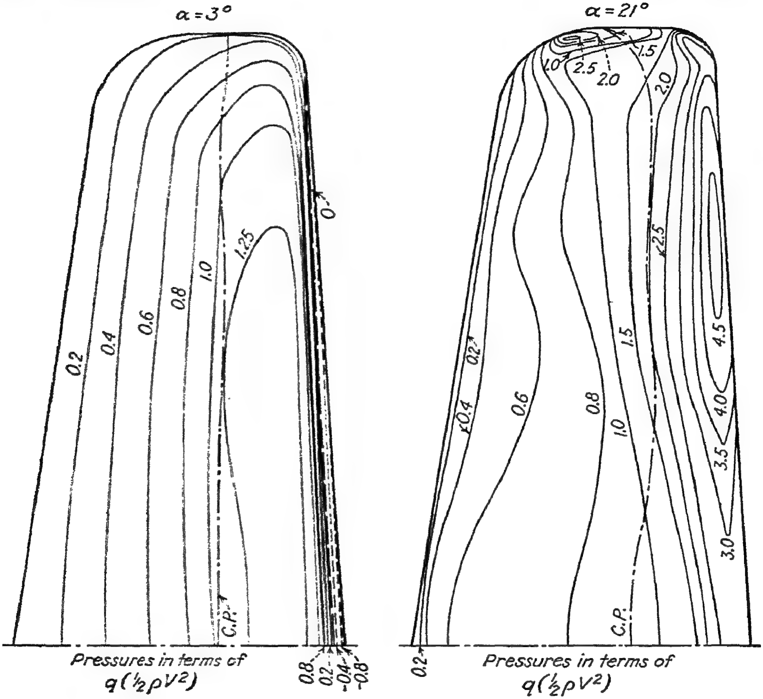

Fig. 172.—Pressure distribution, at two different angles of attack, on a wing tapered in plan and section.

tips, may be used either in conjunction with taper of chord or section or separately. This, too, has been the subject of some wind-tunnel study.[1] The effect on maximum lift is small,

[1] "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, vol. I, p. 63, Berlin, 1921; "La résistance de l'air et l'aviation," by G. Eiffel, p. 174, Paris, 1911; "Wing Tip Research, Pt. II," by J. H. Parkin, H. C. Crane, and J. S. E. MacAllister, *Bull.* 3, Sec. 12, School of Eng. Research, Univ. of Toronto; "Span Load Distribution on Two Monoplane Wing Models as Affected by Twist and Sweepback," by Montgomery Knight and Richard W. Noyes, *Tech. Note* 346.

usually but not always unfavorable, generally amounting to less than 4 per cent unless the warp from center to tip amounts to more than 10 deg. (which would be very unusual). Drag coefficients also are little affected, but so far as there is any effect they are definitely increased throughout the range of lift coefficients by an increase of angle toward the tips. A reduced angle (washout), on the other hand, may improve the drag characteristics slightly. This again is, in part but not in its entirety, an induced-drag effect, as washout of angle in a rectangular wing improves the lift distribution along the span in the same sense as tapering the chord. The best result with washout of which there is record shows a substantially uniform reduction in the drag, of about 3 per cent at all angles, by the use of a twist in the wing starting at the center line and gradually increasing in amount to the tips, the difference between the center and tip angles being 11.1 deg.[1] and the former being the larger. On the average, one might reasonably hope for half that gain. Washout of angle is most important in its beneficial effect on lateral stability, especially when combined with a little sweepback.

Wings of constant section are likely to continue to be made with a constant angle of attack from tip to tip, or if any twist is introduced into the wing it will be put there for its effect on stability and control rather than for any expected increase in efficiency. On a tapered wing there is reason to expect more benefit from twist where the different sections give their best performance at different angles. This holds out promise for profit from a new line of study of tapered-wing performance. In the other pan of the balance, of course, is the fact that no further reduction of induced drag can be expected from the introduction of twist into a wing that has already attained through its taper a substantial approximation to the ideal elliptic distribution of lift along the span.

[1]From the Göttingen tests.

CHAPTER IX

AIRFOIL COMBINATIONS

Although the monoplane has been steadily gaining favor and accounts in 1936 for 80 per cent of the airplanes currently built in the United States, for an even larger share than that of Continental European production, and for two-thirds of the current British crop, machines with two superposed wings still play a substantial part. The star of the biplane has been declining since 1924, but for certain purposes the type has marked advantages and to predict its total disappearance would be far from safe. Where two or more wings are working in close proximity there is of course interaction between them, and the biplane or triplane combination therefore cannot be treated simply as the sum of two or three monoplanes.

Biplane Interference Corrections.—The history of the biplane correction factor largely parallels that of aspect-ratio allowances. In both cases a start was made through wind-tunnel tests directly applied. In both cases a theory was subsequently evolved which, checked against the tests already undertaken, offered a satisfactory basis for subsequent calculations. The biplane problem in its most essential aspects, like that of aspect ratio, has been solved by an induced-drag formula.

Tests[1] on biplane combinations are all in general agreement in their results. They all show a small loss in maximum lift, a

[1] "Experiments on Models of Biplane Wings," by J. R. Pannell and others, *R. and M.* 193; "Model Tests of R. A. F. 19 Wing Section as a Biplane," by J. A. Carroll and F. B. Bradfield, *R. and M.* 648; "Biplane Investigation with R. A. F. 15 Section," Pt. I, by W. L. Cowley and C. N. H. Lock, *R. and M.* 774; *ibid.*, Pt. II, *R. and M.* 857; *ibid.*, Pt. III, by W. L. Cowley, A. G. Gadd, L. J. Jones, and S. W. Skan, *R. and M.* 872; "Biplane Wing Combinations," *Bull.*, p. 136, Expt. Dept., Airplane Eng. Div., U. S. Army Air Service, June, 1918; "Aspect Ratio and Overhang for Biplane Combinations," *Bull.*, p. 18, Expt. Dept., Airplane Eng. Div., U. S. Army Air Service, August, 1918; "The Air Forces on a Systematic Series of Biplane and Triplane Cellule Models," by Max M. Munk, *Rept.* 256; "Preliminary Biplane Tests in the Variable Density Wind Tunnel," by James M. Shoe-

small increase in minimum drag, and a very considerable increase in drag at intermediate lift coefficients. The parallel with aspect-ratio effect is immediately obvious. Qualitatively, at least, putting two wings together in biplane combination produces the same sort of result as cutting down the aspect ratio of a single one. The reduction of maximum lift as compared with that of the single wing ranges, for a biplane with the gap equalling the chord and with no stagger, from 3 to 9 per cent (being least, other

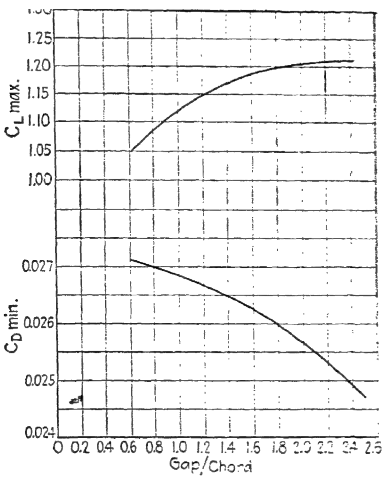

FIG. 173.—Variation of lift and drag characteristics with gap chord ratio.

things equal, at high Reynolds' numbers); 0.94 is a good average correction factor to apply. The factors of gap and stagger must of course be specified in connection with any results, as it is obvious that putting one wing very far ahead of the other or changing the vertical distance between them must modify their mutual interference. The minimum profile drag for the biplane combination is (at high Reynolds' numbers, small-scale tests

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maker, *Tech. Note* 289; "Methods of Experimentation with Models and Utilization of Results," by Lieut.-Col. Robert, *Proc.* Inter. Air Cong., London, 1923; "Études systematiques sur les biplans," by A. Toussaint, Service Technique de l'Aéronautique, Paris, March, 1923; "Biplane Investigation," by J. H. Parkin, J. E. B. Shortt, and J. G. Cade, *Aero. Research Paper* 19, Univ. of Toronto, 1927.

giving a different result) from 3 to 15 per cent higher than that for the monoplane. When the complete set of lift and drag curves for a biplane combination is placed beside that for a single airfoil of the same section, it is also apparent that the effect of the superposition of the two wings is to decrease the slope of the lift curve, increasing the angle of maximum lift, and slightly to flatten off and round over the peak of the curve—again in strict accordance with the aspect-ratio precedent.

All this for a gap equal to the chord. Change that, and everything changes. To give a general idea of the effect of G/c, curves of the variation of maximum lift and minimum drag as the gap/chord ratio changes are plotted in Fig. 173. The values on which the curves are based are taken from British experiments on biplane models of R. A. F. 15 section and aspect ratio 6,[1] unfortunately conducted at very low Reynolds' numbers but accepted in lieu of anything more modern and equally complete.

Biplane Induced Drag.—The theory for biplanes has already been given a simple explanation,[2] though it was at once less simple and less completely logically satisfying than the one that was available in the case of the monoplane. In general, biplane interference can be best expressed by multiplying the induced-drag coefficient for a monoplane wing working at the same lift coefficient by the factor n. The biplane combination is then equivalent to a monoplane of which the aspect ratio is $1/n$ times that of the biplane.

As a prelude to the recital of numerical values, it will be useful to turn back to (30)[3] and rework it. The first step is to expand the general induced-drag formula by substituting $C_L(\rho/2)SV^2$ for L. Thus

$$D_i = \frac{1}{\pi\frac{\rho}{2}V^2}\left(\frac{L_u^2}{b_u^2} + \frac{L_l^2}{b_l^2} + \frac{2\sigma L_u L_l}{b_u b_l}\right) =$$

$$\frac{\frac{\rho}{2}V^2}{\pi}\left(\frac{C_{L_u}^2 S_u^2}{b_u^2} + \frac{C_{L_l}^2 S_l^2}{b_l^2} + \frac{2\sigma C_{L_u} C_{L_l} S_u S_l}{b_u b_l}\right) \quad (57)$$

[1] *R. and M.* 196, *cit. supra.*
[2] P. 95, *supra.*
[3] P. 97, *supra.*

It follows, substituting the expanded form for D_i, that

$$C_{D_i} = \frac{1}{\pi S}\left(\frac{C_{L_u}{}^2 S_u{}^2}{b_u{}^2} + \frac{C_{L_l}{}^2 S_l{}^2}{b_l{}^2} + \frac{2\sigma C_{L_u} C_{L_l} S_u S_l}{b_u b_l}\right) \tag{58}$$

If C_L is assumed the same for the two wings, which is nearly enough true for this purpose so long as there is no decalage, and if it is remembered that the area of a wing is the product of its span by its mean chord, so that $S/b = c$,

$$C_{D_i} = \frac{C_L{}^2}{\pi S}(c_u{}^2 + c_l{}^2 + 2\sigma c_l c_u) \tag{59}$$

Since C_{D_i} is also equal to $nC_L{}^2/\pi R_u$, R_u being the aspect ratio of the upper wing, it follows that

$$n = \frac{R_u}{S}(c_u{}^2 + c_l{}^2 + 2\sigma c_u c_l) \tag{60}$$

This form is perfectly general for all biplanes, whatever their proportions, so long as they lack decalage. In certain particular cases it permits of a further simplification.

Thus, for example, where the upper and lower wings have the same mean chord the equation becomes

$$n = \frac{b_u}{c} \; \frac{1}{c(b_u + b_l)} \; 2c^2(1 + \sigma) = \frac{2b_u}{b_u + b_l}(1 + \sigma) \tag{61}$$

Where, on the other hand, the two wings have the same aspect ratio, chord and span being varied in the same proportion,

$$R_u = R_l = \frac{b}{c}$$

$$S_u = R_u c_u{}^2$$

$$S = R(c_u{}^2 + c_l{}^2)$$

$$n = \frac{c_u{}^2 + c_l{}^2 + 2\sigma c_u c_l}{c_u{}^2 + c_l{}^2} = 1 + \frac{2\sigma c_u c_l}{c_u{}^2 + c_l{}^2}$$

$$1 + \frac{2\sigma \dfrac{b_l}{b_u}}{1 + (b_l/b_u)^2} \tag{62}$$

The values of σ, as computed by Prandtl,[1] are

b_l/b_u	G/b_u						
	0	0.05	0.10	0.15	0.20	0.30	0.40
0.6	0.600	0.526	0.460	0.404	0.357	0.269	0.207
0.8	0.800	0.679	0.581	0.500	0.434	0.329	0.259
1.0	1.000	0.780	0.655	0.561	0.485	0.370	0.290

In Figs. 174 and 175 the values of n have been plotted for the two cases already analyzed, those of equal chords and of equal

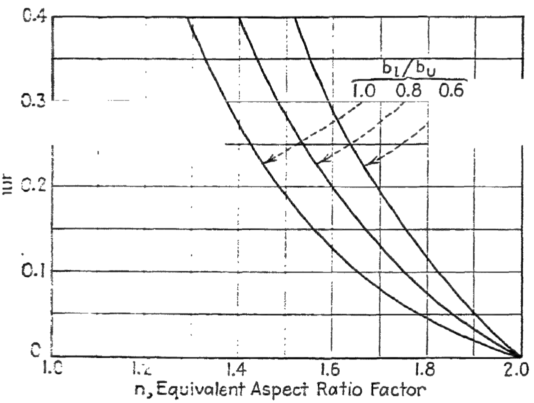

FIG. 174.—Aspect-ratio correction factors (for induced drag) for biplanes with upper and lower chords equal.

aspect ratios. It will be observed that on the first presumption overhang is always bad. If the chords are to be equal, the spans ought to be equal as well to keep the induced drag to a minimum. For any change in that relationship the increase in self-induced drag of the wing of which the aspect ratio is lowered by clipping the span more than offsets the saving due to reduced interference between the two wings. The disadvantage of overhang is, as might well be foreseen, largest when the gap is large and the wings operate most nearly independently of each other.

[1] Prandtl's original values were computed, and they are generally quoted, as functions of the ratio of gap to mean span. They have been modified here to fit the ratio of gap to maximum span.

With equal aspect ratios, on the other hand, overhang is advantageous—again accordant to the indications of common sense. Even this case does not show overhang at its absolute best, for Glauert has shown that the absolute minimum of induced drag for a given combination of spans is obtained when the aspect ratio of the longer wing is considerably lower than that of the shorter one. Though the exact ratio of chords for minimum total induced drag varies considerably with the gap, as a rough general average for ordinary design practice it can be said that the chords should vary as the cube of the spans. The

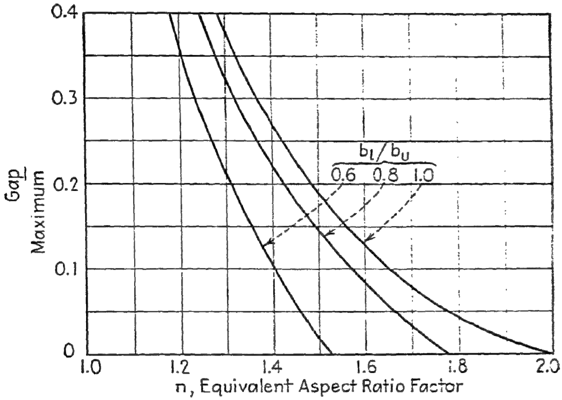

FIG. 175.—Aspect-ratio correction factors (for induced drag) for biplane with upper and lower wings of equal aspect ratio.

conclusion is of limited practical value, as it indicates a lower chord generally too small to be structurally feasible, but it points the road that should be traveled as far as structural limitations will allow.

Among the most practical of all biplane problems is the distribution of a given wing area for maximum efficiency where it is assumed that the ultimate span is for some reason limited. The choice is then between a monoplane of a certain aspect ratio, an equal-winged biplane of double that ratio, and an infinite variety of biplanes with varying span and chord relationships between their upper and lower wings. The equal-winged solution proves to be the best, and very much the best. With a gap of 0.15 of the upper span, the equal-winged biplane has 22 per

cent less drag than the monoplane of like span and total area, whereas the combination with the lower span 40 per cent less than the upper and with the best possible combination of chords for that relation of spans shows but 3 per cent of improvement over the monoplane.

In final summary, then, the use of overhang is justifiable only when (1) the extreme span of the airplane is not a seriously limiting factor in design; and (2) the chords of the wings are varied in at least the same ratio as the span.

For those who may prefer to work their biplane problems in terms of span rather than of aspect ratio, (30) can be used to derive a general expression for k, the factor by which the extreme span of the biplane must be multiplied to get a corrected span for use in such formulas as (18). Thus

$$D_i = \frac{L^2}{\pi\frac{\rho}{2}V^2k^2b^2}$$

Comparing with (57),

$$k^2 = \frac{L^2\ b_u^2}{\left(\frac{L_u^2}{b_u^2} + \frac{L_l^2}{b_l^2} + 2\sigma\frac{L_uL_l}{b_ub_l}\right)} \tag{63}$$

If C_L is considered constant

$$k^2 = \frac{}{\left(\frac{S_u}{S}\right)^2 + \left(\frac{S_l}{S}\right)^2 \cdot \left(\frac{b_u}{b_l}\right) + 2\sigma\ \frac{S_uS_l}{S^2}\ \frac{b_u}{b_l}} \tag{64}$$

where S is the total area of the biplane. As already noted in Chap. V, this comes out to $k^2 = 2/n$ in the equal-winged case.

Lift-curve Slope.—As previously explained,[1] the slope problem is somewhat less simple for the biplane than for the monoplane, the mutual influence of the wings on streamline curvature introducing a new factor. Glauert's presentation[2] takes the form (adjusted here to conformity with the conventions and units used elsewhere in this volume)

$$\left(\frac{dC_L}{d\alpha}\right)_B = \frac{1}{\frac{1}{(dC_L/d\alpha)_\infty} + \frac{n}{\pi R_B} + \frac{\beta}{2}} \tag{65}$$

[1] P. 99, *supra*.

[2] "Aerofoil and Airscrew Theory," Chap. XIII.

where β is a function of G/c ranging from 0.079 when G/c is 0.75 to 0.054 for unity and 0.038 for 1.25, $(dC_L/d\alpha)_\infty$ is the slope of the lift curve for a monoplane of infinite aspect ratio, and R_e is the equivalent monoplane aspect ratio of the biplane, or R_B/n. Radian measure is of course used for $dC_L/d\alpha$. Substituting its mean value of 5.65 for $(dC_L/d\alpha)_\infty$, as before[1]

$$\frac{(dC_L/d\alpha)_B}{\left(\dfrac{dC_L}{d\alpha}\right)_\infty} = \frac{1}{1 + \dfrac{1.8n}{R_B} + 2.8\beta} \tag{66}$$

The average slope of the lift curve in units per degree, assuming a slope for infinite aspect ratio 10 per cent below the theoretical

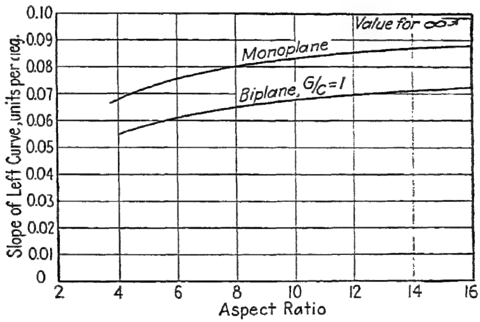

FIG. 176.—Lift-curve slope and aspect ratio, for biplane and monoplane.

value of 2π per radian, is plotted in Fig. 176 for a monoplane and for a biplane with the gap equal to the chord.

Effects of Stagger.—The commonest modification in biplane arrangement, commoner even than the use of overhang, is the introduction of stagger, either positive or negative. It is ordinarily dictated either by structural considerations or by such purely practical matters as the requirement of a wide field of view for the pilot and of easy access to and exit from all of the cockpits. The effects on aerodynamic efficiency of moving one wing forward or backward with respect to the other are of secondary importance, though not entirely negligible. The effects on stability, and especially on spinning characteristics, are much more serious.

[1] P. 94, *supra*.

Although the effects of stagger may be treated at least in part by the induced-drag theory, they are so small that the complications introduced into the application of the theory and the tabulation of the coefficients to take care of them are hardly worth while, and it does quite as well to depend on the results of experiment.[1] They are most apparent in a tabulation of maximum values of the lift coefficients, the maximum increasing steadily with increasing positive stagger and decreasing when the stagger is negative. The rate of change, which is quite uniform throughout the range of probable values of stagger, is about 1 per cent increase of maximum lift for every 10 per cent of increase in the algebraic value of the stagger.

Stagger may, of course, be measured either in terms of the angle between a line connecting the leading edges of the wings and the line perpendicular to the chord of the lower wing or as a percentage of the chord, a 30-deg. stagger being the same thing as one of 57 per cent if the gap and chord are equal. The second method is generally the more convenient.

The effects on drag are almost too small to estimate, lying within the experimental error, so that the several sets of experiments are not entirely consistent. The general tendency, however, is toward a slight increase of drag/lift ratio with negative stagger and an increase of minimum-drag coefficient in the same direction. Neither effect is likely to amount to more than 5 per cent for stagger up to 50 per cent. Combinations with positive stagger give efficiencies approximately equal to those for the orthogonal biplane (one without stagger). The introduction of stagger also has a slight effect in distorting the curves of lift against angle of attack, so that the angle of zero lift is about $\frac{1}{2}$ deg. smaller (algebraically) with a stagger of +25 per cent than with a negative setting of the same numerical amount. The shift of the curve at the point of maximum lift is in the opposite direction and of a trifle larger magnitude, the angle of maximum

[1] *R. and M.* 196, *cit. supra*, p. 105; "Biplane Investigation with R. A. F. 15 Section; Tests at Various Staggers and Gap Chord Ratios" by W. L. Cowley, A. G. Gadd, L. J. Jones, and S. W. Skan, *R. and M.* 872; "Biplane Investigation with R. A. F. 15 Section," Pt. II, by W. L. Cowley and L. J. Jones, *R. and M.* 857; "The Effect of Staggering a Biplane," by F. H. Norton, *Tech. Note* 70; "The Air Forces on a Systematic Series of Biplane and Triplane Cellule Models," by Max M. Munk, *Rept.* 256.

lift with a 25 per cent positive stagger being about 1 deg. larger than with the upper wing set back 25 per cent of the chord with respect to the lower.

All these deviations from mean are insignificant compared with those that appear at angles of attack above that of maximum lift, and especially above 30 deg. At very large angles the upper wing of a biplane is, of course, shielded behind the lower one. Negative stagger increases the shielding; positive stagger

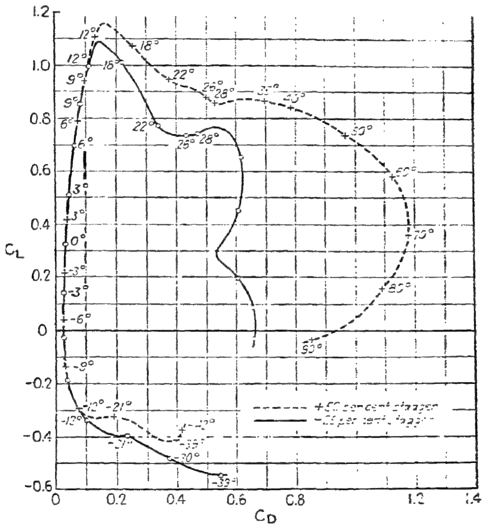

FIG. 177.—Effect of stagger upon aerodynamic forces at very high angles of attack.

decreases it. Whereas the drag of a single airfoil at 90 deg. is approximately equal to its maximum lift at 15 deg. or thereabouts, the shielding effect makes the drag per unit of area for a biplane a good deal less than that. Hence the form of the curves,[1] for biplanes of −25 per cent and +50 per cent stagger, in Fig. 177. The kink in the negative-stagger curve at 60 deg. is due to the shielding effect having attained a maximum at that point and to the gradual emergence of the upper wing out of the shadow of the lower as the angle is further increased. For an orthogonal

[1] Taken from *Rept.* 317, *cit. supra.* See also, for data, at high Reynolds' number, *Tech. Note* 289, *cit. supra.*

biplane, the same kink appears at just over 70 deg. All this is very important in its effect on spinning tendencies.

The most important aerodynamic effects of stagger are shown in the center-of-pressure motion, in the distribution of loads between the wings, and in stability characteristics, but those matters will be taken up somewhat later.

Decalage.—Decalage, or the setting of the upper and lower wings at different angles of attack, is shown by tests[1] to have even less effect than stagger, although the induced-drag theory would indicate that there would be some benefit from the use of a small amount of negative decalage, or the setting of the lower wing at a larger angle than the other in order that the two might give equal lifts. This is not checked by experiment, for a positive decalage of 1 deg. appears to have a very slight beneficial effect, of the order of 1 per cent, on minimum drag/lift ratio, while further increase of decalage, or the reversal of its direction so that the lower wing is set at a larger angle than the other, proves definitely harmful, although the effects are still small.

It has often been suggested that it might be possible to reduce biplane losses by using different airfoil sections for the upper and lower members of the combination. Neither theory nor experiment lends appreciable support to such an idea. Tests indicate substantial constancy of the biplane correction factors for all sections and all combinations of sections. While there have been no tests on combinations with the upper and lower wings of widely different form, with a section like the Göttingen 387 in combination with one like the R. A. F. 15, for example, the present indications are that standard correction factors may safely be applied to a fictitious monoplane result taken as the arithmetical mean of the results from tests on models of the airfoils represented in the upper and lower wings, respectively.

In making the induced-drag calculations for a biplane combination made up of two airfoils of differing section, the induced drag may be found separately for the upper and lower wings, allowing for the difference in lift coefficients of the two sections at the same angle of attack.

[1] *R. and M.* 196, *cit. supra*, p. 110; "Stable Biplane Arrangements," by J. C. Hunsaker, *Engineering*, Jan. 7 and 14, 1916; *Rept.* 317, *cit. supra;* "Pressure Distribution Tests on a Series of Clark Y Biplane Cellules with Special Reference to Stability," by Richard W. Noyes, *Rept.* 417.

For a more accurate method reference may be made to the sources already cited.[1]

Distribution of Load between Wings.—Even though the upper and lower wings are of the same section, the lift of a biplane combination is not in general evenly distributed between the two elements, nor do they work at the same efficiency. The cause of the difference is obvious from the simplest physical reasoning,

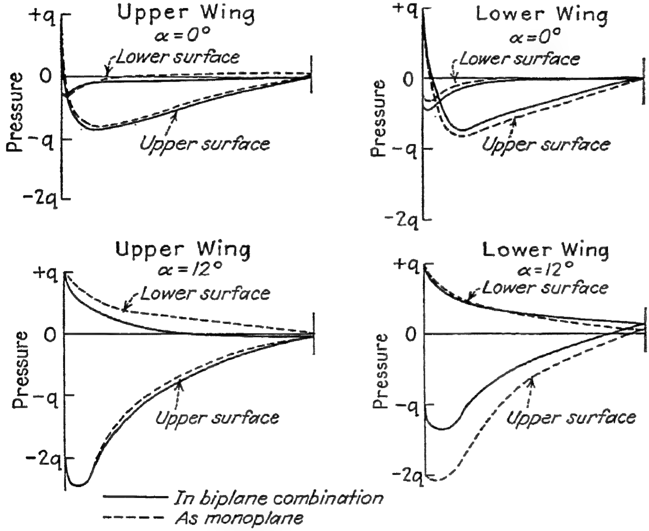

FIG. 178.—Pressure distribution on the wings of a biplane.

and even more so from the pressure plots in Fig. 178,[2] where the pressure distributions on the upper and lower wings of a biplane at two angles of attack are compared with the corresponding plots for the same wings tested as monoplanes at the same angles. Manifestly the interference effects are largest on that side of a wing which is nearest to the obstruction causing the interference. The disturbance of flow is therefore greatest on the upper surface of the lower airfoil and on the lower surface of the upper one, and since the upper surface is the more impor-

[1] "General Biplane Theory," by Max. M. Munk, *Rept.* 151; "Aerofoil Theory," by H. Glauert, *R. and M.* 723.

[2] Based on "Pressure Distribution Tests on PW-9 Wing Models Showing Effects of Biplane Interference," by A. J. Fairbanks, *Rept.* 271.

tant, both in producing lift and in controlling efficiency, it would be expected that the lower wing would be the worst sufferer from biplane effect and would have the lower lift. Experiment[1] proves this actually to be the case for the ordinary orthogonal biplane combinations at normal flight attitudes, although the difference of loading between the upper and lower wings when there is no

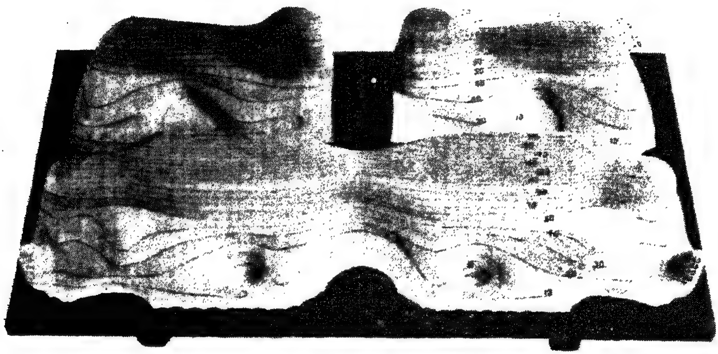

FIG. 179.—Pressure distribution over a biplane in flight at a high angle of attack.

stagger is much less than is often supposed. Small enough to have no appreciable effect on the induced drag, it is still large enough to be of great importance in distributing the loads for structural analysis.

[1] "Forces and Moments on Upper and Lower Planes of a Biplane," by J. R. Pannell and E. A. Griffiths, *R. and M.* 196; "The Contribution of the Separate Planes to the Forces on a Biplane," by W. L. Cowley and H. Levy, *R. and M.* 589; "Biplane Investigation with R. A. F. 15 Section," by W. L. Cowley and C. N. H. Locke, *R. and M.* 774; *ibid.*, Pt. II, by W. L. Cowley and L. J. Jones, *R. and M.* 857; "The Air Forces on a Systematic Series of Biplane and Triplane Cellule Models," by Max M. Munk, *Rept.* 256; "The Distribution of Loads between the Wings of a Biplane Having Decalage," by Richard M. Mock, *Tech. Note* 269; "An Extended Theory of Thin Airfoils and Its Application to the Biplane Problem," by Clark B. Millikan, *Rept.* 362; "Working Charts for the Determination of the Lift Distribution between Biplane Wings," by Paul Kuhn, *Rept.* 445; "Relative Loading on Biplane Wings," by Walter S. Diehl, *Rept.* 458; "Relative Loading on Biplane Wings of Unequal Chords," by Walter S. Diehl, *Rept.* 501.

Figure 179[1] illustrates, by a relief map, the variation of pressure intensity over the whole extent of a biplane's wings. The interruption in the upper part of the figure is the space through which the fuselage passed. The appearance of the peak of pressure drop at the wing tip near the trailing edge, a phenomenon already encountered and explained in the general monoplane case, will be noted.

Departures from the simple orthogonal form may change the distribution considerably. If the biplane has a large negative stagger, for example, it is obvious that the interference on the lower wing will be decreased while that on the upper will be increased in amount, and that the lower wing will therefore tend to carry an increasing share of the load. This effect may even overbalance the other one so that the lower wing will furnish more than half of the lift if the negative stagger is large. Similarly, at very large angles of attack, near to or beyond that of maximum lift, the shielding effect already noted makes its appearance and the upper wing's share of the total lift is again cut down at these attitudes. Some of these points, notably the changing distribution of load with changing stagger, can be allowed for by methods based on pure theory, and Glauert and Munk have done much to rationalize the whole question of lift distribution.

The conventional practice of many years, and one that still prevails to a large extent, is the plotting of a ratio of the lift coefficient for the upper wing to that for the lower against the lift coefficient for the biplane. Diehl has shown,[2] however, that the presentation of the data can be very much simplified if it is assumed that the lift coefficients simultaneously existing for the two wings under any given condition can be expressed by the equations

$$C_{L_u} = C_{L_B} + (K_1 + K_2 C_{L_B})\left(\frac{S_l + S_u}{2S_u}\right) \tag{67}$$

and

$$C_{L_l} = C_{L_B} - (K_1 + K_2 C_{L_B})\left(\frac{S_l + S_u}{2S_l}\right) \tag{68}$$

[1] From the N. A. C. A.

[2] *Repts.* 458, 501, *cit. supra.*

where K_1 and K_2 are experimentally determined constants dependent on the geometrical properties of the combination; and that the assumption is substantially supported by the results of experiment. It even holds very well (which is surprising) in the immediate neighborhood of the maximum lift.

Diehl's curves for the corrections are too numerous to reproduce here as a whole, but their most important indications can be numerically approximated. For a biplane without stagger, K_1 ranges from -0.01 for a combination of very thin airfoils to -0.07 where the basic section is thick and the gap small. If it were taken as -0.025 in every such case where the gap equals the chord and varied as the inverse square of the gap/chord ratio, no serious error would be likely to result. Where there is stagger, 50 per cent of positive stagger reduces K_1 to zero, 50 per cent of negative stagger doubles the arithmetical value that it would have in an orthogonal biplane, and the effect of other amounts can be found by interpolation. If the upper span exceeds the lower, K_1 should be changed by -0.001 for every 1 per cent of overhang [defined by the formula $(1 - b_l/b_u) \times 100$] up to 20. With any amount of overhang from 20 to 60 per cent, the overhang correction to K can be held constant at -0.02. Thereafter, with still further increase of overhang, the correction diminishes. It is somewhat surprising to find the apparent relative disproportion of lift between the two wings at zero lift for the combination actually increasing with small amounts of overhang, so that 20 per cent of overhang has as much effect on the distribution under that condition as has reducing the gap/chord ratio by a third. Diehl has shown that the work of Munk affords a partial, though only a partial, explanation.

For the basic combination, aspect ratio 6, gap/chord unity, and no stagger or overhang, K_2 is 0.05. It increases by 0.017 for every 10 per cent of positive stagger, 30 per cent of stagger doubling it, and after that correction is made it has to be varied in inverse proportion to the gap/chord ratio and in inverse proportion to the aspect ratio. For an aspect ratio of 10 and a gap of 1.4 times the chord, for example, K_2 for an unstaggered biplane would be $0.05 \times 1/1.4 \times 6/10$, or 0.021. Overhang again has a pronounced effect, every 1 per cent increasing K_2 by about 0.003 up to 30 per cent, the effect remaining constant

from 30 to 50 per cent and then falling off with further increase.

In summary, on the unstaggered biplane with wings of equal area the lower wing carries a small positive lift, counterbalanced by a down load of equal magnitude on the upper wing, when the lift of the combination is zero—no doubt due, as suggested by Munk, to the action of the wings as a venturi tube of very mild contraction and to the consequent decrease of pressure on the upper surface of the lower wing and the lower surface of the upper one, producing a positive lift on the first and a negative lift on the second. As the lift increases, the upper wing gains ground, until the lift coefficients on the two become equal at about $C_L = 0.5$ for the unstaggered biplane or at about 0.15 for one with +25 per cent stagger. Curves of $C_{L_u} - C_{L_B}$ are plotted in Fig. 180 for the standard case and for four others, each varied from it in one particular.

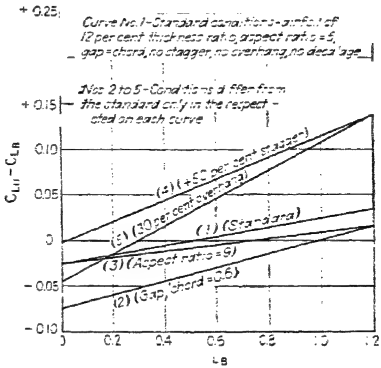

Fig. 180.—Correction factors for distribution of lift between upper and lower wings.

As the angle of maximum lift is actually attained, the lift curves for the upper and lower wings take very different forms, the curve for the lower member of the combination flattening out, and either attaining its maximum value at, or holding a value virtually equal to the maximum up to, an angle sometimes as high as 30 or 35 deg. The maximum lift for the upper wing comes about where it would for a monoplane. The distribution of lift at very large angles of attack would then fall far short of satisfying Diehl's hypothesis, but the matter is unimportant, for it is not under those conditions that the high structural loads that make an intimate knowledge of load distribution most important are encountered. A representative set of lift curves for the upper and lower wings of a combination (unstaggered), are given in Fig. 181.[1]

[1] From *Rept.* 417, *cit. supra.*

Though most of the problems that arise in practice fall within the area covered on the last two pages, the designer occasionally finds it expedient to use decalage or to make the chords of the two wings unequal, though most designers do the latter a good deal less frequently than the interests of efficiency, as herein set forth, might dictate. It is necessary to be prepared for the analysis of those cases as well as of the more common ones, and Diehl's studies have been extended to deal with them. Decalage obviously has a major effect on K_1, since it introduces a direct and opposite offset of the two wings from their respective zero-lift angles under the condition that makes the lift zero

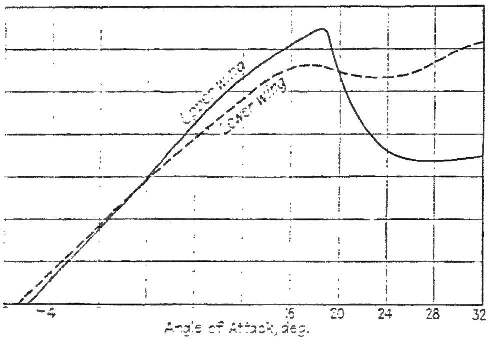

Fig. 181.—Lift characteristics for the upper and lower wings of an orthogonal biplane.

for the combination. When the gap/chord ratio is unity the correction to K_1 is 0.063 per degree of decalage, increasing to 0.072 with the gap chord reduced to 0.6. Increases of the ratio above unity, on the other hand, have little effect. Positive decalage (the upper wing set at the greater angle) serves also to decrease the value of K_2, by about 0.020 per degree. The effect on K_2 is small compared with that on K_1, and it would be a fair approximation (as decalage almost never exceeds $2\frac{1}{2}$ deg.) to say that the introduction of decalage serves to shift any such curve as those of Fig. 180 parallel to itself, positive decalage moving it downward and negative decalage upward, by 0.055 for every degree of decalage.

Though Diehl has gone into the question of load distribution where the upper and lower chords are unequal at such length

as to give that case a memorandum all to itself,[1] his studies can be summarized closely enough for most practical purposes in a couple of sentences. If the stagger is measured by the inclination of a line connecting the points one-third of the way back on the chord, rather than the leading edges; if the amount of stagger is specified as a percentage of the lower chord; if the basic values of K_1 and K_2 as corrected for stagger by the rules already given then are multiplied by the ratio of the two chords, C_u/C_l; and if the corrections for overhang and decalage are multiplied by the same ratio; then the lift distribution will be satisfactorily approximated.

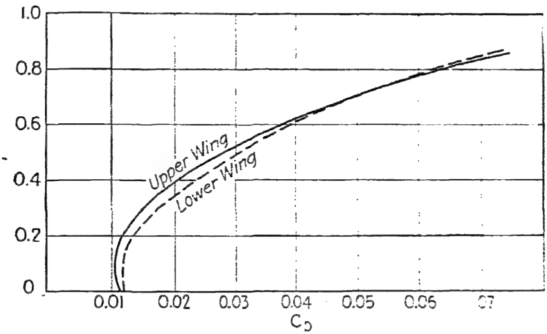

FIG. 182.—Comparison of the polars for the upper and lower wings of an orthogonal biplane.

To give a rough idea of what all this means in design practice and of the importance that it has in stress analysis, it can be set down that at maximum lift the lift coefficient for the upper wing may range between one equal to the average for the biplane and one 15 per cent above it; that at one-quarter maximum lift either the lower or the upper wing may run up to 15 per cent above the biplane figure; and that in a vertical dive at terminal velocity (the highest speed attainable after diving a great distance) the upper wing carries a negative lift and the lower wing a positive one that may under the worst conditions slightly exceed the normal load sustained in steady horizontal flight.

The distribution of lift between the upper and lower wings of a biplane is of considerable importance to the designer in

[1] *Rept.* 501, *cit. supra.*

preparing to calculate the strength of the structure. The difference of drag, although of less practical significance, is still interesting to the student. The differences between the characteristics of the two wings are best shown in Fig. 182,[1] where polar curves have been plotted for the biplane combination's two elements, each tested in the presence of the other. The section was an R. A. F. 15, the Reynolds' number 240,000. It will be observed that, although the upper wing has a drag considerably below that of the lower one at small angles, it loses ground at large lift coefficients.

Pitching Moments.—The moment curve for a biplane follows roughly the same rule as that for a monoplane in that as a general rule it plots against lift coefficient nearly as a straight line and that when moments are taken about a certain "aerodynamic center" the straight line becomes nearly vertical, the moment nearly constant at all angles. The constancy is somewhat less exact than in the case of the single airfoil, and in final calculations for a biplane's stability and balance it is best to work from an actual experimental curve of C_M against C_L for a model of the actual combination to be used or something very near it, but as a first approximation the constant-moment assumption can be used. The assumption of the location of the central axis at the 25-per-cent point of the chord, however, had better be abandoned. The aerodynamic center tends to lie a little farther forward on biplanes than on monoplanes, and 23 per cent is a better fix for its average location. We shall assume that point chosen in all further discussion of the moments and their values.

Before proceeding further with respect to any axis, however, the "chord" of a biplane needs some further definition. In the case of an orthogonal combination it makes relatively little difference where it is taken along the line between the two wings, for the moments of the normal forces on the wings about the mean-chord axis of the combination would be independent of the vertical location of the mean chord, and the longitudinal forces are so small that a vertical shift of the chord by as much as 5 per cent of its own length has but little effect on the size of the moments—less effect than a 1 per cent shift along the axis. With a staggered biplane it is another story.

[1] Based on *R. and M.* 774, *cit. supra.*

It seems reasonable, and indeed it is often specified, that the mean chord should be so located that its distances from the two wings vary inversely as their total lifts. That has the disadvantage of making the mean chord dependent on angle of attack, approaching one of the wings rapidly as zero lift is approached, and it is far simpler and therefore preferable to take a fixed position, determined directly from moment data from the wind tunnel, and then to make such corrections to the moments at particular angles as the test data may show to be necessary.

There are fewer data for these purposes than one might wish, and experimental errors are large enough to make those that do exist[1] somewhat conflicting in their indications. In general, it seems best where the wings are of equal area to take the mean chord midway between them if there is negative stagger and $57\frac{1}{2}$ per cent of the way from the lower chord to the upper if the stagger is positive. Where areas are unequal the vertical coordinate is fixed by the formula

$$\frac{\bar{y}}{G} = \frac{pS_u}{pS_u + S_l} \tag{69}$$

where $\bar{y}$ is the distance of the mean chord from the lower chord, G the gap, and p a weighting factor which is given the value of 1.0 with negative stagger or none, 1.35 when the stagger is positive.

With a mean chord so selected, the biplane can be considered to have a pitching-moment coefficient, with respect to the 23-per-cent point on the mean chord, numerically equal to the moment about the 25-per-cent point for a monoplane of the same section. This holds for all staggers and for all gap/chord ratios within the ordinary range of practice. Though all these rules are approximations at best, they will be found to give the moment for a biplane correct to within 0.010 throughout the range of lift coefficients in most cases.

Rather surprisingly, decalage, even when accompanied by stagger, proves to have little or no effect on the location of the aerodynamic center. The same moment axis can still be used. Still further, decalage in an unstaggered combination

[1] "The Effects of Staggering a Biplane," by F. H. Norton, *Tech. Note* 70; "Stable Biplane Arrangements," by J. C. Hunsaker, *Engineering*, Jan. 7 and 14, 1916; *Repts.* 256, 417; *Tech. Note* 289; *R. and M.* 774, 857, 872, all *cit. supra*.

has no effect on the magnitude of the moment, which can still be taken as equal to the moment for the basic airfoil with respect to a slightly different axis.[1] When stagger and decalage appear in combination, however, the numerical values of the moments undergo a pronounced change.[2] Figure 183[3] shows it directly through the moment curves, about the 23-per-cent point of a mean biplane chord located vertically in accordance with the rules on page 271, for a combination with a large positive stagger and with varying amounts of decalage, both positive and negative. The basic airfoil was a Clark Y.

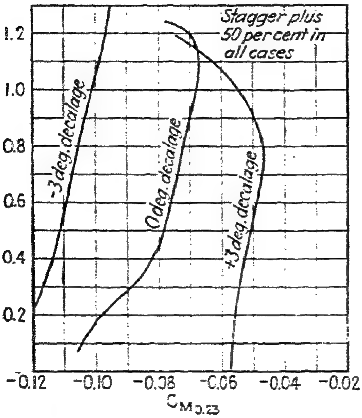

FIG. 183.—Pitching moments c biplane combination with a large positive stagger and varying decalage.

Very roughly, it appears that the diving moment is increased by 0.01 for every degree of negative decalage when there is a positive stagger of 50 per cent. The effect would increase as the stagger increased, and with this particular section a combination of 75 per cent stagger and about +6 deg. of decalage would suffice to make the moment about the aerodynamic center positive and so to make the biplane cell stable in pitch without independence on a tail, precisely like a single airfoil with a "stable" center-of-pressure travel. The lower wing, behind the upper one and at a smaller angle, itself serves the role of the reflexed trailing edge on a stable airfoil. The use of large amounts of decalage is generally a somewhat inefficient stabilizing expedient. The tailless biplane with negative decalage and heavy stagger has certain devoted adherents,[4] but they have won little sympathy.

[1] See p. 271, *supra*.

[2] First observed by Eiffel. "La résistance de l'air et l'aviation," by G. Eiffel, Paris, 1911; "Stable Biplane Arrangements," by J. C. Hunsaker, *Rept.* 417, *cit. supra*.

[3] Based on *Rept.* 417.

[4] Most particularly A. A. Merrill, who has built and made public display of a number of examples of the type.

The moment characteristics for the members of a combination of unit gap/chord ratio, individually considered, are generally nearly enough alike to be treated as identical except for final stress analysis. Even large amounts of stagger, either positive or negative, have surprisingly little effect on the moment curve or center-of-pressure travel. The center of pressure tends to lie a little farther forward on the upper wing than on the lower one, but by so little as to amount to a difference in diving moment of only about 0.02 at high lift coefficients, somewhat less than that at small ones. As the gap is decreased the effect of mutual interference on center-of-pressure position mounts very rapidly, to a moment spread of about 0.07 when $G/C = 0.50$. Where the individual moments are needed they can be considered as equidistant from the monoplane value, so that with G/C equal to unity the upper wing can be expected to have a diving moment 0.01 less than the monoplane airfoil of the same section, the lower wing 0.01 more than the monoplane.

Triplane Interference.—Triplane effects are in general like those on biplanes but, of course, exaggerated in degree because of the increased interference, the upper and lower wings working under substantially the same conditions as they would in a biplane, while the middle one is subject to disturbance from all sides. Since triplane arrangements are exceedingly rare in current design practice, tests are less numerous than for the biplanes, but several sets of studies exist.[1]

Triplane interference, like that in a biplane combination, can be considered as acting primarily to increase the induced drag, and there should therefore be for each triplane combination a value of n somewhat higher than that for a biplane of corresponding gap/span ratio. The theory of aerodynamic induction has been applied to the superposition of three wings as well as of two,[2] the theory indeed being much the same except that it is necessary to make separate allowance for each of the two com-

[1] "Triplane Combinations," by J. C. Hunsaker and T. H. Huff, *Engineering*, July 21, 1916; "Triplane Tests," by C. Wieselsberger, *Tech. Note* 178, translated from *Technische Berichte*, vol. III, No. 7; "Forces and Moments on Triplanes," by E. A. Griffiths and C. H. Powell, *R. and M.* 250; "Triplane Investigation with R. A. F. 15 Section," by W. L. Cowley and H. Levy, *R. and M.* 432; Munk, *Rept.* 256, *cit. supra.*

[2] "Der induzierte Widerstand von Mehrdeckern," by L. von Prandtl, *Technische Berichte*, vol. III, p. 309.

ponents of force induced on each wing by the action of the other two wings. Calculated values of n are given in Table VII for a few cases.

TABLE VII

Gap Span	n
0.05	2.46
0.1	2.17
0.15	1.97
0.2	1.82

On the average, the maximum-lift coefficient of a triplane combination with the gap equal to the chord is 90 per cent of that for the monoplane.

As might be anticipated, the upper wing carries the largest share of the load in a triplane combination having three wings of equal area, while the load is lightest on the middle one of the three elements. Over the ordinary working range of angles of attack and on an orthogonal triplane of unit gap/chord ratio the upper wing carries 37 per cent of the total load in round numbers, the lower 33 per cent, and the middle one 30 per cent. As on the biplane, these percentages of course vary somewhat with changing angle of attack. To put it in other terms, the upper wing carries 10 per cent more than the average intensity of lift; the lower one 2 per cent less than the average; the middle one 8 per cent less. Diehl's method of making the corrections by addition rather than by taking ratios[1] would no doubt work here as well as for the biplane, but there has been so little demand for triplane data that the appropriate constants and curves have not been worked out.

The effect of stagger in a triplane is very much as in a biplane, a redistribution of lift among the wings with the share of the upper wing increasing as positive stagger is introduced. As in the case of the biplane, again, the performance of the whole combination is relatively little affected, the principal change being in maximum lift, which is little influenced by positive stagger but decreases when the upper wing is moved backwards with reference to the lower ones. There are, of course, a great variety of possible arrangements of three superposed wings which do not fit into any simple scheme of stagger and gap nomenclature. The two gaps may differ, the three wings may be of

[1] P. 265, *supra*.

different span or chord, and the middle wing may be set ahead of or behind the other two.[1] None of these unusual combinations, however, appears to produce any very remarkable results.

Tandem Arrangements.—There remain the tandem combinations, the arrangement of one wing behind another. Since a tandem pair may be made up with the rear airfoil raised or lowered with respect to the front one, the tandem combinations merge into the biplanes and multiplanes with extreme stagger but among the wing arrangements likely to be met with in practice there is seldom any difficulty in deciding whether the vertical or the longitudinal relative displacement between the wings is the dominant factor affecting the performance of the group.

Tandem combinations are subject to serious structural disadvantages, heavy forces having to be transmitted along the body which connects the front and rear wings, and that alone would be sufficient to account for their unpopularity. The majority of wind-tunnel tests on such combinations either date back to the very early stages of aerodynamic research or were undertaken for other purposes, as, for, example, in connection with propeller investigations.[2]

The performance of a normal tandem combination is always poorer than that of the individual wings of which it is composed if they themselves are of reasonably good form, as might be foreseen from the slightest consideration of induced-drag theory. Since if the forward member of a combination is giving lift it must also be producing downwash, the axes of profile lift and drag for the rear members of the combination are rotated through a corresponding angle, and the component of lift on the rear wings measured as drag (*i.e.*, parallel to the direction of the undisturbed wind stream in advance of the combination) is thereby increased. If the airfoils were set close together, the mean downwash, determining the induced drag, would be $\frac{1}{2}\epsilon$ on the first airfoil, $\frac{3}{2}\epsilon$ on the second, $\frac{5}{2}\epsilon$ on the third, and so on. The mean value for a whole series of n airfoils would be

[1] The effect of the last-mentioned modification is studied in "Triplane Tests," by C. Wieselsberger, *Tech. Note* 178.

[2] "Nouvelles recherches sur la résistance de l'air et l'aviation," by G. Eiffel, pp. 149–174, Paris, 1914; "Tandem Aerofoils," by J. L. Nayler and W. L. LePage, *R. and M.* 804; "Multiplane Interference Applied to Airscrew Theory," by R. McK. Wood, F. B. Bradfield, and M. Barker, *R. and M.* 639.

$n(\epsilon/2)$, and it would therefore appear that the induced drag would be proportional to the number of airfoils. A triple tandem combination of airfoils of aspect ratio 6 would be equivalent to a single wing of aspect ratio 2, precisely as though the three had been merged into a single one of three times the chord. That does not tell the full story, however, for when two or more wings are brought very close together in tandem the profile drag as well as the induced portion is modified by the turbulence of the flow off the trailing edge of the forward element. The reduction of air speed immediately behind the forward airfoil also affects the magnitude and distribution of the forces.

In practice, however, even with a close spacing, the tandem combination does somewhat better than the single wing of aspect ratio reduced as just indicated. As the separation between the forward and rear airfoils is increased, the interference losses should, of course, be reduced, since the downwash of the forward member of the combination dies out to an appreciable degree before the air reaches the rear one. A tandem combination with a large separation may equal a biplane of the same aspect ratio in aerodynamic performance, but it is far less desirable structurally and is indeed subject to such serious drawbacks, in respect both of the bracing of the wings and of the design of the body which connects them, as to make it virtually unknown in present-day design practice. The only serious attempts to use tandem arrangements since 1920 have been in certain machines of very great weight, for which it was desired to keep the overall dimensions to a minimum, and they have not been markedly successful.

The combination of two wings in tandem with a clear gap between them of twice the chord of one wing is found to have an induced drag about 1.7 times that of a single airfoil (so equalling the performance of a biplane with G/b equal to 0.08). Experiment shows that the longitudinal gap makes less difference than might be expected, but any change of angular relation between the wings makes itself felt quite strongly. Because of the existence of downwash from the front wing it is, of course, necessary to set the rear member at the larger angle of attack, or to use negative decalage, in order to secure equality of lift between the two. The amount of decalage really ought to vary with the angle of attack, but a good average value is 3 deg.

Unfortunately, the combination offering greatest aerodynamic efficiency has serious disadvantages on the score of stability.

Vertical displacement of one airfoil with respect to the other has but little effect on the forces so long as the vertical shift of position does not exceed one-third of the longitudinal gap between the elements—a range hardly likely to be approached in actual airplane design if tandem combinations are to be used at all.

As the number of airfoils set in tandem increases, the performances of the rear ones in the series tend to become worse, as each

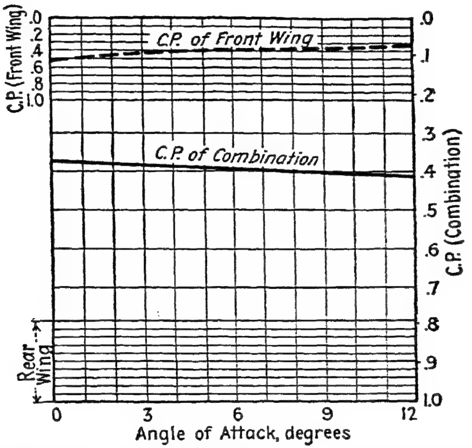

FIG. 184.—Movement of the center of pressure on a tandem monoplane.

airfoil adds its increment of downwash. A point is finally reached, however, at which the angle between the air that passes directly over the airfoils and the undisturbed stream is such that the downwash is damped out between each two airfoils as much as it is increased by passing over a single airfoil, and thereafter the individual efficiencies of further additions to the series remain constant. It has been shown by MacKinnon Wood[1] that the fourth airfoil in a series and all those which follow it experience substantially identical reactions.

Center of Pressure on Tandem Combinations.—The center-of-pressure movement on a tandem combination is quite different

[1] *R. and M.* 639, *cit. supra.*

from that on a single airfoil of the same section. If the front and rear elements are set with parallel chords, the airfoils being of typical form with slightly concave lower surfaces, the center of pressure moves backward slowly but steadily as the angle of attack is increased, giving a stable travel. The approximate curve of center-of-pressure travel for such a tandem combination is plotted together with that for one of the elementary airfoils in Fig. 184. The explanation is that the rear airfoil, working in the downwash from the front one, has a permanent effective decalage and always furnishes less than half the lift. As the angle of attack is increased, however, the relative increase of lift is a little larger for the rear wing than for the front one, and the center of pressure of the combination therefore tends to move back toward the rear airfoil, notwithstanding the fact that the center for each individual airfoil is moving forward at the same time. In one particular case an increase of angle of attack from 0 to 6 deg. increased the lift of the front airfoil by 117 per cent and that of the rear one by 149 per cent. To put it in another way, the front wing furnished 58 per cent of the total lift at the smaller angle and only 55 per cent at the larger. This difference of rate of relative change of lift between two surfaces in tandem, incidentally, has a vital bearing on the stabilizing action of the ordinary horizontal tail surfaces of an airplane.

CHAPTER X

VARIABLE-LIFT AIRFOIL ARRANGEMENTS

It was until 1931 the practically universal rule, and it is still very frequently the case, that the wing of an airplane is rigid and unchanging in form, the section always being the same and the lift coefficient being capable of alteration only by change of angle of attack. During the last few years, however, a rapidly increasing number of machines of all types have been built with wings capable of undergoing some modification during flight with a view to change of lift. Many alternative schemes for accomplishing that end have been tried in the wind tunnel, and a gradually increasing number have advanced to the point of incorporation in a finished aircraft.

There are great theoretical attractions in a provision for modifying the lift. Since the minimum speed of an airplane is fixed by the formula

$$V_{\min} = \sqrt{\frac{W}{L_{c_{\max}} S}} \tag{70}$$

it can be reduced only by increasing the lift coefficient of the airfoil or decreasing the wing loading per unit of area. Other things being equal, it is always desirable to make the minimum speed as low as possible for ease and safety of landing. A low $V_{\min}$, too, promotes a quick take-off and a steep climbing angle. Unfortunately, however, other things do not remain equal. It has been made abundantly clear that the changes in airfoil section which lead to very high lift coefficients are not conducive to a maximum of efficiency, and the reduction of wing loading also brings trouble in its train, since it involves a decrease of efficiency at high speed. If an airplane with rigid wings has a maximum speed of 120 m. p. h. and a minimum of 60, the angle of attack when flying at maximum speed must be that at which the lift coefficient has one-quarter of its maximum value, and a good representative wing of moderate thickness and aspect ratio 6 will

give a drag/lift ratio of about 0.042 under that condition. If the maximum speed is kept the same and the minimum reduced to 40 m. p. h., the lift coefficient at maximum speed has but one-ninth of the maximum value, and the drag/lift ratio of the wing when flying level with the throttle fully open rises from 0.042 to about 0.062, with a corresponding increase of power required to drive the airplane at a given maximum speed. If the power remained constant, therefore, the decrease of minimum speed by a reduction of wing loading would entail a drop at the other end of the speed range as well.

If now the wing loading can be kept high and an airfoil giving only a moderate maximum-lift coefficient is used for ordinary flight, but so constructed that it can be modified for a great increase of lift on demand, the low minimum speed can be secured without any sacrifice elsewhere except such as may result from the added weight and resistance of the mechanism used in producing the change of lift.

Variable-lift devices can be classified under three general headings, dependent respectively on variation of angle, of area, and of airfoil section.

Variable Angle.—The first of the three alternatives may be quickly disposed of. The attempt to modify the lift of an airfoil combination by changing its angle with reference to the fuselage, mounting the wings so that they can be rotated around axes parallel to their span, rests on a wholly fallacious belief that there is some restraint on the variation of angle of attack of an ordinary airplane during flight. As a matter of fact, the pilot is, of course, able to vary the angle of attack at will by the use of his controls, tilting the airplane as a whole to any desired angle with reference to its path. There is no possible gain by rotating the wings with respect to the fuselage, except that if the fuselage could be kept traveling always with its axis parallel to the flight path its resistance would be slightly reduced under some conditions. Any gain in performance secured in that way, however, would be more than counterbalanced by the loss due to the addition of the gear necessary for rotating the wings, to say nothing of the difficulty of incorporating provision for such rotation and still keeping the structure sufficiently strong and rigid. Variable-angle airplanes have been built, but they belong to the unsophisticated past of aeronautical engineering.

Variable Area.—The variation of wing area, free of intermingling with any other change, also calls for but little consideration here. It is rather a structural than an aerodynamic problem. If the wing area can be increased 20 per cent in preparation for landing, the lift will be increased 20 per cent, and the minimum speed cut down by about 9 per cent. There is no mystery in that, but the actual construction of a machine in which the wings can be reefed and extended again without unsafe sacrifice of strength and rigidity and without running into excessive weight is a very difficult problem. No wholly adequate solution has as yet been found. If it could be found in either of two ways, it would be far more advantageous to extend the span, so reducing the induced drag, than to extend the chord.

Much more promising is a combination of change of area and of sectional form. To extend the area by increasing the chord is considerably simpler mechanically than to run extensions of span out from the tips, and an extension of chord almost necessarily changes the section somewhat. All the variable-area devices that seem to show promise of early application are so planned that the change of section and increase of area will work together in increasing the lift, and in most cases the change of section is the dominant element. They are then best considered as a special category of variable-section arrangements, and they will be so treated here.

Variation of Section.—The only one of the three general types of variation that has really come into extended use is the change of airfoil form.

In the light of what has so far been said about airfoil sections and their properties, it is evident that the increase of lift coefficient by change of sectional form can best be accomplished either by increasing the mean camber or by some device that will postpone or avert the separation of flow on the upper surface. The latter alternative belongs in the realm of invention. Several antiseparation devices have been used, and they will be discussed in due order.[1] To work on the general curvature of the wing, keeping the thickness constant and increasing the mean camber, is more obvious.

To the flexure of the airfoil as a whole from one smooth contour to another, certain practical obstacles arise. One study

[1] Pp. 297, 315, *infra*.

of the problem[1] has assumed the use of a flexible wing rib, permitting deformation from a normal section of zero mean camber to a high-lift form of about 8 per cent, the maximum lift being increased 105 per cent by the change. This increase of lift taken in itself is hardly a fair criterion of merit, because of the extraordinarily low maximum lift of the normal section as used for high-speed flight. A better index is the ratio of the maximum lift of the deeply cambered section to the minimum drag of the symmetrical one—corresponding to one of the criteria of high-speed performance already suggested for single airfoils. That ratio reached a value of 108 for tests at a Reynolds' number of 68,000, a condition under which no single rigid airfoil has given a ratio of more than 80.

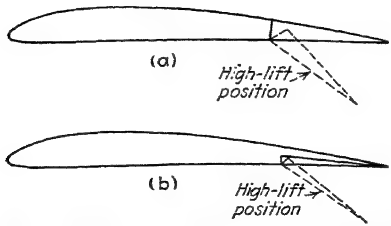

FIG. 185.—Simple (*a*) and split (*b*) trailing-edge flaps.

Despite this very encouraging apparent efficiency, the flexure of the wing as a whole to change its section has seemed so likely to produce structural trouble and to permit of vibration that no serious attempt has been made to incorporate such a wing in a service airplane. A much simpler form of variable-camber device, and the one that has been most extensively employed in actual design, is the flap gear.

Flap Gears.—A flap is nothing but a hinged portion of the trailing edge. It may be simple, as in Fig. 185 *a*, or split, as in *b*. In the former case, it amounts at first sight merely to the extension of the ailerons in toward, or even all the way in to, the plane of symmetry. There is, however, an additional control mechanism permitting the depression or elevation of the flaps on both sides of the machine together and through the same angle. Sometimes the same movable sections serve as flaps and as ailerons, the control linkage allowing either of their simultaneous

[1] "The Parker Variable Camber Wing," by H. F. Parker, *Rept.* 77.

depression by identical amounts or of their opposite angular displacement, for purposes of lateral control, from the attitude to which they may at the moment have been set. That system gives a maximum efficiency of flap action, making the trailing edge available for use over the whole span, but it is mechanically complicated and bad for lateral control. Much commoner is the total separation of function, the trailing edge being hinged in several sections and the outer portion of each wing being used as an aileron while the inner one serves as a variable-lift device alone. Only the inner portions, and often less than a quarter of the span in the aggregate, can be jointly depressed at the pilot's will. Another alternative, and one of which the future is likely to see much, is the replacement of the ailerons by an entirely different type of lateral control, so again leaving the trailing edge of the wing free to carry depressible flaps all the way from tip to tip. No really satisfactory substitute for the aileron has been found as yet, but there is hope.

The amount of wind-tunnel research available on flaps is so large as to be almost bewildering[1]—though in some respects not

[1] "Experiments on an Aerofoil Having a Hinged Rear Portion," by J. L. Nayler, E. W. Stedman, and W. J. Stern., *R. and M.* 110; "Aerofoils with Flaps" (4 parts), by J. L. Nayler, L. W. Bryant, and other, *R. and M.* 152; "Experiments on an Aerofoil with Flap Extending along the Whole Length," by E. A. Griffiths, *R. and M.* 319; "Investigations of the Performance of a Variable Cambered Wing," by W. L. Cowley and H. Levy, *R. and M.* 652; "Model Experiments with Variable Camber Wing," by R. G. Harris and F. B. Bradfield, *R. and M.* 677; "Experiments on an Aerofoil Having a Hinged Rear Portion When Forming the Upper Member of a Biplane Combination," by J. L. Nayler, E. W. Stedman, and L. W. Bryant, *R. and M.* 110; "Forces and Moments on a Biplane with Wings of Variable Section," by J. R. Pannell and N. R. Campbell, *R. and M.* 196; "The Direction and Velocity of the Air Flow behind a Biplane with Wings of Variable Section," by J. R. Pannell and N. R. Campbell, *R. and M.* 196; "Model Tests of 64 Section Biplane Wings with Flaps," by F. B. Bradfield, *R. and M.* 772; "Some Experiments on Thick Wings with Flaps," C. D. Hanscom, Jour. Soc. Auto. Eng., p. 271, March, 1921; "Zap Flaps and Ailerons," by Temple N. Joyce, *Trans.* Am. Soc. Mech. Eng., Aero. Eng. Sec., 1933; "The Effect of Partial-span Split Flaps on the Aerodynamic Characteristics of a Clark Y Wing," by Carl J. Wenzinger, *Tech. Note* 472; "The Effect of Split Trailing-edge Wing Flaps on the Aerodynamic Characteristics of a Parasol Monoplane," by Rudolf N. Wallace, *Tech. Note* 475; "Wing-tunnel Measurements of Air Loads on Split Flaps," by Carl J. Wenzinger, *Tech. Note* 498; "The Effects of Full-span and Partial-span Split Flaps on the Aerodynamic

nearly enough has even yet been done. It is supplemented by a small amount of very useful theoretical work.[1]

The geometrical effect is best suggested by Fig. 186, where an airfoil is shown with such a flap set at −5, 0, +15, and +45 deg. (the positive sign denoting depression of the trailing edge).

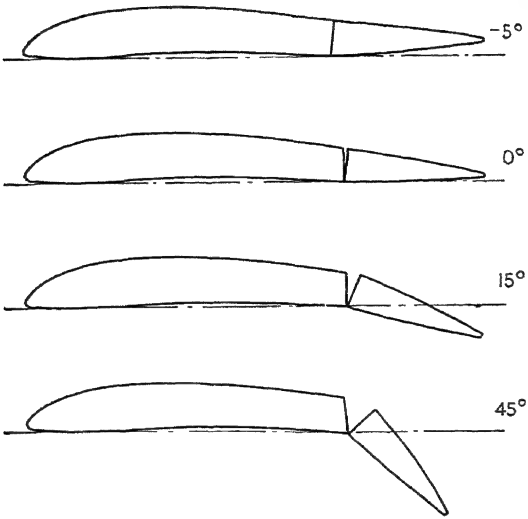

FIG. 186.—Range of motion of a simple flap.

Manifestly the effect of turning the flaps down is to make the lower surface deeply concave, while increasing the effective camber of the upper side. The first effect would seem at first sight to be the important one, as the form of the forward part

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Characteristics of a Tapered Wing," by Carl J. Wenzinger, *Tech. Note* 505; "A Simple Method for Increasing the Lift of Airplane Wings by Means of Flaps," by Eugen Gruschwitz and Oskar Schrenk, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Oct. 28, 1932, translated in *Tech. Memo.* 714; "Lift and Drag of the Bristol Fighter with Fairey Variable Camber Wings," by E. T. Jones, L. E. Caygill, R. G. Harris, and H. M. Garner, *R. and M.* 1085.

[1] "Theoretical Relationships for an Aerofoil with Hinged Flap," by H. Glauert, *R. and M.* 1095; "The Theoretical Relationships for an Aerofoil with a Multiple-hinged Flap System," by W. G. A. Perring, *R. and M.* 1171; "Analytical Determination of the Load on a Trailing-edge Flap," by Robert M. Pinkerton, *Tech. Note* 353.

of the upper surface, the part which primarily controls the lift, is not directly affected. When the flap is pulled down to an extreme position it acts as an obstacle interposed in the stream flowing along the lower surface, producing an effect which may be crudely described as a "banking up" of the air on the lower side of the airfoil, with a reduction of velocity and an increase of pressure, and consequently of intensity of lift.

As a matter of fact, however, pressure-distribution tests such as those plotted in Fig. 187[1] for an angle of attack of 7 deg. and

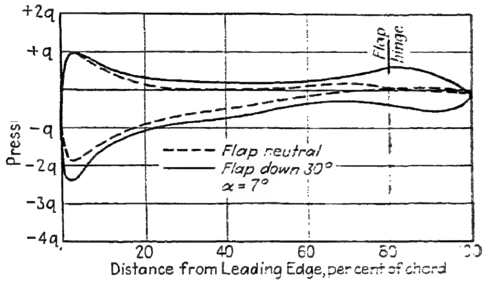

FIG. 187.—Effect of flap depression on pressure distribution.

two flap angles give a different answer. The airfoil used in these particular tests happens to have been of symmetrical biconvex section, and the pressure distribution with the flap neutral was therefore a little out of the ordinary, but the indications on the effect of pulling the flap down would be the same with practically any section. They show the increased lift coming more from the upper surface than from the lower one, and the apparent influence of the flap on the upper surface, as shown by increase of pressure drop at a given angle of attack, extending forward to within 1 or 2 per cent of the chord of the leading edge. The explanation is clear in the light of the consideration that has been given to boundary layers and their phenomena.[2] The lowering of the flap produces a dead-air region, with lowered pressure, immediately behind it. Instead

[1] From "Pressure Distribution over a Symmetrical Airfoil Section with a Trailing-edge Flap," by Eastman N. Jacobs and Robert M. Pinkerton, *Rept.* 360.

[2] Chap. VI, *supra.*

of having to rise substantially to atmospheric pressure at the trailing edge, the upper-surface pressure rises only to a level well below the atmospheric. Available as a result is a positive pressure gradient between the leading and trailing edges to help accelerate the air along the upper surface. The velocity of flow is increased, the pressure drop is correspondingly increased, and the tendency to boundary-layer stagnation with rising pressure that produces separation is reduced. The streamline flow along the forward half of the upper surface is both more perfect and more rapid when the flap is pulled down to a large angle, and both improved quality and smoothness and increased velocity act to increase the upper surface's contribution to lift.

Obviously the chord of the section as measured in a straight line across airfoil and flap from leading to trailing edge is considerably decreased when the trailing edge is swung downward and therefore forward. This decrease of chord is not taken into account in calculation, the coefficients of force for variable-camber wings always being referred to the area as measured in the normal form with the flaps neutral. Manifestly, what interests the designer in any variable-lift device is the possible variation of total lift on the wings, and that lift will be directly proportional to the lift coefficient as reported from the wind tunnel only if the area used for purposes of calculation is kept constant throughout.

The usual effect of the simple flaps that have been tried on monoplane airfoils is to increase the maximum lift of the basic section by from 40 to 60 per cent. As might be expected, the increase is least for those airfoils which already had high maximum-lift coefficients. A wing with a convex lower surface and a very low normal maximum lift will be helped much more by the flaps than will a section already including in its normal contour a deeply concave lower side and a high upper camber. There is some tendency to a convergence of the maximum lifts of all airfoils when they are fitted with flaps, as anything that looks like an airfoil can have its maximum-lift coefficient brought up as high as 1.65 with a proper flap set at the best angle, while the maximum is not likely in any case to exceed 2.3, a total spread of but 40 per cent of the smaller value. The designer planning to use trailing-edge flaps on the wings of his machine may therefore feel free to select a basic airfoil of somewhat lower mean camber

than he might use if the section had to be considered as rigid and permanent in form.

The angle of flap setting yielding the highest lift coefficient varies somewhat with form of the basic airfoil and considerably with the width of the flap. The angle is always very large, however, ranging in the experiments that have been made from 30 deg. depression below neutral in the case of an airfoil with a concave lower surface and a flap having a width of 40 per cent of the chord up to 75 deg. with a flap width of but 10 per cent of the chord. Both limits are unusual, and the angle of flap setting for maximum lift generally lies between 45 and 60 deg.

In actual practice the angles used generally are not quite that large. From 25 to 40 deg. is the common limit of the angle of depression. Within such limits, it is still possible to secure from 60 to 90 per cent of the increase in lift that can be shown in a wind-tunnel test with the flaps all the way down.

The effects of a flap are, within wide limits, insensible to its dimension along the chord. Though the absolute optimum dimension for most sections is approximately 30 per cent of the wing chord, the flap width can be set anywhere between 20 and 40 per cent of the chord without affecting the maximum lift by more than 5 per cent. It can even be reduced to 10 per cent of the chord without decreasing the increase of maximum lift due to the flap by more than 15 to 20 per cent of its highest possible value. As we have already noted, the angle of depression of the flap for best results has to be increased as the chord is reduced.

When flaps cover less than the full span, as is the common case, Wenzinger has shown[1] that the effect is not very far from being lineally proportional to the length that they do cover. Since the ailerons are the usual disturbing element the tendency is to concentrate the jointly depressible flap segments near the center of the span, and under that condition the performance is a little better than a strict application of the lineal-proportionality rule would indicate. To be perfectly specific, flaps extending over half the semispan on each side of the plane of symmetry give 60 per cent of the maximum-lift increase that could be obtained if they covered the entire span. The division of the flaps into two distinct parts with an open gap between them across the fuselage, on the other hand (also a common case

[1] *Tech. Note* 472, *cit. supra.*

in practice), loses effectiveness in something a little more than strict proportionality to the width of the gap.

Though all this has been written with special reference to the simple flap, most of it applies equally well to the split form, which is the more widely used in practice. The theory of action is the same, except that the split flap has the advantage of leaving the basic form of the upper surface undisturbed while lowering the pressure at its trailing edge (since the trailing edge lies in the dead-air zone immediately behind the depressed flap) and so

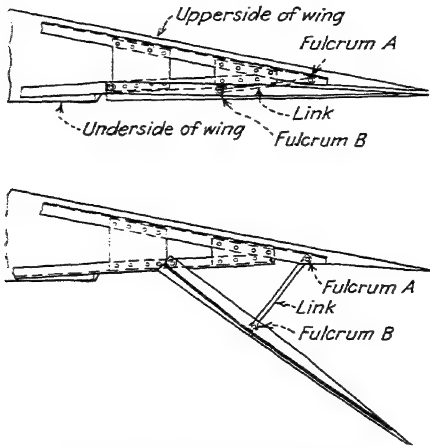

FIG. 188.—The Zap balanced flap.

providing the acceleration of upper-surface flow and the reduced liability to separation that have already been discused. As might be expected in view of this continued availability of the whole extent of the upper surface, even after the flap is fully depressed, the split flap shows somewhat more increase of maximum lift than does the simple one. Its typical range of improvement over the basic airfoil is from 55 to 75 per cent, as against the 40 to 60 of the simple case. Best flap widths are substantially the same for the two types. Best angles of depression are not very different, though tending to run a little (5 or 10 deg.) higher for the split form.

The general discussion applies with essential force, too, to the type of flap that moves backward as it comes down, so increasing

the area by a small amount at the same time that it changes the section. The Zap flap, illustrated in Fig. 188,[1] is the best known and most used example of that sort, and its advantage is not so much in its increase of wing area and consequent increase of effect (though it runs the maximum lift from 5 to 10 per cent above the best figures attainable with an ordinary split flap) as in the balance of flap loads and the resultant reduction of operating forces that the linkage provides. Against that, of course, has to be set a certain complication of crossheads and the like, a considerably more elaborate operating mechanism than the simple hinge of the ordinary flap, whether split or simple—which, however, frequently has to be complicated in its own turn by the introduction of hydraulic or electrical devices for operating a control that comes on large airplanes to require a force beyond the possibility of manual application.

Since the depression of the flap increases the mean camber, it might be expected to change the zero-lift angle and to shift the lift curve to the left, maintaining the same slope and keeping the curve parallel to itself. This it does. The deepest mean camber of a Clark Y airfoil in its normal form is 4.8 per cent, while the section distorted by a lowering of a simple flap through 45 deg. has a maximum effective camber of 18 per cent. The lift curve ought then, according to the rough rule that holds so well for ordinary airfoils,[2] to be transplanted 13 deg. to the left. Actually the shift is but 10 deg. at zero lift, but the lift curve with the flap down proves to be deformed in that region, and at lift coefficients lying in the straight-line portion of both curves the lateral separation is almost exactly what the prediction calls for—as shown in Fig. 189, where the curves are reproduced. It will be noted there, too, that the maximum lift with the flap down comes at a somewhat smaller angle of the basic section than with the flap up. That again is what might be expected, and the prediction's confirmation is welcome to the designer, for a

[1] Taken from Joyce, *Trans. Am. Soc. Mech. Eng.*, *cit. supra*, where the action of the Zap flap and the problems of its practical application are discussed in some detail. See also, for a more general treatment of the effect of shifting the flap hinge backwards along the wing chord to any degree whatever as the flap comes down, "The Aerodynamic Characteristics of a Model Wing Having a Split Flap Deflected Downward and Moved to the Rear," by Fred E. Weick and Thomas A. Harris, *Tech. Note* 422.

[2] See p. 193, *supra*.

reduction of the angle of maximum lift simplifies the landing problem and reduces the height of the landing gear that has to be used to enable the airplane to make contact with the ground at a proper angle of the wings. It is also helpful in connection with lateral stability and control where the flaps are used only on the central portion of the wing, as it serves to produce stalling near the center of the wing earlier than at the tips.

Less welcome is the discovery that the use of flaps, as indeed of most other high-lift devices, tends to sharpen the peak of the lift curve very notably and to increase the chance of finding a discontinuity there.

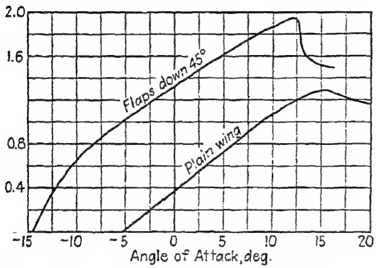

FIG. 189.—Effect of flap depression on lift characteristics.

Flaps are somewhat less effective in increasing the lift on a biplane combination than on a monoplane, as the depression of the flap on the upper wing interferes with the free flow of air over the upper surface of the other member of the combination. The fitting of a flap gear on the upper wing alone, the upper and lower wings being of equal size, increases the lift only about 10 per cent over that of the basic combination of rigid airfoils, while the depression of flaps on the upper and lower wings together and by like amount showed an increase at maximum lift of around 35 to 40 per cent, two-thirds as much as for the monoplane. The optimum flap angle is smaller for the biplane than for the single wing, the maximum lift being attained with the flaps down only about 30 deg. Some improvement in results can be secured if the flaps are interconnected so that those on the lower wing, having nothing below them with which they can interfere, turn down farther than the upper ones. One particular investigation,[1] in which a careful study was made of this point, showed that the absolute maximum of lift could be attained with the upper flap at 22 deg. and the lower one at 48 deg. for a thin section with a nearly flat lower surface. Certainly the angle of the

[1] Unpublished. Carried out at the Mass. Inst. of Tech. for the Wright Aeronautical Corp.

lower flap ought to exceed that of the upper one by at least 50 per cent.

Although the primary function of a flap gear is to increase the maximum lift, it may also have a favorable effect on efficiency under certain special conditions. The best flap angle for any particular condition of flight can readily be determined by plotting profile-drag coefficient against lift coefficient for a number of flap settings on the same sheet, as has been done in Fig. 190 for a particular airfoil of approximately 8½ per cent maximum thickness and 2.8 per cent deepest mean camber.[1] From the envelope of such a sheaf of curves plot of optimum flap angle against lift coefficient can be constructed. As a rule, where the basic section has a mean camber of 3 per cent or more the lowest possible values of C_D for lift coefficients up to 0.2 or 0.3 are to be obtained with the flap turned up to a very slight negative angle, not exceeding 2 or 3 deg. Where the basic section is symmetrical there is, of course, no advantage in the use of negative flap angles, and with cambers of less than 3 per cent negative settings are likely to be helpful only at lift coefficients below 0.2. The minimum coefficient of power required to drive the wing through the air, $D_C/L_C^{3/2}$, is generally reached when the flap is down slightly, something between 3 and 8 deg. The most advantageous angle of depression in this case is, of course, largest for basic airfoils of least mean camber. In no case is the beneficial effect of the optimum flap setting on any characteristic except maximum lift likely to exceed 5 per cent if the basic section is a reasonably good one.

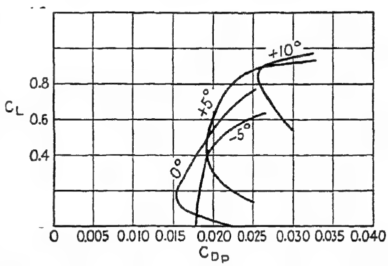

FIG. 190.—Effect of flap angle on drag characteristics.

Small though the gains may be, they ought to be taken into account. With the flap at last taking the place in airplane design that it had deserved long before it received it, airfoil sections ought no longer to be judged on the basis of their performance as rigidly fixed outlines. Every section that shows promise ought to be minutely studied with a flap attached. The

[1] From *R. and M.* 152.

ratio of maximum lift to minimum drag ought henceforth to be the ratio of maximum lift with the flap down 40 deg. to minimum drag with it up 2 deg. (or whatever may be the appropriate angles for the particular section). And if that is done, though unfortunately it is far from having been done as yet and there is a lamentable lack of data from which to draw definite conclusions, the probability is strong that mean cambers of over 2 per cent or perhaps even those exceeding 1 per cent will become as obsolete in good design as thicknesses of less than 8 per cent are now. The low-ambered section will come into its own.

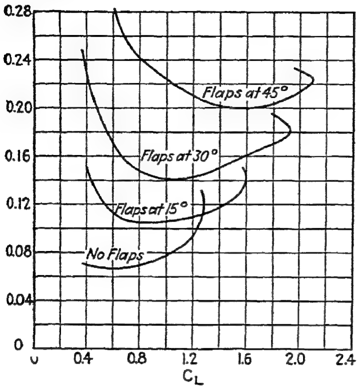

FIG. 191.—Effect of flap angle on gliding angle.

When the flap is pulled far down to give maximum lift the drag is, of course, very high. In Fig. 191, D/L is plotted against C_L for a straight Clark Y airfoil and for the same basic section with a split flap having a width of $0.25c$ and pulled down to 15, 30, and 45 deg.[1] Whereas the basic section for which the results

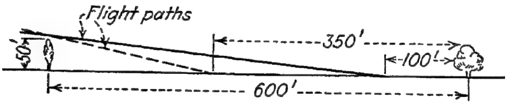

FIG. 192.—Paths of approach to landing with and without flaps.

are there plotted has a D/L at maximum lift of 0.12, the D/L with that combination of flap angle and angle of attack which gives the absolute maximum values to the lift coefficient is up to 0.225. This low efficiency may, however, be an actual advantage, for it makes it possible to steepen the inclination of the airplane's glide path when coming in for a landing without gaining speed and so to descend over obstacles with a reduced horizontal travel. The depressed flap acts as an air brake, making it possible to dive without gaining velocity, and the result is crudely shown in Fig. 192, showing two different possible flight

[1] From *Tech. Note* 498, *cit. supra.*

paths coming into a field 600 ft. wide and bordered with obstacles 50 ft. high. The airplane descending in a glide on a slope of 1 ft. in 10 will have used up practically the whole field before touching the ground at all, while if the angle can be steepened to a drop of 1 ft. in 5 the permissible length of run after striking the ground will be more than tripled. In addition to this gain, the flap gear reduces the actual velocity at the time of landing and so cuts down the length of run. Still further, as will be shown in another place, the flaps simplify the landing maneuver and improve the lateral control during the approach.

One of the most stubborn of the obstacles in the way of large-scale adoption of the flap was the presumption of a bad effect on longitudinal balance of the airplane and on the effectiveness of the tail surfaces. The depression of the flaps changes the downwash, and that, combined with the extremely disturbed state of the air flowing off the trailing edge of a wing of such irregular form as is given by the extreme depression of the flap, has been thought likely to render the tail surfaces somewhat ineffective.

In all this there is some truth, though far less than was feared. The downwash with the flaps down is not only increased, as compared with that for the plain wing, in direct proportion to the lift as theory would promise, but in a somewhat larger ratio. Furthermore, and more important, it is subject to more irregular variation with location, and there is more likelihood of sudden changes of flow condition around the tail surfaces with small changes of angle of attack. The drop of total head in the wake is much increased, which means a reduced velocity of flow across the controls. In a typical case (Clark Y with a split flap at 60 deg., flap width of $0.2c$, angle of attack of 10.7 deg., a little beyond that of maximum lift)[1] the total head in the wake one-half chord length behind the trailing edge falls to a minimum of 50 per cent of the value in the undisturbed stream, as against a corresponding minimum of 78 per cent for the plain wing at the same angle of attack. At two chord lengths behind the trailing edge, approximating to the forwardmost position in which the tail surfaces are likely to be placed, a better streamline flow has been regained, but the wake effect is still quite evident and the minimum of total head at that distance is 69 per cent with the

[1] "Investigation of Full-scale Split Trailing-edge Wing Flaps with Various Chords and Hinge Locations," by Rudolf Wallace, *Rept.* 539.

flap down and 84 per cent for the plain wing. The difference is significant, but not significant enough to interfere with the flap's acceptance. It seems of enough importance to some designers to induce them to provide separate flaps for the right and left wings and leave a gap between them through which the air can presumably flow undisturbed to the controls. The practice is a questionable one, for disturbance is always at its very worst, and most liable to the sudden changes that are most disastrous to stability, in the neighborhood of such discontinuities of section as exist at the inner end of a flap. Where it can be made structurally convenient, it would be better to carry the flaps straight across the center line and to increase the area of the tail surfaces as much as may be necessary to compensate for the diminished velocity of the air flow past them when the flap is down.

The depression of a flap of course throws the center of pressure to the rear. The effect of the "banking up" of the air on the lower surface of the wing is largest just forward of the hinge, and the effect on the upper surface, though it extends from one edge to the other, is of greatest relative importance at points well back on the chord. The center of pressure at the maximum-lift attitude with a flap down 45 deg. is from 7 to 10 per cent farther back than at maximum lift on the basic airfoil. Put in terms of moments, as is most convenient for stability calculations, the flap increases the coefficient of diving moment around the 25 per cent point by approximately 0.2—the amount being surprisingly independent both of flap width and of flap angle at least as long as the angle is above 40 deg. Even a 15-deg. flap angle has half as much effect on pitching moment as can be found with the same flap at 60 deg.

With the flap set at a negative angle the wing naturally shows the characteristics of one with a reflexed trailing edge. The diving moment is reduced to the point where it reaches zero and changes sign, and the center-of-pressure travel is reversed, the center of pressure remaining forward of the 25-per-cent point and moving forward as the angle of zero lift is approached.

Most serious, where the flaps extend over the whole span, is the influence on aileron control. The aileron itself is merely a flap which is pulled down to increase the lift on one side and pulled up to decrease that on the other, so giving a rolling moment. If flaps on both sides were pulled down to the point

of maximum lift, any further change of angle in either direction would decrease the lift at that point, and differential movement of the ailerons would produce no rolling moment. It is largely for that reason that most of the airplanes which incorporate flap gears have them so rigged that only the part of the flaps near the plane of symmetry of the airplane is used for increasing lift, the outer portions being retained for service as ailerons alone.

Automatic Flaps.—Though it is the practically universal rule for flaps to be manually operated, they can be made to operate themselves, the control being by a spring which tends to pull the movable element down at all times.[1] As the speed of the airplane is reduced by the pilot the angle of attack is increased; the center of pressure moves forward on the normal section; the aerodynamic moment about the hinge of the flap is therefore reduced; and the spring holds the flap down to a larger angle. The pilot is relieved of the necessity of giving any attention to the flap control, an advantage against which there has to be set the possibility of disconcerting and even dangerous vibrations of the free flap if its design and damping are not exactly right.

To determine the angle at which an automatic flap will ride for various angles of attack of the basic section of course requires a knowledge of the moments of the force on the flap about its hinge. For one case, that of an airfoil of slightly concave lower surface having a flap with a width of 22 per cent of the chord, calculations have been made on the assumption that the spring permits the flap to take the neutral position at the angle corresponding to one-sixth of the maximum-lift coefficient, and that the initial tension in the spring and the direction of its pull are so adjusted that its moment about the hinge remains constant for all flap angles. This is a condition which can at least be very closely approximated in practice. Figure 193 shows the variation of flap angle with lift coefficient under those particular conditions. It will be observed that the flap comes down far enough to give the absolute maximum of lift, provided that no stops are interposed to limit its travel.

For small flap angles the center of pressure of the flap lies quite near the hinge, both for the simple and for the split types,

[1] For a description of a particular application of this sort and data on its effects, consult "Flap Gear for Airplanes," by A. Hessell Tiltman, *The Automobile Engineer*, February, 1927.

and the hinge moments are correspondingly small. As the angle increases the center of pressure moves back, and at flap angles of 40 deg. or more it can be assumed with fair accuracy to lie 40 per cent of the flap chord behind the hinge.[1] The average intensity of load on the flap, at its most effective angle and at the angle of attack of the wing corresponding to maximum lift, is from 10 to 30 per cent less than the maximum lift per unit of area on the

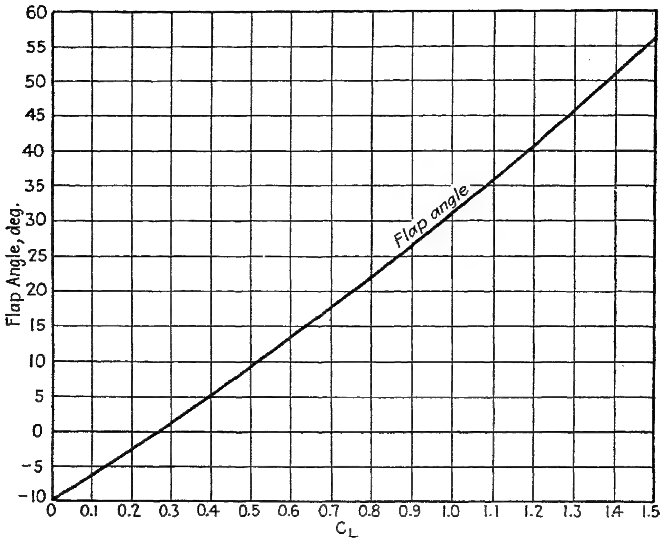

FIG. 193.—Variation of angle of an automatic (spring-controlled) flap.

wing with the flap down—the ratio of average load on the flap to average lift on the wing being highest with the narrowest flaps. The total load on a flap with a width of 20 per cent of the chord may then be, even in steady flight at minimum speed and with no allowance for the effects of rough air or of accelerations, fully 15 per cent of the weight of the airplane on which it is mounted.

Flaps at the Leading Edge.—A flap along the trailing edge may be either replaced or supplemented by one hinged to the leading edge of the airfoil, so approximating more nearly to the original ideal of a perfectly flexible wing of continuously variable curva-

[1] "Wind-tunnel Measurements of Air Loads on Split Flaps," by Carl J. Wenzinger, *Tech. Note* 498.

ture. The lift is, of course, increased by turning the leading-edge flap down to increase still further the mean camber of the wing, as indicated in Fig. 194. The maximum effect is much less, both on lift and on drag, than that of a flap in the more usual position, but such tests as have been made[1] show it to be by no means without effect.

Taken alone, with no rear flap to move in conjunction with it, a leading-edge flap increases the maximum lift of a typical airfoil about 15 per cent, the best angle of setting for high lift being approximately 10 deg. On the same airfoil (in biplane combination) a trailing-edge flap alone produced a 38 per cent increase. When the two flaps were employed together, the maximum lift was pushed up to 46 per cent above the normal, an 8 per cent

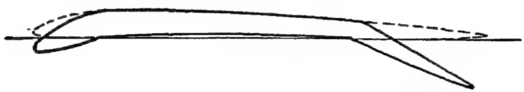

FIG. 194.—Wing with leading- and trailing-edge flaps.

additional gain by using a leading-edge flap in conjunction with that at the trailing edge. With the two flaps used together the optimum angles of setting are a little less than when either one is used alone, but the change is but 1 or 2 deg. in each case. To recite all these effects *in extenso* is somewhat academic, however, as the leading-edge flap offers too unpleasant a structural problem to be undertaken for the sake of so limited an aerodynamic gain.

Slotted Wings.—As an expedient for securing temporary increases of lift, the principal rival of the trailing-edge flap is the slot, originated by F. Handley Page and first publicly described by him in 1921.[2] The slot is an opening through a wing, connecting the upper and lower surfaces and running parallel to the span. Its primary object is the elimination of the collapse into instability of the flow over the upper surface of an airfoil at high angles of attack, thus making it possible for the curve of lift against angle of attack to go on rising along a substantially

[1] "Investigation of the Performance of a Variable Cambered Wing," by W. L. Cowley and H. Levy, *R. and M.* 652; "Model Experiments with Variable Camber Wings," by R. G. Harris and F. B. Bradfield, *R. and M.* 677.

[2] "The Handley Page Wing," by F. Handley Page, *Aero. Jour.*, June, 1921. The same device was independently developed, at almost the same time, by Lachmann in Germany.

straight line to angles well beyond those at which maximum lift is reached by the ordinary airfoil. It may be either left continuously open or opened under some conditions of flight and closed under others. In the latter case the closure may be effected either by passing a plate across the surface of the slot to stop the flow of air through it or (much more commonly) by moving the part of the wing that lies ahead of the slot forward and backward, leaving an open slot in the one case and fairing into the main portion of the wing to make a single profile in the other. There is another alternative in that the forward part of the wing may be moved either mechanically, under the control of the pilot, or automatically. Automatic operation, which is the general rule, is very simple, requiring little more than that the movable portion should be left free to travel forward either in roller guides or by swinging on a parallelogram linkage. The portion of the

FIG. 195.—Slotted wing.

wing contour immediately around the nose, the portion defined by the movable segment when it is in the slot-closed position, carries as a whole a positive pressure at low angles of attack and a negative pressure (with respect to atmospheric) at high angles. At low angles the air forces push it back against the main body of the wing. At high angles, where the slot may begin to display its effect in suppressing burbling and increasing maximum lift, they suck it forward to the limit of the travel that its stops allow.

A typical slotted airfoil is shown in Fig. 195. It gives a maximum lift a little more than 50 per cent in excess of that of the basic section, at the same time increasing the angle of maximum lift by about 7 deg. Even greater increases of lift have been attained in a few instances.

In its essentials, the action is very simple.[1] The part of the airfoil forward of the slot acts somewhat as an independent wing of very small area. When the airfoil as a whole is set at a very

[1] For discussions of the theory, see "Theory of the Slotted Wing," by A. Betz, *Berichte und Abhandlungen der wissenschaftlichen Gesellschaft für Luftfahrt*, January, 1922, translated in *Tech. Note* 100; "On the Two-dimensional Flow around Slotted Wing Sections," by Busuke Hudimoto, *Mem.* of Coll. of Eng. of Kyoto Imperial Univ., March, 1934.

large angle of attack, as in Fig. 196, the small forward portion is at a small angle to the air in its immediate neighborhood, and there is therefore a smooth streamline flow over its upper surface. The downwash of the air flowing off from the trailing edge of the small forward element (considering it as acting as an airfoil in its own right) turns the lines of flow down along the upper surface of the larger part of the airfoil, thus resisting that collapse of the flow in the neighborhood of the region of maximum sharpness of curvature of the upper-surface contour which is characteristic of the rigid airfoil of smooth outline at a very large angle. Furthermore, the convergence of the slot forces an increase of the air velocity through it as the upper surface is approached, and that combines with the effect of the circulation about the auxiliary

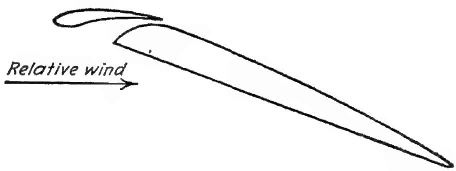

FIG. 196.—Slotted wing at its angle of maximum lift.

airfoil to give an accelerated flow along the upper surface of the wing behind the slot. Acceleration of flow acts, as always, to hurry the boundary layer along and to resist the boundary-layer stagnation that produces separation. Hence the appearance of separation is delayed to unaccustomedly large angles of attack. Hence, as already noted out of the record of experience, the increase of lift with increasing angle is led to continue far beyond the ordinary range, the maximum lift then attaining values correspondingly above those commonly expected.

In the relation of theory to fact in detail, pressure distribution offers, as usual, a fertile resource. Figure 197[1] shows the distribution around both portions of a slotted airfoil at an angle of attack of the basic section of 25 deg. Though the angle is far beyond that at which burbling would ordinarily appear, the pressure diagram for the main wing maintains the form that would normally be characteristic at about 12 deg., except that the pressure drop on the rear half of the upper surface is greater than it would

[1] From "Pressure Distribution on a Slotted R. A. F. 31 Airfoil in the Variable-density Wind-tunnel," by Eastman N. J

be under those conditions. The distribution on the auxiliary airfoil is qualitatively the same, the diagram immensely compressed by the shortness of the chord. The peak of pressure drop rises even higher on the auxiliary airfoil than on the main one (partly because of its deep curvature and high mean camber), actually reaching $-5.7q$ when α for the basic section is 31 deg.

With this goes a high but not an extravagant intensity of loading on the auxiliary airfoil. The coefficient of average load nor-

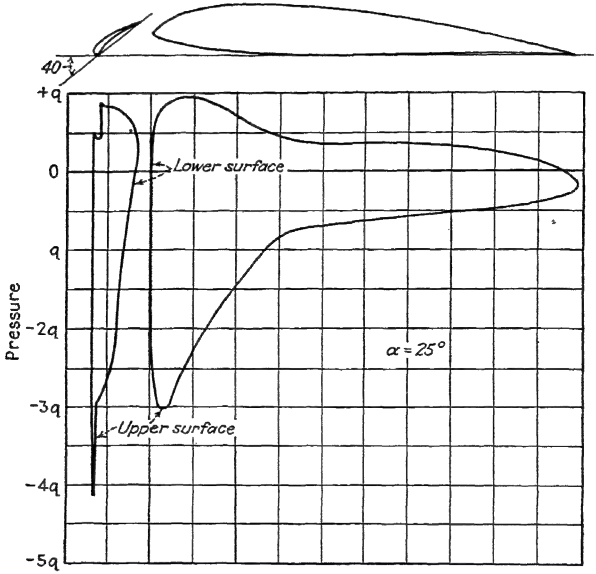

FIG. 197.—Pressure distribution on a slotted wing.

mal to its chord in this case reached a maximum of 2.0—about equal to the average $C_{L_{max}}$ for a slotted wing as a whole.

Pressure distribution serves also to explain, and to predict in detail, the slot's automatic functioning. Figure 198,[1] for example, shows in their proper position and direction, and scaled for magnitude, the vectors of total resultant force on the auxiliary airfoil with the slot closed. Obviously if the auxiliary be sup-

[1] "Based on "Forces on Leading Wing of N-22 Airfoil with Handley-Page Automatic Slot," *Rept.* 371, Navy Yard, Washington, D. C., 1928.

ported on rods that run in guides parallel to the chord, it will move forward as soon as the resultant of its aerodynamic loads is inclined forward of the perpendicular to the chord. Specifically, in Fig. 198, 9 deg. would be just on the dividing line, and just above that angle the slot would begin to open. Where swinging links are used in place of rigid rods and rollers the angle of opening depends, of course, upon the mechanics of the linkage as well as upon the aerodynamic factors, and the movement may be made to start with the vector inclined by a considerable amount in either direction from the vertical.

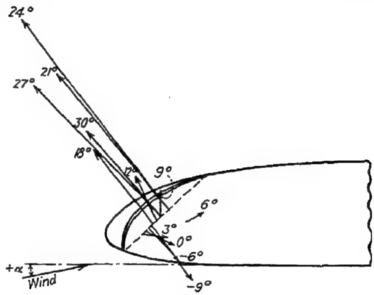

FIG. 198.—Forces on the movable portion of a slotted wing.

Hence, as already noted in the light of the Handley Page experiments, the increase of lift with increasing angle is led to continue far beyond the ordinary range, the maximum lift then attaining values correspondingly above those commonly expected.

Upon slots as upon flaps there has been a vast amount of wind-tunnel work,[1] and much information upon the effect of their form and position is available as a result.

[1] "Experiments with Slotted Wings," by G. Lachmann, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, June 15, 1921, translated and published as *Tech. Note* 71; "Problem of the Slotted Wing," by W. Klemperer, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Oct. 31, 1921; "Results of Experiments with Slotted Wings," by G. Lachmann, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, May 26, 1924, translated and published as *Tech. Memo.* 282; "The Handley Page Slotted Wing," by H. Glauert, *R. and M.* 834; "Tests of Four Slotted Aerofoils, Supplied by Messrs. Handley Page, Ltd.," by F. B. Bradfield, *R. and M.* 835; "Some Experiments on a Slotted Aerofoil," by H. B. Irving and A. S. Batson, *R. and M.* 929; "Full-scale and Model Measurements of Lift and Drag of Bristol Fighter with Handley-Page Slotted Wings," by E. T. Jones and L. E. Caygill, *R. and M.* 1007; "Model Experiments on R. A. F. 31 Aerofoil with Handley Page Slot," by H. B. Irving, *R. and M.* 1063; "Slotted-wing Airplanes," by E. Everling, *Zeitschrift des Vereines deutschen Ingenieure*, May 7, 1927, translated as *Tech. Memo.* 432; "Wind-tunnel Experiments on the Design of an Automatic Slot for R. A. F. 34 Section," by F. B. Bradfield and F. W. G. Greener, *R. and M.* 1204; "Practical Tests with the Auto-control

The most striking thing about the results of the tests is the limited sensitiveness of the lift coefficient to niceties of form. It appears to be a necessary condition of good performance that the slot should decrease in width from the lower toward the upper surface of the airfoil (to produce the velocity increase at the exit

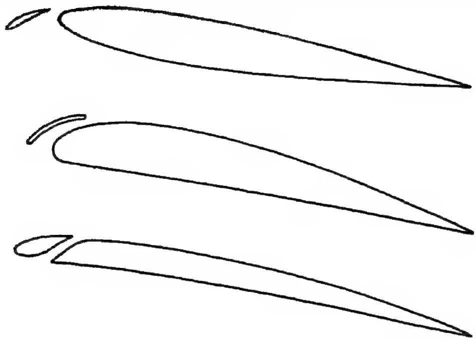

Fig. 199.—Alternative forms of slot.

that serves to scour the upper surface), but within the range covered by that limitation forms as widely different as the three shown in Fig. 199 have proved eminently successful in increasing lift coefficients and angles of maximum lift. Almost any kind of a slot near the leading edge suffices to increase the maximum lift of such a section as the R. A. F. 15 by from 45 to 65 per cent. On

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Slot" (a lecture, followed by an excellent discussion), by G. Lachmann, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Aug. 28 and Sept. 15, 1930, translated as *Tech. Memos.* 593 and 594; "The Use of Slots for Increasing the Lift of Airplane Wings," by Fr. Haus, *L'Aéronautique*, June, 1931, translated as *Tech. Memo.* 635; "The Aerodynamic Characteristics of a Slotted Clark Y Wing as Affected by the Auxiliary Airfoil Position," by Carl J. Wenzinger and Joseph A. Shortal, *Rept.* 400; "Flight Tests to Determine the Effect of a Fixed Auxiliary Airfoil on the Lift and Drag of a Parasol Monoplane," by Hartley A. Soulé, *Tech. Note* 440; "The Characteristics of a Clark Y Wing Model Equipped with Several Forms of Low-drag Fixed Slots," by Fred E. Weick and Carl J. Wenzinger, *Rept.* 407; "The Effect of Multiple Fixed Slots and a Trailing-edge Flap on the Lift and Drag of a Clark Y Airfoil," by Fred E. Weick and Joseph A. Shortal, *Rept.* 427; "Wind-tunnel Tests of a Clark Y Wing with a Narrow Auxiliary Airfoil in Different Positions," by Fred E. Weick and Millard J. Bamber, *Rept.* 428; "Wind-tunnel Tests on Combinations of a Wing with Fixed Auxiliary Airfoils Having Various Chords and Profiles," by Fred E. Weick and Robert Sanders, *Rept.* 472.

the thick sections which already have high maximum-lift coefficients the benefit is not so marked, the increase of lift for a Clark Y being only 42 per cent with the most effective of many slots tested, while for the R. A. F. 19, a section with a mean camber of 10 per cent and a basic maximum lift of 1.62, it was only about 10 per cent. In other words the slot, like the flap, tends to equalize the differences in maximum lift between the thick and thin sections and those with concave and those with flat lower surfaces. A slotted R. A. F. 6 gave a maximum-lift coefficient of 1.88, for example; an R. A. F. 15, 1.67; a Clark Y, 1.84; an R. A. F. 19, the high-lift section already referred to, 1.80; and a Göttingen 387, 1.76. All of these figures are nearly enough

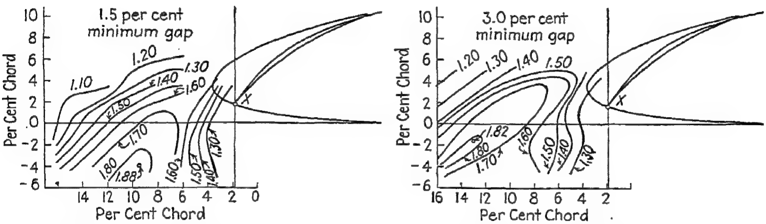


FIG. 200.—Relation of maximum lift to slot form (the contours indicate the maximum lift for any position of the point X on the movable portion of the wing).

the same so that their order might have been reversed by small changes in the forms of the slots used or by a change in the Reynolds' number at which the tests are used. The slot eliminates at least one of the inherent disadvantages of the low-lift wing, although there are as yet no experiments to show how it would act on an airfoil of extreme low-lift type with zero mean camber.

The width of the slot, like the form of the auxiliary airfoil at the leading edge, seems to make but little difference within wide limits, but the minimum distance across the gap between the auxiliary and principal elements of the airfoil lies between 2 and 3 per cent of the chord on the most successful slotted wings.

Though extensive changes can be made without serious consequences, there is, of course, a best possible slot for any section. Generally speaking, as the gap between the trailing edge of the auxiliary portion and the main wing is diminished the leading edge of the auxiliary portion has to be moved backward and

downward for best results. Particular applications of that rule appear from Fig. 200, where the best positions of the auxiliary airfoil are illustrated for gaps, or slot widths, of 1.5 and 3.0 per cent of the chord, respectively. The basic section in these particular tests,[1] with the slot closed, was a Clark Y. The ideal gap in this case was 2 per cent, but the maximum lift lay always between 1.8 and 1.9 as the gap was changed from 1 per cent to 3.

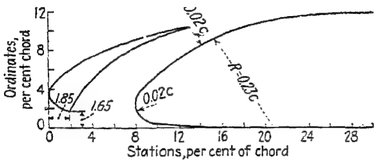


FIG. 201.—Ideal arrangement for a fixed slot.

Except in respect of drag with the slot closed it proves helpful to round off the leading edge of the main wing, so giving the slot more distinctly the form of a venturi tube. Where the slot is to be left open at all times such rounding is advantageous from every point of view. Rounding off the leading edge of the lower surface of the auxiliary airfoil forward of the slot, on the other hand, is bad unless the radius of the rounding is exceedingly small. The best arrangement discovered[2] for a fixed slot, based on a Clark Y section as an envelope of the two segments of the wing

FIG. 202.—Best combination of Clark Y airfoil with fixed auxiliary.

and bridging over the slot, is shown in Fig. 201. Once again the minimum gap is 2 per cent. The maximum lift with this form was 5 per cent less than for the best of the open-and-shut arrangements such as Fig. 200 illustrated, and the ratio of maximum lift to minimum drag was 12 per cent lower than for the plain Clark Y wing.

Somewhat better, if a fixed slot has been determined on, is the addition to a good wing section of a small auxiliary airfoil above and ahead of its leading edge. The best form and placing of such an auxiliary for the Clark Y are shown in Fig. 202.[3] "Slotted wing" begins to misfire as a descriptive phrase for a combination

[1] From *Rept.* 400, *cit. supra.*

[2] In *Rept.* 407, *cit. supra.*

[3] From *Rept.* 472, *cit. supra.*

like that, but the theory and the objective remain the same. The arrangement shown in Fig. 202 has 5 per cent less maximum lift than that in Fig. 201, but the drag is also far lower and the ratio $C_{L_{max}}/C_{D_{min}}$ is actually a third higher than for the unadorned

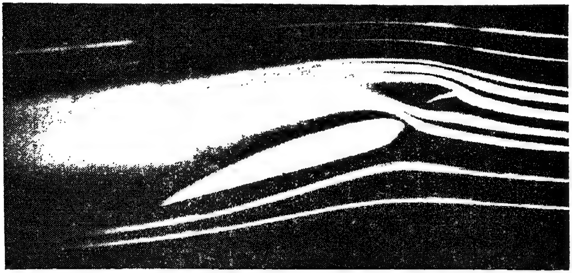


FIG. 203.—Flow of air around an airfoil with a fixed auxiliary element.

Clark Y at the same Reynolds' number. The high-speed performance with such an auxiliary ought then to be good, but the drag coefficients at values of C_L from 0.5 to 1.0 are high and the minimum power consumption considerably higher than for the simple airfoil without auxiliary. As a general rule, there is little

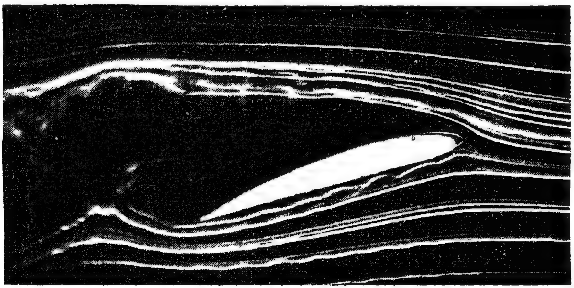


FIG. 204.—Flow around a simple airfoil, at the same angle of attack as Fig. 203.

to be said for fixed slots except where simplicity and low first cost are considerations far outweighing performance.

Figures 203 and 204 compare the air flow around a fixed-auxiliary-airfoil combination with that around a normal unslotted airfoil at the same large angle of attack. The action of the auxiliary airfoil, which is essentially the same as the action of a slot, in preventing burbling is obvious.

An interesting halfway house between the fixed and the movable slot is the pilot plane, much investigated in Great Britain.[1] Much like Fig. 202 in general appearance, a pilot-plane combination differs from it mechanically in that the auxiliary surface is pivoted and free to swing between stops. At low angles of attack, it lies with the air stream and substantially parallel to the chord of the basic airfoil. At large angles its trailing edge swings up some 20 or 30 deg., or as far as a stop will allow, and the combination takes on a combined form much more like that of a true slotted wing. The increase of maximum lift and reduction of drag at high angles due to freedom to pivot are much less than

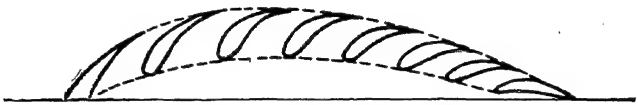


FIG. 205.—Multi-slotted airfoil.

one might expect, and although the best pilot-plane arrangements show somewhat better results than the best fixed-auxiliary combinations of the type of Fig. 202 the improvement is nowhere more than 5 or 10 per cent. Everything taken into account, mechanical complication included, it appears that the designer's wisest choice will always lie between the extreme of simplicity represented by the fixed auxiliary well ahead of the leading edge and the extreme of aerodynamic efficiency furnished by the automatically opening slot with the movable portion well below the leading edge of the fixed one, as in Fig. 200.

Experiments have been extended to cover the case of two or more slots variously spaced along the airfoil chord, and even an arrangement with seven slots,[2] shown in Fig. 205. It gave the extraordinary maximum-lift coefficient of approximately 3.5, but it is hardly fair to compare such an arrangement with a single

[1] "Wind Tunnel Tests on a R. A. F. 15 Aerofoil with Pilot Planes," by F. B. Bradfield and K. W. Clark, *R. and M.* 1145; "Wind Tunnel Tests of Aerofoils with Pilot Planes," by F. B. Bradfield and K. W. Clark, *R. and M.* 1213; "Wind Tunnel Tests of a R. A. F. 30 Wing Fitted with a Self-setting Slotted Plane," by F. B. Bradfield and S. Scott Hall, *R. and M.* 1225.

[2] "The Handley Page Wing," *Aero. Jour.*, p. 270; "Results of Experiments with Slotted Wings," by G. Lachman, *Tech. Memo.* 282; "The Effect of Multiple Fixed Slots and a Trailing-edge Flap on the Lift and Drag of a Clark Y Airfoil," by Fred E. Weick and Joseph A. Shortal, *Rept.* 427.

airfoil. The form of the basic airfoil has almost disappeared, and it has essentially been replaced by a heavily staggered multiplane with large negative decalage. It would seem more logical to take the chord used in figuring the area of the section, and so the lift coefficients, as the sum of the chords of the numerous elements, rather than the distance across an envelope curve. The maximum lift was reached with the chord of the basic section at an angle of attack of nearly 45 deg. The actual construction of an airfoil broken up into small bits by a large number of slots would, of course, present very serious problems. Where there is a serious effort to keep some form and solidity for the basic airfoil, as in Fig. 206,[1] the maximum lift is unlikely to exceed that

Fig. 206.—Airfoil with two slots.

obtainable with a single slot in the same airfoil by more than 10 per cent.

Though slots are ordinarily placed at the leading edge, the theory of their action makes it apparent that some benefit may be expected even if they are cut through well back on the chord. The increase of maximum lift diminishes steadily with increasing distance from the nose, but the addition to minimum drag with the slot open is also diminished, and in a larger ratio. A few cases, with their maximum-lift coefficients, are shown in Fig. 207,[1] where it appears that the variation of maximum lift with slot coordinate for the fixed type of slot is virtually linear. The unslotted wing takes the place on the curve of one with a slot at the trailing edge, and for every 10 per cent of the chord from the actual slot location (measured to the forward edge of the slot on the upper surface) to the trailing edge the maximum lift is decreased by 4 per cent. The ratio of maximum lift to minimum drag is highest for the wing with the slot farthest back, exceeding the ratio for the unslotted wing by 4 per cent. The increase of minimum drag by 65 per cent above the drag of the basic section in the case of the wing with a fixed leading-edge slot proves too heavy a burden for even its high maximum lift to carry when it is efficiency ratios that are sought.

[1] From *Rept.* 427, *cit. supra.*

The most important effect of the slotted wing on airplane performance is in lowering the landing speed. Its possibilities of improving take-off distance and rate of climb will be discussed elsewhere.

While the very high angle at which maximum lift is attained with a slotted wing has the advantage of reducing the liability

FIG. 207.—Slotted and flapped wings and their maximum lifts.

of reaching a stalling attitude without intention on part of the pilot, there is a counterbalancing drawback in the difficulty of getting the airplane on to the ground at minimum speed. In making a landing on an ordinary landing gear, the path of travel must be very nearly horizontal just prior to the instant of impact with the ground, and if the minimum speed is to be attained the angle of the wing chord to the horizontal must then approximate to the angle of maximum lift. With that angle increased to 24 deg., a fair average value for a slotted wing, it would be necessary either to set the wings at so large an angle to the fuselage that

the machine would fly in a very abnormal and inefficient attitude at high speed, or to increase the landing angle by building the landing gear up as indicated in Fig. 208. Neither alternative is very acceptable.

Combinations of Slot and Flap.—As a means of overcoming this difficulty the combination of a leading-edge slot, which increases the angle at maximum lift, with a trailing-edge flap, which decreases it, has been tried, the slot and flap being operated

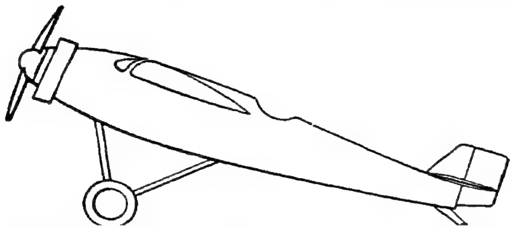


Fig. 208.—Landing-gear form necessary for three-point landing at minimum speed with a slotted wing.

in conjunction. A logical extension from that, in turn, has been the incorporation of another slot about two-thirds of the way back on the chord at the leading edge of the flap and opened by the movement of the flap.[1] The opening and closing of the rear

[1] "The Handley Page Slotted Wing," by H. Glauert, *R. and M.* 834; "Tests of Four Slotted Aerofoils, Supplied by Messrs. Handley Page, Ltd," by F. B. Bradfield, *R. and M.* 835; "On the Use of a Slotted Trailing Flap on Aerofoils of Various Cambers," by F. B. Bradfield, *R. and M.* 865; "Tests on an Aerofoil with Two Slots Suitable for an Aircraft of High Performance," by F. Handley Page, *Flight*, Jan. 28, 1926; "Experiments on Airfoils with Aileron and Slot," by A. Betz, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. III, translated in *Tech. Memo.* 437; "The Effect of Multiple Fixed Slots and a Trailing-edge Flap on the Lift and Drag of a Clark Y Airfoil," by Fred E. Weick and Joseph A. Shortal, *Rept.* 427; "Model Experiments with Rear Slots and Flaps on Aerofoils R. A. F. 31 and R. A. F. 26," by H. B. Irving, A. S. Batson, and A. L. Maidens, *R. and M.* 1119; "Wind-tunnel Tests of a Wing with a Trailing-edge Auxiliary Airfoil Used as a Flap," by Richard W. Noyes, *Tech. Note* 524; "Technical Aspects of the 1934 International Touring Competition" (an admirable collective critique of the variable-lift mechanisms used on the various airplanes entered in a competition in which the lowest possible minimum speed was a major factor), by R. Schulz and W. Pleines, *Luftwissen*, Sept. 15 and Oct. 15, 1934, translated in *Tech. Memo.* 760.

slot can, of course, be mechanically accomplished by carrying the flap hinges on outriggers, so that the hinge line is below the flap or ahead of its leading edge or both. The results of several combinations are included in Fig. 207.

The effect of the introduction of the flap is to shift the lift curve sidewise and decrease the angle of maximum lift, precisely as with an unslotted wing. The value of the maximum benefits notably. The Clark Y, a section which in its basic form has a maximum-lift coefficient of 1.29, shows an increase to 1.95 with a trailing-edge flap down 45 deg. and both slots closed or to 1.81 with two open slots of the fixed type and no flap. The opening of the rear slot with the flap at 45 deg. increases the figure to 1.98, while a front slot sends it up to 2.18 with the rear one closed. With both slots open and the flap down 45 deg. the lift goes to 2.26, a final overall increase of 75 per cent over the original maximum. With the best form of movable slot and no constraint to make the auxiliary airfoil fair into the general wing contour, a coefficient of 2.40 could no doubt be attained.

The final maximum in this case was reached at an angle of attack of 19 deg., the addition of the flap having less effect than might have been hoped. The problem of so constructing as to give a proper landing angle still remains.

A trailing-edge slot for use in conjunction with a flap can be of almost any form without fatal damage to its efficiency, provided only that no part of the flap projects appreciably above the upper surface of the wing. That the slot should grow progressively narrower from the lower to the upper surface and that the forward boundary of the slot should meet the upper surface at an angle not exceeding 45 deg., so that the air emerging from the slot may encounter the main stream along the upper surface with a minimum of shock, are desirable qualities but not essential ones. The best form so far discovered is included, second from the bottom, in Fig. 207.

A slotted flap, detached as it is from the main body of the wing, has the form of an independent airfoil. Instead of developing separation on the upper surface at the flap hinge even at very moderate flap angles, as does a simple flap, it benefits by the flow of air through the slot to maintain a streamline flow along its upper surface up to a flap angle of 20 or 30 deg. Then it burbles. With further increases of angle the lift on the lower

surface of the wing continues to increase, and the flap continues to render some assistance (though a gradually diminishing amount) in accelerating the flow along the upper surface and reducing the separatist tendencies forward of the flap, but the lift contributed by the upper surface of the flap itself falls off rapidly. The net effect of all this is that for a wide range of flap angles the maximum lift remains nearly constant.

The diving moment around the 25-per-cent point is affected in very much the same fashion by the slotted flap as by the unslotted one. A 20-deg. flap angle, with a flap width of 20 per cent of the wing chord, increases the diving moment by about 0.15, an angle of anything from 30 to 60 deg. by about 0.20. These are averages for the working range of angles of attack, with special emphasis on the angles near maximum lift. They can be nothing more than averages, for the actual moments with the flap down vary considerably with airfoil angle. The aerodynamic center, or axis on the chord with respect to which the moments are constant, when the flap is down is at about 21 per cent of the chord in place of the canonical 25 per cent.

The next step is to make the rigid airfoil complete in itself, and then to add to it an auxiliary airfoil in such relationship to the trailing edge of the main wing that it becomes essentially a slotted flap. The auxiliary member may be rigidly mounted, free only to be rotated about an axis near its leading edge,[1] or it may be retractable into a pocket in the lower surface of the main airfoil. In the latter case it becomes a Fowler wing,[2] at the moment appearing the most interesting of the devices for combining variation of area and of camber.

The Fowler wing provides lift coefficients as high as 3.2, based on the area of the rigid wing with the flap retracted. For performance calculations that is the basis that ought in general to be employed, but it must always be borne in mind that variable-area wings call for added structure to support their

[1] "Wind-tunnel Tests of a Wing with a Trailing-edge Auxiliary Airfoil Used as a Flap," by Richard W. Noyes, *Tech. Note* 524.

[2] "Variable Lift," by Harlan D. Fowler, *Western Flying*, Nov., 1931. "Wind-tunnel Tests of the Fowler Variable-area Wing," by Fred E. Weick and Robert C. Platt, *Tech. Note* 419; "Aerodynamic Characteristics of a Wing with Fowler Flaps, Including Flap Loads, Downwash, and Calculated Effect on Take-off," by Robert C. Platt, *Rept.* 534.

extensible portions, and that from a structural point of view it is the extended area rather than the retracted one that is critical. It is then of interest to determine the lift coefficient based on the area with flap extended as well as on the basic area of the rigid

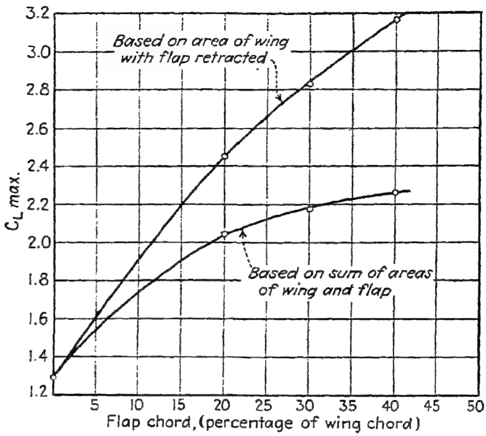


FIG. 209.—Maximum lift with Fowler flap, and variation with flap size.

portion, and the maximum so computed proves to rise to about 2.3. Curves of maximum lift on both bases are plotted against width of flap in Fig. 209. The basic section for which both these curves were developed was the Clark Y, and in each case, what-

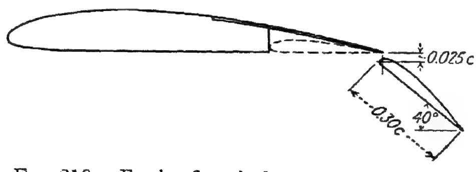


FIG. 210.—Fowler flap, in best extended position.

ever the width of the flap, its best possible attitude and setting were used in obtaining the points plotted on the curves.

The optimum flap position for maximum lift, in the particular case of a flap having a width of 30 per cent of the wing chord, is illustrated in Fig. 210. Whatever the flap width, it seems to remain true that best results are obtained with its leading edge directly under the trailing edge of the basic wing and a vertical

gap between the fixed and movable surfaces of about 2.5 per cent of the chord. The results were quite insensitive to movement of the flap forward of the trailing edge, a forward movement of 5 per cent of the chord reducing the maximum lift by only 9 per cent, but very sensitive indeed to any movement to the rear. A shift of 2 per cent backward from the optimum position, allowing a horizontal gap of that width between the fixed and movable airfoils, produces a 12 per cent drop in maximum lift. The sensitivity to vertical movement is approximately as great as that to horizontal movement to the rear of the optimum location, and the effects of closing the vertical gap between the two surfaces are more rapidly damaging than those of further opening it. With an optimum gap of 2.5 per cent of the chord, either a reduction of the gap to a shade under 1 per cent or an increase to 7 per cent costs 10 per cent in lift.

The work so far done on Fowler combinations has not clearly established the extent to which intermediate settings of flap can be used with benefit, but the probability is that there is no advantage in moving the flap at all unless it is to be moved to a very substantial extent and depressed at least 10 degrees. Whereas an auxiliary surface permanently installed behind the trailing edge of the wing, and adjustable only in angle, would undoubtedly make it possible to secure improvements in minimum thrust and power coefficients by very minor changes in its angle of setting, there seems to be little probability of any such gain with the Fowler arrangement. For every purpose except that of increasing the maximum lift or reducing the take-off run, the fully retracted position of the flap is likely to give the best results.

The most serious drawbacks so far appearing to this type of variable-lift device are the limitation that it puts on wing span and its bad pitching-moment characteristics. If the very high lift coefficient obtainable with the Fowler flap is to be used to permit correspondingly large reduction of wing area it becomes structurally impracticable to build the wing of high enough aspect ratio to maintain the span that might have been considered desirable in a rigid wing of constant area. Reduction of span means increase of induced drag, and sacrifice of ceiling and climb characteristics in general. It seems best to consider the Fowler mechanism, then, rather as a device for reducing the minimum

speed of flight and reducing the take-off run, keeping the basic wing area substantially unchanged from what it might have been with a simple flap, than as a device for reefing area and increasing maximum speed.

The diving moments produced by the operation of the flap are, as might have been foreseen in view of the rearward extension of the wing, immense. The diving-moment coefficient is increased by approximately 0.6, as compared with that for the basic wing, when the flap has a chord of 30 per cent of the basic chord. With the flap chord increased to 40 per cent for extreme maximum lift the coefficient may be increased as much as 0.75. To keep the airplane in equilibrium under that condition requires either very much larger tail surfaces than are commonly employed, or a stabilizer adjustable through a very large angle in flight. In a typical case with normal tail surfaces, the full extension of the 40 per cent flap would require a change of lift coefficient of as much as 1.0 over the whole extent of the horizontal surface to maintain the machine in balance.

Automatic Boundary-layer Control.—The slot is itself a form of boundary-layer control, accelerating the flow in the boundary along the upper surface. In the quest for a greater efficiency in that capacity, all manner of experiments have been tried.[1] All depend upon the interconnecting of two or more points of an airfoil's surface, to the end that air may flow through the interior from a high-pressure point to a low-pressure one and either accelerate the boundary layer in a critical region or suck it off from the surface or bring a supply of air to flow along the upper surface of an element (such as a flap) that would otherwise be in a dead-air region. All these schemes can be disposed of in somewhat cavalier fashion. Practically all of them effect some increase in maximum lift. None, to this time, has shown any measure of superiority over the simplest combinations of hinged or auxiliary flaps with slots straight through the wing.

[1] "Wind-tunnel Tests on an Airfoil Equipped with a Split Flap and a Slot," by Millard J. Bamber, *Tech. Note* 324; "Experiments with an Airfoil Model on Which the Boundary Layer Is Controlled without the Use of Supplementary Equipment," by I. H. Abbott, *Tech. Note* 371; "Wind-tunnel Tests of a Hall High-lift Wing," by Fred E. Weick and Robert Sanders, *Tech. Note* 417; "The Effects of Equal-pressure Fixed Slots on the Characteristics of a Clark Y Airfoil," by Albert Sherman and Thomas A. Harris, *Tech. Note* 507.

The field remains a beguiling one for the inventor, but the precedents offer little encouragement except the somewhat negative encouragement of knowing that whatever crop the soil may be capable of bearing is yet to be harvested.

Boundary-layer Control by Pressure or Suction.—The theory has already been explained.[1] It has already been explained why such apparently similar effects can be obtained by blowing air backwards along the upper surface or by sucking it continuously off from the innermost stratum of the upper-surface boundary layer into the interior of the wing. There remains the record of experience with attempted practical application.

The experimental record is still incomplete. The work that has been done is still scattered.[2] It is too early to become dogmatic about what boundary-layer control can or cannot

[1] P. 141, *supra*.

[2] "Experiments with an Airfoil from Which the Boundary Layer Is Removed by Suction," by J. Ackeret, A. Betz, and O. Schrenk, *Tech. Memo.* 374; "Wind-tunnel Experiments on the Effect on the Maximum Lift of Withdrawing and Discharging Air from the Upper Surface of an Airfoil," by W. G. A. Perring and G. P. Douglas, *R. and M.* 1100; "Tests of Pneumatic Means for Raising Airfoil Lift and Critical Angle, by E. N. Fales and L.V. Kerber, *Jour.* Soc. Automotive Eng., May, 1927; "Increasing Lift by Releasing Compressed Air on Suction Side of Airfoil," by F. Seewald, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Aug. 16, 1927, translated as *Tech. Memo.* 441; "Experiments with a Wing from Which the Boundary Layer Is Removed by Pressure or Suction," by K. Wieland, *Zeitschrift für Flugtechnik und Motorluftschiffahrt,* Aug. 16, 1927, translated as *Tech. Memo.* 472; "Preliminary Investigation on Boundary Layer Control by Means of Suction and Pressure with the U. S. A. 27 Airfoil," by Elliott G. Reid and M. J. Bamber, *Tech. Note* 286; "Wings with Nozzle-shaped Slots," by Richard Katzmayr, *Berichte der aerodynamischen Versuchsantalt in Wien*, vol. I, 1928, translated as *Tech. Memo.* 521; "Wind-tunnel Tests on Airfoil Boundary-layer Control Using a Backward-opening Slot," by Montgomery Knight and Millard J. Bamber, *Tech. Note* 323; "Experiments with a Wing Model from Which the Boundary Is Removed by Suction," by Oskar Schrenk, *Luftfahrtforschung*, June 11, 1928, translated as *Tech. Memo.* 534; "Experiments with a Wing from Which the Boundary Layer Is Removed by Suction," by Oskar Schrenk, *Zeitschrift für Flugtechnik und Motorluftschiffahrt,* May 15, 1931, translated as *Tech. Memo.* 634; "The Use of Slots for Increasing the Lift of Airplane Wings," by Fr. Haus, *L'Aéronautique,* June, 1931, translated as *Tech. Memo.* 635; "Wind-tunnel Tests of Airfoil Boundary-layer Control Using a Backward-opening Slot," by Millard J. Bamber, *Rept.* 385; "Large-scale Boundary-layer Control Tests on Two Wings in the N. A. C. A. 20-foot Wind Tunnel," by Hugh B. Freeman, *Rept.*

accomplish. But, as in several previous cases of the same order, it is possible to make certain general statements. Some of them are certainties. The others, on the evidence so far accumulated, are at the very least strong probabilities.

Thus: both pressure and suction are highly effective in increasing maximum lift; the effect continues to increase almost indefinitely with increasing intensity of pressure or suction; maxi-

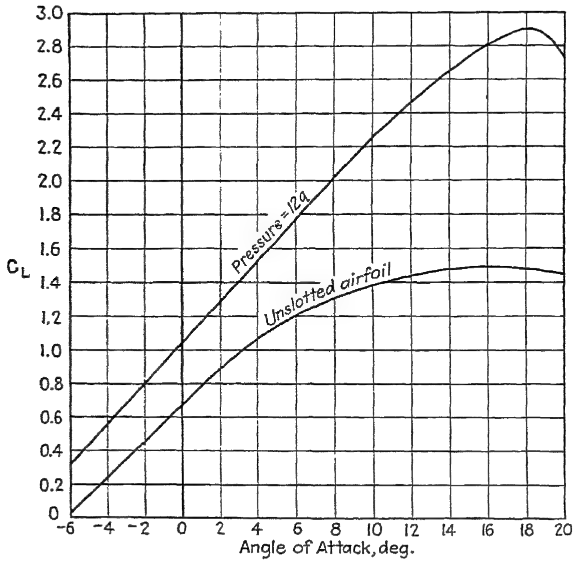


FIG. 211.—Effect of pressure boundary-layer control on lift characteristics.

mum-lift coefficients up to 3.2, considerably higher than are attainable by any other known device, have been secured by both the pressure and the suction methods and with all sorts of airfoils, even up to one of 30 per cent thickness; the relative advantage is greatest on the thicker sections, partly because their performance is so extravagantly bad under ordinary conditions and becomes as good as for sections of much less thickness after they have been made separation proof by boundary-layer treatment, partly because the larger passages available inside the thick wing reduce the internal friction in the air ducts and so help to keep down the power consumption; the suction method

is much more efficient, judged only by the power having to be expended in pumping air to secure the results, than the positive-pressure system but may prove to be inferior to the other alternative when full account has been taken of the effects on drag.

The increase of maximum lift comes accompanied by changes in the lift curve. The angle of zero lift is decreased, by as much as 3 deg. for very high internal pressures; the curve becomes more truly a straight line, beginning to fall off only within

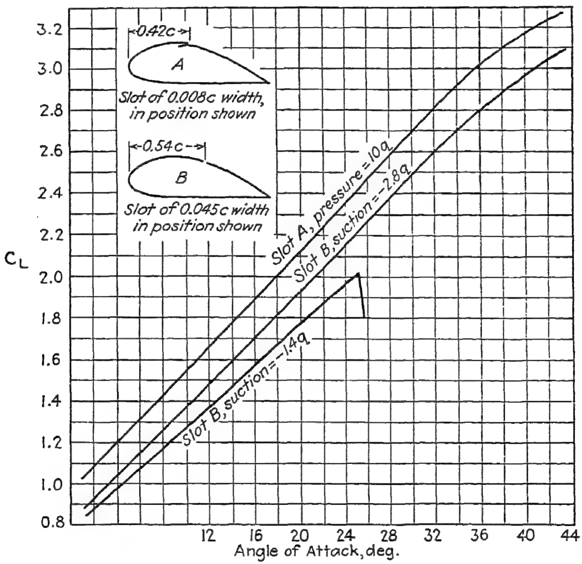


FIG. 212.—Comparison of pressure and suction methods of boundary-layer control.

2 or 3 deg. of the maximum; as a natural consequence the peak of the curve is somewhat sharpened, though actual discontinuity is rare; the angle of maximum lift is increased, but this is a much smaller factor than with the leading-edge slot and the increase is unlikely to exceed 4 deg. except on airfoils of extreme thickness. By way of illustration, the lift curves for a plain airfoil (of 15 per cent maximum thickness and $5\frac{1}{2}$ per cent deepest mean camber) and for the same airfoil with a pressure of $6\rho V^2$ intensity (equivalent to 306 lb. per sq. ft., or a head of

60 in. of water, at 100 m. p. h.) applied through a slot 0.1 in. wide halfway back on the upper surface, are plotted in Fig. 211.[1] Unfortunately there are no data to give a really fair comparison between the suction and pressure methods on an airfoil of that class, but Fig. 212 does it for a section of 30 per cent maximum thickness with the slot forms and locations shown.

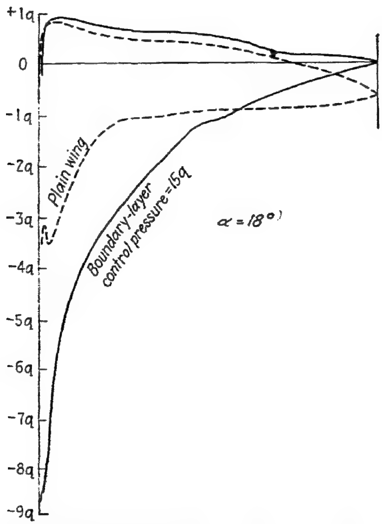


Fig. 213.—Effect of boundary-layer control on pressure distribution.

Pressure distribution confirms expectation so nicely that it is unnecessary to pore at length over its results. Figure 213[2] shows the pressures around the contour of a U. S. A. 27 wing at 18 deg. angle of attack, with and without boundary-layer control by pressure. The maximum pressure drop over the leading edge is increased by 160 per cent. The existence of atmospheric pressure exactly at the trailing edge, rather than a pressure about $0.5q$ below atmospheric as on the normal wing at the same angle, testifies to an absence of separation. The lift,

[1] Taken from *Rept.* 385, *cit. supra.*

[2] From "Preliminary Investigation on Boundary Layer Control by Means of Suction and Pressure with the U. S. A. 27 Airfoil," by Elliott G. Reid and M. J. Bamber, *Tech. Note* 286.

as approximately measured by the area enclosed within the pressure diagram, is approximately doubled.

The success of boundary-layer control in preventing separation is photographically attested in Figs. 214 and 215,[1] where the

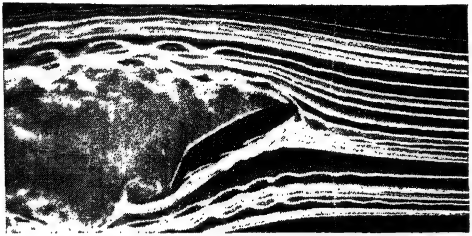


FIG. 214.—Flow of air around a flapped airfoil at a large angle of attack.

flow around a flapped airfoil at a large angle of attack is indicated under normal conditions (214) and with suction applied to the boundary layer (215).

Figure 216 and its immediate successor are good ones on which to fix attention as summarizing the whole tale of boundary control as so far written. They show the best form and location

FIG. 215.—The same airfoil as in Fig. 214, but with boundary-layer control by suction applied, showing the suppression of burbling.

of slot for pressure discharge, its forward (or upper) boundary ground to a razor edge to expel the air as nearly as possible tangentially to the surface, its location around the point of maximum camber (though the slot may be moved forward to within 10 per cent of the chord of the leading edge without appreciable loss of effect, and loses its value, on the other hand, very rapidly if it is moved back to beyond 55 per cent); and

[1] From the N. A. C. A.

the best for suction. After much experimenting with forwardly directed slots meeting the wing surface at a very small angle to scoop off the boundary air, like those used for pressure but faced in the opposite direction, the best means of applying suction has been found[1] to be a simple wide opening in the upper surface of the wing. The best width of slot is from 2 per cent of the chord up to 6 per cent (or perhaps even wider, the available data not showing), with little difference of value within that range. The best location, at least for the airfoil used in these tests, is at 55 per cent from the leading edge, and a shift of 10 per cent in either direction doubles the power expenditure for a given effect. The pressures used with this type of suction opening are, of course, very much lower than with the pressure

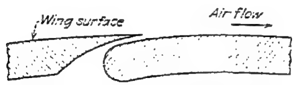


FIG. 216.—Ideal form of slot for boundary-layer control by pressure.

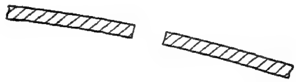


FIG. 217.—Ideal form of slot for boundary-layer control by suction.

slit, as it is the volume of air projected onto or withdrawn from the surface that is really the major controlling factor, and the pressure head required for a given volumetric flow increases rapidly as the size of the opening is reduced. Experiment indeed indicates that the volume of air having to be withdrawn is less with a wide slot than a narrow one, while the volumes handled with a suction slit and with a pressure slit of equal width are substantially the same for equal maximum-lift coefficients. With the best arrangement so far developed, the volume of air having to be withdrawn at 100 m. p. h. to get a $C_{L_{\max}}$ of 3.0 is 1.2 cu. ft. per sec. per sq. ft. of wing area. On a wing of 5-ft. chord, that is equivalent to the continuous withdrawal of a layer of air just half an inch thick from the upper surface (assuming a mean velocity equal to the speed of flight—not a bad approximation, as the upper-surface increase due to circulation and the reduction due to the boundary-layer velocity gradient roughly counterbalance each other). Blasius' formula[2] indicates a boundary-layer thickness, 55 per cent of the way back on a

[1] Schrenk, *Luftfahrtforschung*, 1928; Freeman, *Rept.*; both *cit. supra.*

[2] P. 127, *supra.*

5-ft. chord at a Reynolds' number of 3,000,000, of 1/8 in.[1] All the boundary-layer content, and more, has to be inhaled into the wing to attain maximum-lift coefficients of 3.0 or better.

In dealing with power consumption, it is the common practice to convert it into a theoretical increase in the drag coefficient and then to plot the total of the actual drag and the power-consumption supplement. The use of pressure boundary control reduces the measured drag, sometimes to less than zero, as the

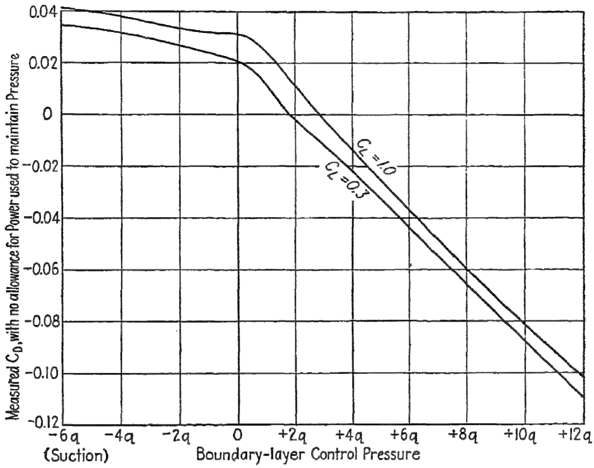


FIG. 218.—Effect of boundary-layer control on drag characteristics.

direct reaction of the backwardly ejected air offsets a part of the true drag and in that degree replaces the thrust of the airplane's screw propeller by a form of jet propulsion. Somewhat the same advantage can be attained from the suction method by ejecting the withdrawn air straight astern out of the tail of the fuselage, but the reactive force in that case is likely to be small unless additional power is first expended in compressing the air, as the thrust from such a stream is equal to Qv, where Q is the mass of air handled in unit time and v the velocity of its ejection. Boundary-layer control by pressure, unlike control by suction, characteristically requires a high-velocity flow. The high velocity has to be paid for in a large expenditure of power in a

[1] It would be somewhat thicker, as the location is in the turbulent zone.

compressor, and brings with it as incidental benefit a large reactive thrust. That thrust's magnitude is indicated in Fig. 218, where it appears that internal pressures of $3q$ at a high angle of attack and $2q$ at a low one were enough to reduce the measured drag of a typical airfoil to zero.

For small pressures the jet reaction, combined with the reduction of the skin friction by acceleration of the boundary layer,

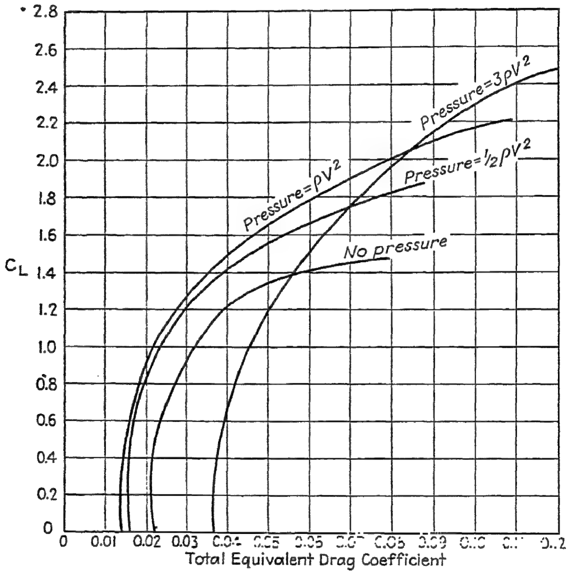


FIG. 219.—Effect of boundary-layer control on equivalent drag including the equivalent of the power expended in pumping air.

may actually exceed the apparent addition to drag representing the power expenditure, and the total apparent drag coefficient may be reduced to below the value for a plain airfoil. The equivalent polars, including the compressor-power drag, are plotted in Fig. 219 for several different pressure intensities at a slit halfway back on the upper surface of an airfoil of 15 per cent thickness.[1] The wing tested had an aspect ratio of only 1.6, with large end plates to reduce the tip losses, and their friction

[1] Based on *Rept.* 385.

is included in the drag, making the drag coefficients abnormally high all the way along the line. The curves show the possibility, with pressures corresponding to a head of from 2½ to 5 in. of water and with a discharge rate of less than 3 cu. ft. of air per sec. per sq. ft. of wing area at a speed of 200 m. p. h., of reducing the total equivalent wing drag and wing power consumption by as much as 25 per cent at high speeds and by almost as much in the region of minimum power consumption by the wings.

Though the power consumption for a given lift increase by the suction system is only about a third what it is by the pressure-increase method, the inferior magnitude of the jet reaction may more than counterbalance the saving. Though various reports of laboratory studies have declared firmly in favor of the suction alternative as the more efficient, the data so far accumulated leave the case not proved. Much more work is needed on drag coefficients with boundary control in action, on many different airfoil sections. Studies on a section as abnormal as that shown in Fig. 212 cannot be accepted as conclusive.

In one respect, to be sure, the advantage is clearly with the suction method. One of boundary control's primary functions is the increase of maximum lift and reduction of landing speed. Low landing speed is most needed after an engine failure, when there can be no careful choosing of an airport. If the pressure or suction blower is driven by the engine the high maximum lift may be missing just when it is most needed. Some sort of auxiliary drive has to be considered for that contingency, and for economy's sake the smallest possible power consumption is to be sought. For the most efficient suction slot in the airfoil of 30 per cent thickness, the horsepower having to be delivered by the blower to give a lift coefficient of 3.0 at a 50-mile landing speed would be $0.022S$, S being the wing area. Allowing a reasonable blower efficiency, the auxiliary power needed to hold the lift coefficient up to that level on an ordinary light two-passenger plane would be about 3.5 hp. The practical problem of providing that much power on terms that will make it invincibly secure against the consequences of a main power-plant failure is a serious one, and bears much of the responsibility for the failure of boundary-layer control to make its way into service airplanes, but the problem of assuring the 8 or 10 hp. that the

pressure type of control would demand would be more serious still.

Rotor-airfoil Combinations.—One more form of boundary-layer device remains to be considered. A draft can be made upon the Magnus effect.[1] By the incorporation of a rotating cylinder within the airfoil contour, usually near the leading edge,[2] the boundary layer can be accelerated along the upper surface and the circulation strengthened. The resultant airfoil section is, as would be guessed from Fig. 220, inevitably bad in its low-angle drag characteristics. Rotation of the nose cylinder has an appreciable effect on the boundary-layer structure, though the influence becomes very inconspicuous as the trailing edge is approached, but the lift is much less affected than by other and simpler mechanisms.

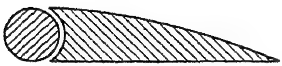


FIG. 220.—Airfoil with rotor nose.

[1] See p. 77, *supra*.

[2] "Preliminary Investigation of the Effect of a Rotating Cylinder in a Wing," by E. B. Wolff, *Der Ingenieur*, Dec. 6, 1924, translated as *Tech. Memo.* 307; "Tests for Determining the Effect of a Rotating Cylinder Fitted into the Leading Edge of an Airplane Wing," by E. B. Wolff and C. Koning, *Der Ingenieur*, Mar. 6, 1926, translated as *Tech. Memo.* 354; "Determining the Velocity Distribution in the Boundary Layer of an Airfoil Fitted with a Rotary Cylinder," by B. G. van der Hegge Zijnen, *Rept.*, A129; Rijksstudiedienst voor de Luchtvaart, Amsterdam, 1926, translated as *Tech. Memo.* 411.

CHAPTER XI

EFFECTS OF SURFACE TEXTURE, GROUND INFLUENCE, AND OTHER MISCELLANEOUS AIRFOIL PHENOMENA

Whatever the basic form of an airfoil, there are certain secondary factors which may superpose their effects on the performance as given in an ordinary wind-tunnel test of an ordinary model. They may act originally either on the airfoil itself or on properties of the fluid stream in which it is submerged. One of the most obvious of possible disturbing influences in the first class is that of surface texture.

Surface Texture.—A varnished-fabric wing can, with sufficient care, be brought almost to the smoothness of plate glass; or by improper procedure in finishing it may be left as rough as fine sandpaper. Such changes in texture have already been seen[1] to affect the skin friction, and to have a controlling influence over the point of transition from laminar to turbulent flow in the boundary layer. They may affect the circulation and the general flow structure about the airfoil as well. Wind-tunnel tests are needed, and have been made, covering not only the results of small changes in the texture of the surface as a whole but also the consequences of the deliberate introduction of grooves in some part of the surface in the hope of so modifying the flow of air as to attain a result in some respect favorable.[2]

[1] P. 153, *supra*.

[2] "An Experiment on the Effect of Changing the Roughness of the Surface of a Model Aerofoil," *R. and M.* 72; "The Effect of Grooves in the Surface on the Performance of an Aerofoil," by Henry M. Mullinix and Robert W. Fleming (an unpublished thesis in aeronautical engineering at the Mass. Inst. Tech.); "Grooved Aerofoils," by J. H. Parkin, H. C. Crane, and J. S. E. McAllister, *Aero. Research Paper* 11, *Bull.* 4, School of Eng. Research, Univ. Toronto; "Wings with Rough Surfaces," by L. von Prandtl, C. Wieselsberger, and A. Betz, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. I, p. 69; "Effect of Roughness on Airfoils," by Oskar Schrenk, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. III, translated as *Tech. Memo.* 430; "Measurement of Profile Drag on an Airplane in Flight by the Momentum Method" by

The early experiments possess little more than historical interest. Intimately connected as it is with the state of boundary-layer flow, the effect of surface condition is subject to scale effects of the first magnitude. Now that data at Reynolds' numbers of from 1,000,000 to 8,000,000 are available, everything taken at lower levels may be largely disregarded.

Full-scale data are, of course, to be preferred over those taken from models. The difficulty of correlating the degrees of roughness that appear on the two scales is extreme. Of the full-scale figures, taken at Reynolds' numbers of from 2,500,000 to 4,000,000, those of Martin Schrenk[1] are most comprehensive. They indicate minimum profile drags roughly in accordance with the following indices:

Polished duralumin, no projecting rivets	1.00
Plywood, varnished and polished	1.10
Fabric, six coats of dope, polished	1.15
Duralumin, painted, no projecting rivets	1.10
Duralumin, corrugated parallel to chord	1.7
Plywood, treated with linseed oil only	1.4
Fabric, two coats of dope	1.7
Coarse fabric, untreated, stretched taut	2.0
Coarse fabric, untreated, slack	3.5
Plywood, with poppy seed sprinkled in varnish	2.9

Though the conclusions of other experimenters can hardly be called specifically at variance with those of Schrenk, they generally point to differences somewhat less pronounced than he found. Trials on large wings in the N. A. C. A. wind tunnel[2] showed an increase of about 30 per cent in drag as a result of corrugating the skin,—the peaks and troughs of the corrugations of course running, as in Schrenk's experiments, parallel to the chord of the wing. An increase of about 15 to 20 per cent with a corrugated surface could be attributed to simple increase

Martin Schrenk (a particularly exhaustive study covering many different changes of surface condition), *Luftfahrtforschung*, May 18, 1928, translated as *Tech. Memos.* 557 and 558; "Tests of Large Airfoils in the Propeller Research Tunnel, Including Two with Corrugated Surfaces," by Donald H. Wood, *Rept.* 336; "The Aerodynamic Characteristics of Airfoils as Affected by Surface Roughness," by Ray W. Hooker, *Tech. Note* 457; "Effect of the Surface Condition of a Wing on the Aerodynamic Characteristics of an Airplane," by S. J. DeFrance, *Tech. Note* 495.

[1] *Tech. Memo.* 557, *cit. supra.*

[2] *Rept.* 336, *cit. supra.*

of skin friction in strict proportionality to the increase of surface area. In the variable-density tunnel, Hooker[1] found the increase in drag above that of the most highly polished surface that could be obtained to be less than 10 per cent except for models of which the relative irregularities of surface as compared with the span and chord were extreme (as for a wooden model finished with only two coats of shellac), less than 20 per cent even then. De France's comparative trials[2] on a small commercial airplane showed a difference of slightly more than 10 per cent in the profile drag of the wings as ordinarily finished for commercial purposes and after being given the highest possible polish.

Schrenk further found change of form of the profile-drag curve, with an improved flatness in the wings of smoothest surface. The wings with corrugated covering showed a par-

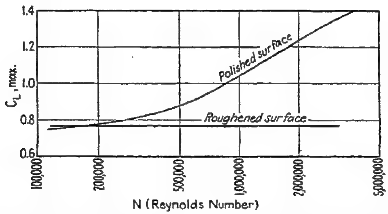


FIG. 221.—Effect of surface condition on maximum lift.

ticularly rapid increase in profile drag as the lift coefficient increased, a phenomenon which Schrenk attributed to the increasing divergence of the lines of air flow from the parallel to the chord as the angle of attack is increased, especially near the wing tips, and to the consequent compulsion of the stream-lines to abandon their easy passage along the troughs of the corrugations and to start climbing up and down their contours in pursuing the course obliquely outward toward the wing tips into which the tip-vortex structure directs them when the lift is large. Though there are some possible explanations for the discrepancy between the two cases, Schrenk's reasoning, logical as it sounds, would be on a more certain footing if it had not so totally failed of confirmation by Wood's studies of the effect of corrugation.

[1] *Tech. Note* 457, *cit. supra.*
[2] *Tech. Note* 495, *cit. supra.*

Effects on maximum lift seem in some cases to be more pronounced than those on drag. Corrugation of the surface appears to decrease the maximum by some 3 to 5 per cent. A metal model rubbed down with coarse emery, leaving scratches from 0.001 in. to 0.0025 in. in depth, had 20 per cent less lift than a highly polished one. A model covered with a layer of carborundum grains 0.005 in. in diameter (geometrically equivalent to a layer of gravel of 1/16 in. diameter adhering to the upper surface of a full-sized wing, or to Schrenk's poppy seed) had its maximum lift halved, its minimum drag doubled. It was in connection with this last test that the amazing nature and magnitude of the scale effect became apparent. Figure 221[1] shows the variation of $C_{L_{max}}$ with N for the polished airfoil and the carborundum-coated one, and becomes all the more remarkable when it is remembered that the turbulence factor[2] in the variable-density tunnel is high, and that Reynolds' numbers prevailing during tests there are equivalent to values about twice as high in a non-turbulent flow. Presumably the rough-surfaced model had already passed through its region of rapidly changing maximum lift, attained its ultimate value, before reaching the lowest N covered in this series of tests. Presumably the polished model in its turn would stabilize at some Reynolds' number above 3,550,000 (or above 7,000,000 in non-turbulent air), at some value of $C_{L_{max}}$ above (but presumably not much above) 1.4.

Some parts of an airfoil's contour are, of course, more sensitive to surface condition than others. True to expectation, the upper surface proves much more responsive than the lower one, the part nearer the leading edge much more so than the sections farther back along the chord. The rear half of the upper surface can in fact be deliberately roughened almost to the poppy-seed level without noticeable ill effect,[3] but the slightest irregularities near the leading edge show themselves immediately through increased drag and decreased lift. On a symmetrical section the importance of location proved to be such that roughening of a narrow strip exactly at the nose reduced the maximum lift by 13 per cent, while an exactly similar treatment of a similar strip 1.5 per cent of the chord back of the nose on the upper

[1] Based on *Tech. Note* 457, *cit. supra.*

[2] See p. 138, *supra.*

[3] Oskar Schrenk, *Tech. Memo.* 430, *cit. supra.*

surface had only a third as much effect.[1] The "nose" is, of course, actually a part of the upper surface under maximum-lift conditions, as the stagnation point moves around the leading edge as the angle of attack increases (except when the leading edge is perfectly sharp), to take a position well below the forward extremity of the chord when the angle becomes very large.

In general conclusion, it appears that roughness of surface is always bad, that the saving of power or improvement of speed that can be secured by substituting the best possible surface for

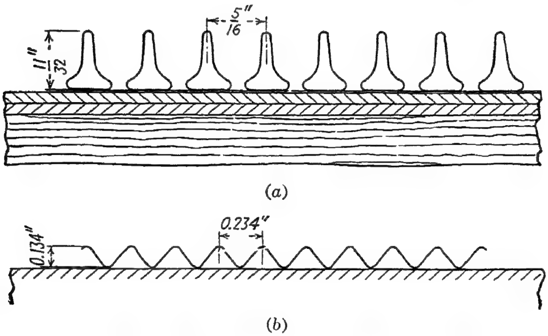


FIG. 222.—Alternative forms of wing radiator.

an ordinary commercial finish and by keeping it faithfully polished is small but not too small to consider or to take some trouble over in a commercial operation or a high-performance military ship, that a very little real carelessness in finish or in maintenance may easily increase the profile drag by from 10 to 30 per cent and reduce the maximum speed by 3 per cent or more. (The ratio between the percentage change in wing drag and that in speed no doubt seems excessive, but it is correct. The explanation, if one is required, follows in a later chapter.) The state of the surface near the trailing edge is of comparatively little importance, but labor expended on perfecting the finish on the upper surface in the forward third of the chord is never likely to be wasted.

Wing Radiators.—One particular form of roughness that sometimes appears in high-performance aircraft with liquid-cooled

[1] *Tech. Note* 457, *cit. supra.*

engines is the wing radiator, a deep corrugation of the wing surface to make tubes through which the cooling water may flow. Such specific information as exists on wing-radiator resistance indicates the tremendous importance of the precise form of the corrugations and of the way in which they are blended into the smooth strips that run around the leading and trailing edges. An American design[1] with sharp wedge-shaped tubes and almost rectangular air passages between them, as shown in Fig. 222*a*, increased the profile drag of the smooth airfoil by 0.011, enough to more than double the drag of a high-performance wing. A British one,[2] shaped as in Fig. 222*b* to leave nearly triangular passages between for the flow of air, raised the drag by but 0.004 or scarcely more increase than comes from the gentle sinusoidal corrugations that characterize some systems of sheet-metal construction.[3] Obviously, everything depends on care in detail.

Rivet Heads and Drag.—Another particular surface irregularity of particular practical importance is the protruding rivethead, almost omnipresent in metal construction. Though systems of countersinking the heads to leave a flush surface exist, their employment has been common only on the bottoms of seaplane hulls. The general rule is the exposure of hundreds if not thousands of heads, on both wing and fuselage. Such tests as exist[4] suggest that the matter deserves more attention from designers than it has received.

To string nine rows of rivets parallel to the span on the upper surface and the same on the lower, the rows uniformly spaced along the chord and each row composed of brazier heads of $\frac{5}{16}$ in. diameter and $\frac{1}{16}$ in. high and spaced 1 in. apart, is found to increase profile drag at high Reynolds' numbers by 0.0018, or something over 20 per cent of the minimum. The protuberances on the upper surface are responsible for roughly three-quarters of the effect, and those nearest the leading edge for most of it.

[1] "Full-scale Investigation of the Drag of a Wing Radiator," by Fred E. Weick, *Tech. Note* 318.

[2] "Wind-tunnel Tests on Gloster and Supermarine Wing Radiators," by R. G. Harris, L. E. Caygill, and R. A. Fairthorne, *R. and M.* 1311.

[3] See p. 326, *supra*.

[4] Martin Schrenk, *Luftfahrtforschung* and *Tech. Memos.* 557 and 558, *cit. supra*; "The Effect of Rivet Heads on the Characteristics of a 6- by 36-foot Clark Y Metal Airfoil," by Clinton H. Dearborn, *Tech. Note* 461.

In accordance with the teaching of other surface-condition experiments, it appears that if the first three rows of rivets could be eliminated the total of drag increase would be reduced by more than a half.

To countersink all the rivets on the wings ought to raise the maximum speed about 2 per cent and would probably be scarcely worth while. To countersink only one-sixth of them, the first third of the rows on the upper surface, would give about half that much increase of speed for much less than half the expense, and becomes much more interesting as a practical means of performance increase.

The effect of the rivets on maximum lift is small but measurable, something between 1 and 2 per cent. Probably that, too, comes mostly from the disturbance of the upper surface near the leading edge.

Protuberances Extending along the Span.—Rivets are not the only offenders against the continuity of an airfoil's surface. Fittings of widely various form project out of the wing to anchor its bracing members. Cover plates surmount butt joints in sheet-metal structures and create a bump equal in height to the plate's thickness. Commonest of all, pipe lines for fuel or for the transmission of pressure from a pitot or venturi tube to the air-speed meter run along the leading edge, often for a large part of the span. A general survey[1] has sought to relate the effect of all such intrusive elements to their size, shape, and position.

The results confirm those from studies of general surface roughness. The forward part of the upper surface is the critical region. Everything else is relatively insignificant except at low angles of attack, where obstructions on the lower surface may play a significant, though always a minor, part.

Figure 223 tells the story of what happens when a narrow barrier is raised at right angles to the surface to a height of $0.005c$ (say $\frac{3}{8}$ in. on the average airplane). Anywhere from the leading edge to 5 per cent of the way back on the upper surface, that tiny obstruction can cut the maximum lift by 40 per cent. Take

[1] "Airfoil Section Characteristics as Affected by Protuberances," by Eastman N. Jacobs, *Rept.* 446; "Wing Characteristics as Affected by Protuberances of Short Span," by Eastman N. Jacobs and Albert Sherman, *Rept.* 449.

it 20 per cent of the way back along the chord, and its influence on maximum lift is reduced by three-fourths. Put it anywhere on the lower surface, more than 3 per cent of the chord from the leading edge, and its influence is almost negligible—always less than 2 per cent. These figures are drawn from tests in which the basic airfoil was symmetrical in section and of 12 per cent thickness. With a different section there would be quantitative differences in the results, but the general shape of the curves and order of magnitude of the effects would always be much the same.

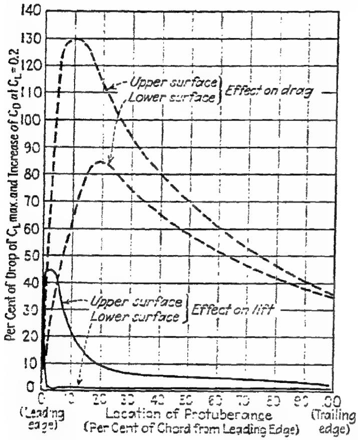


FIG. 223.—Effects on maximum lift and minimum drag of a protuberance covering the whole span to a height of 0.005c.

If there were no more to it than that, the problem of disposing of auxiliaries and bestowing pipe lines for easy accessibility would be simple. They could be strewn all over the lower surface of the wing with a light heart. But unfortunately the airplane flies at times at high speed and a low angle of attack, and there, as shown by the dotted curve in Fig. 223, a very different set of rules prevails. The best place for the obstruction now is at the leading edge or a shade below it. Shift it anywhere from 5 to 25 per cent back on the upper surface, and the total drag of the wing is more than doubled. Put it from 12 to 35 per cent of the way back on the lower one, and the results are almost as bad.

All this is natural enough. The effect on both lift and drag obviously ought to be a minimum (it ought in fact to fall to zero if the air flow were perfectly steady and if the obstruction were reduced to a blade of infinitesimal thickness) for a location precisely at the stagnation point, where the flow divides between the upper and lower surfaces. The stagnation point moves, for a symmetrical section, between the exact leading edge at zero lift and a point that may be as much as 1 or even 2 per cent of the chord back on the lower surface at the angle of maximum lift.

For a location just far enough behind the stagnation point to be just in the region of fully established streamline flow, the disruptive effects are clearly at their worst. As the protuberance is moved still farther back, the damage that it can do is proportionately reduced. It appears then that on the whole, to have a minimum of bad effect under any condition, any such essential obstruction as a necessarily exposed pipe ought to attach to the section contour at, or just a hair's breadth above, the stagnation point for maximum lift. For a symmetrical section that means about $1\frac{1}{2}$ per cent behind the leading edge on the lower surface. For a cambered airfoil, not quite so far back, perhaps from $\frac{1}{2}$ to 1 per cent. Better still, however, suppress all obstructions by carrying them into the interior of the wing.

Generally speaking, the effect of a protuberance is directly proportional to its height, but for protuberances immediately behind and above the stagnation point the linear rule breaks down. In that strategic location, the most infinitesimal irregularities of surface produce astounding drops in maximum lift. Specifically, for the symmetrical section used in these tests a protuberance of a height of $0.0004c$ (equivalent to $\frac{1}{32}$ in. on the typical transport plane) cut the maximum lift by 15 per cent. The need for the most extraordinary care in construction to secure a perfect smoothness of contour of the leading edge becomes obvious.

In the same fashion, protuberances near the leading edge of the upper surface and covering only a short section of the span prove to have a destructive effect out of all proportion to their length. An obstruction rising to a height of $0.005c$ at the 5-per-cent point on the upper surface and covering only 1 per cent of the span at its center line is still good for a 10 per cent drop in maximum lift. Roughly, the effect on maximum lift mounts lineally with length of the obstruction up to that point, then turns a right-angle corner and mounts lineally again up to a total loss of 40 per cent when the obstruction is coextensive with the span. Effects on drag at angles of attack within the ordinary operating range follow approximately a linear law of variation with length. A fair approximation to the increase of drag due to small protuberances, wherever installed, can be made by multiplying their height as a fraction of the chord by their length as a fraction of the span, and that product in turn by 1.2, and

adding the result to the drag coefficient of the wing (which is equivalent to assuming them to have the same drag that they would if they stood by themselves in an undisturbed air stream remote from the interference of the wing). The resultant error, with protuberances of the size encountered in practice, is unlikely ever to exceed 0.0008 in the total drag coefficient, and if greater accuracy than that is desired the curves of Fig. 223 must be used and corrected for height and length.

As a particular practical example, this series of tests included a measurement of the effect of a series of narrow plates rising obliquely from the upper surface of the wing at intervals and simulating the fittings to which the flying and landing wires of biplanes attach. Though the maximum height was but $0.027c$, and the maximum area directly affected extended over only 4 per cent of the span, and the attachment was 15 per cent back from the leading edge to represent a typical front-spar position, the maximum lift was decreased by 6 per cent and the minimum drag increased by 30. Here again the effect on maximum lift, as tested by the curve of Fig. 223 with the appropriate correction factors, is quite out of proportion to the span covered by the obstruction—a little more than double the effect that would be calculated by a simple linear rule. The increase of drag was about three times what would be found by the simple rule just given, the discrepancy's explanation lying in the small angle at which the fittings project from the surface and in their consequent disturbance of flow along the wing over a length out of all proportion to their own area. From this develops the supplementary rule that when a narrow obstruction projects out of the surface of the wing at an acute angle and overhangs the surface the approximation for additional drag should be further multiplied by the cosecant of the angle between the axis of the projection and the surface of the wing.

In any location except immediately behind the leading edge, the effects of an obstruction on both lift and drag can be reduced virtually to zero by giving it a proper fairing on both sides. In the first 10 per cent of the chord, however, even careful fairing is of limited value (limited to reducing the effect of the naked obstruction by about two-thirds) and any sort of a bump or irregularity of curvature of the surface has to be regarded with the greatest alarm.

Effect of Fabric Sag.—A wing as actually constructed is unlikely to reproduce the airfoil section as originally designed with absolute perfection at every point. Particularly in evidence, as a recurrent and seemingly inevitable deviation from design, is the effect of "fabric sag" (incorrectly so called, as it results from too much tension in the wrong place rather than from the existence of slack in the fabric) between the ribs of fabric-covered wings. With all possible care used in applying the fabric, the section truly existing midway between ribs may be 3 or 4 per cent thinner than that at the ribs, and of somewhat different form.

Regarding the corrugated form that "sag" gives a wing as viewed from the front, it looks important. It is mentioned here, however, only for the purpose of exorcising a bogey. Tests on several different types of distortion by sag have shown[1] an effect immeasurably small. The designer may disregard it, as long as the curve of the sag is a smooth one and no structural members press against the fabric from inside the wing to produce unfairness or a sudden change of curvature.

Ground Effect.—Another of the subsidiary influences on airfoil performance, temporary and occasional but none the less important, is that of ground interference. When an airfoil is moving parallel to and a short distance above the ground, the downward flow of air off the trailing edge is less free than ordinarily, and the flow pattern is modified in consequence.

The subject is one that has been extensively investigated in the laboratory,[2] by several different methods. Practically all of the

[1] "Tests on Model Aerofoil of R. A. F. 14 Section, to Compare an Aerofoil of Uniform Section with One Modified to Represent the Sag in the Fabric of an Actual Wing," by W. L. Cowley and L. F. G. Simmons, *R. and M.* 323; "Characteristics of an Airfoil as Affected by Fabric Sag," by Kenneth E. Ward, *Tech. Note* 428.

[2] "Auftrieb und Widerstand einer Tragfläche in der Nähe einer horizontalen Ebene," by A. Betz, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, No. 17, 1912; "Ground Influence on Aerofoils," by Arthur E. Raymond (a thesis in aeronautical engineering at Mass. Inst. Tech), *Tech. Note* 67; "Variation in Resultant Pressure upon Landing Due to the Proximity of the Earth," by Albert A. Merrill, *The Ace*, p. 19, December, 1920; "Cushioning Effect on Aeroplanes Close to the Ground," by W. L. Cowley and C. N. H. Lock, *R. and M.* 754; "Ground Plane Influence on Airplane Wings," by A. F. Zahm and R. M. Bear, Washington Navy Yard *Rept.*, 173, 1921; "Wing Resistance near the Ground," by C. Wieselsberger, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, No. 10, 1921; "Der

tests are in at least qualitative agreement. All alike show an increase in maximum lift when the airfoil is close to the ground and a general tendency to a decrease of drag. The minimum of D/L is greatly improved by ground effect.

The explanation has already been provided.[1] It is still another case of induced drag. As for the maximum lift, the increase there is due to the reaction between the ground and the airfoil. The freedom of escape of the air from under the trailing edge being interfered with, the velocity of flow over the lower surface is reduced, and the pressure on that surface is increased.

The increase is only slightly apparent in a monoplane wing set at its angle of maximum lift and with its center of pressure one-

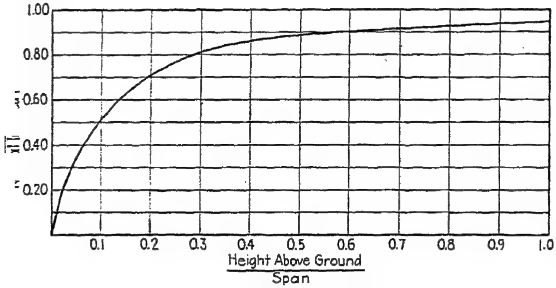


FIG. 224.– Reduction of induced drag by ground effect.

half chord length above the ground, the gain in that case being of the order of 2 or 3 per cent. If the distance is reduced to one-fourth chord length, the maximum lift goes up about 10 per cent, but the structure of most airplanes is such that it is physically impossible for a wing to come as close to the ground as that. Only on monoplanes with the wings set under the fuselage or on biplanes with unusually short landing gears and the lower wing correspondingly close to the ground, therefore, may it be expected that ground effect on maximum lift will be of much importance.

Einfluss der Erdbodennähe auf den Flügelwiderstand," by L. von Prandtl, C. Wieselsberger, and A. Betz, "Ergebenisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. II, p. 41, Berlin, 1923; "A Full-scale Investigation of Ground Effect," by Elliott G. Reid, *Rept.* 265; "Effect of the Ground on an Airplane Flying Close to It," by E. Tönnies, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Mar. 29, 1932, translated as *Tech. Memo.* 674.

[1] See p. 101, *supra*.

The profile drag is but little changed by the presence of the ground. The induced drag can be computed by Wieselsberger's reflection, or imaginary-multiplane, method as described in Chap. V, and the results of computation agree well with those of experiment. In Fig. 224 there have been plotted the correction factors by which the normal induced drag of a wing must be multiplied to determine the induced drag at various heights above the ground.

The relative gain by ground effect proves, rather surprisingly, to be considerably greater for a biplane than for a monoplane flying at a height equal to the average height of the biplane's upper and lower wings. The correction factor in that case can be found to a satisfactory order of approximation by taking the height of the lower wing, instead of the mean height, and applying Fig. 224. A biplane of aspect ratio 6, flying with the lower wing at 10 per cent of its span above the ground, has the aerodynamic characteristics of a monoplane of aspect ratio 6.6 under normal conditions—ground effect just a little more than counterbalancing the interference between the wings of the biplane. For an airplane flying at twice its minimum speed or less there would be a very useful reduction in resistance and consequent economy in power if the flight path could be kept very close to the ground or water surface, while if full power were used the maximum speed would be increased by the ground effect, especially on machines of moderate speed-range ratio. This has been confirmed by free-flight test experience, increases in maximum speed of from 1 to 3 per cent being obtained when flying as close as possible to the surface of level ground or smooth water. The gain is largest for low-wing monoplanes, for biplanes, and in general for machines of moderate power and performance upon which induced drag plays a large part at maximum speed.

Oscillations of Wind Direction.—Another important influence on airfoil behavior is that of air-flow texture, and in particular of oscillations in the air stream. It was first experimentally demonstrated by Katzmayr[1] that the effect of periodic oscillation in the direction of the air stream meeting an airfoil, such oscillations being artificially produced in a wind tunnel, considerably reduced

[1] "Effect of Periodic Changes of Angle of Attack on the Behavior of Airfoils," by R. Katzmayr, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Mar. 31 and Apr. 13, 1922, translated and published as *Tech. Memo.* 147.

the drag measured for the airfoil and even at times changed it from a positive to a negative value. Further experimental study of the effect was subsequently undertaken in France, where Katzmayr's results were fully confirmed.[1] To give a concrete example, it was found that the introduction of an oscillation through a total amplitude of 20 deg. (10 deg. either side of normal) in the direction of the wind stream reduced the minimum-drag coefficient for a typical airfoil from 0.000041 to −0.000045, the coefficient in the oscillating stream being independent of the frequency of the oscillation. The lift is very little affected. A negative drag coefficient seems at first sight to suggest perpetual

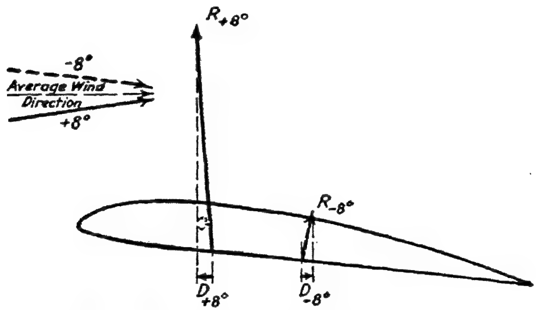


FIG. 225.—The geometry of the Katzmayr effect.

motion, as it indicates that if the model were released or if an airplane were flown under corresponding conditions it would advance straight upwind indefinitely without needing any propeller thrust to impel it. What it actually means, of course, is that energy is in some way being drawn from the pulsating air stream to do the work of propelling the airfoil or the complete airplane against the wind direction.

Katzmayr Effect.—The explanation of the Katzmayr effect becomes clear from an analysis such as is diagrammed in Fig. 225. As the air stream oscillates with reference to the fixed airfoil, the angle of attack of course changes, and the directions of the axes to which the true lift and drag should be referred change correspondingly with the rotation of the stream. If all forces are

[1] "Experimental Investigation of the Effect of an Oscillating Airstream (Katzmayr Effect) on the Characteristics of Airfoils," by Toussaint, Kerneis, and Girault, *Tech. Note* 202.

referred to the mean wind direction, as they always are in wind-tunnel measurements, there is an upwind component of lift for one extreme of wind direction, a downwind component for the other. The downwind component exists when the true angle of attack is small and the lift itself is small, while the upwind inclination goes with a large lift force. The average of the two is therefore directed against the wind, a negative drag, and if instead of merely taking two points the forces are traced and integrated through an entire period of oscillation the reduction in drag can be calculated. In Fig. 225 the resultant forces and their components parallel to the mean wind direction have been plotted only for two extremes of angle. Such calculations have been carried through for several cases[1] and show excellent agreement between the calculated and the experimental values of the drag. The Katzmayr effect thus loses any air of mystery and appears as the normal and inevitable consequence of the summation and averaging of all the forces instantaneously acting.

The Katzmayr effect is so far of academic interest, but it would become of real and practical significance if periodic variations in the vertical direction of natural winds could be shown to exist and if means existed whereby an aircraft pilot could detect them. The inertia of the airplane would act in place of the fixed support in the wind tunnel to keep the flight-path direction and the directions of the effective axes of lift and drag constant if the frequency of the pulsation were high enough. The apparent negative mean drag would then be a propelling force. Such pulsations of direction would afford a possible explanation of the soaring flight of birds. They afford the most reasonable and promising explanation in the case of the reported soaring of sea birds in the south Pacific, where the long swells constrain a periodic variation of the vertical component of velocity of the winds blowing along the surface of the sea at low altitudes. To be sure, however, the very existence of soaring without the use of continuously rising currents of air still appears open to doubt.

[1] "Note on the Katzmayr Effect on Airfoil Drag," by Shatswell Ober, *Tech. Note* 214; "A Note on the Katzmayr Effect, That Is, the Effect on the Characteristics of an Aerofoil Produced by an Oscillating Airstream," by W. L. Cowley, *R. and M.* 969.

CHAPTER XII

PARASITE DRAG

Parasite drag, as the name implies, is that part of the resistance of an aircraft coming from elements of the structure which experience no useful aerodynamic reaction. The drag of a fuselage, for example, or that of a wire is a net aerodynamic loss. In that respect they are unlike the drag of a wing, which is the price that has to be paid for the beneficial part of the aerodynamic reaction on the same surface, the lift that supports the machine in flight. Strictly speaking, only the induced drag is directly interlaced with the lift. The profile drag of the wing, like the drag of a landing gear, is pure waste energy. Logically it may be considered a part of the parasite drag, and for certain purposes it is convenient so to treat it, but it has already been discussed under the heading of airfoil performance and for the moment we shall put it aside.

The essential structure and constitution of drag have been canvassed at length in the preceding chapters. There is, in every case, skin friction. In every case except a purely imaginary ideal one there is eddy-making resistance as well, the resistance that exists only because of the departure of the air flow from an ideal streamline motion and that depends for its magnitude on the degree of the departure. Though eddy-making resistance is never entirely missing, the flow around a good streamline form approximates closely enough to the ideal to keep it exceedingly small as compared with the friction.

The absolute minimum of non-frictional drag, less even than for the most perfect of streamline forms, goes with the case of the flat plate traveling through the air parallel to its own plane. Its total resistance can be read directly from Fig. 92, the curve of skin friction. For the parts of an airplane, other than wings and tail surfaces, friction may contribute from 5 to 75 per cent of the total.

As the flat plate set edgewise to the wind is the extreme case in one direction, so the flat plate perpendicular to the wind direction

is the extreme in the other. Frictional drag there becomes a negligible part (less than 1 per cent) of the whole.

Drag of Flat Plates.—The object in airplane design almost always is to keep as far away from the flat plate normal to the wind direction as possible, since that is a form of very high resistance; yet the drag coefficient of such a plate plays a part in performance calculations, in that "equivalent flat-plate area" is often used as an index of the total amount of parasite resistance offered by a given machine, and it is of much more than merely academic interest from other points of view as well.

Since questions of air resistance have long been recognized as of importance in the design of engineering structures, attempts to find the coefficients of resistance of normal flat plates date back for some two centuries, and some very useful resistance determinations have been made in free air.[1] Only with the general use of the wind tunnel, however, has it become possible to secure reliable values consistent within a maximum spread of 10 per cent or less. Wind-tunnel studies by Eiffel[2] and Hunsaker[3] show that the resistance coefficient of a square or circular flat plate averages approximately 1.2 in the absolute units here used as standard. These figures, like other aerodynamic force coefficients, of course vary with Reynolds' number, appearing to approach a maximum of 1.25 for a circular disk at very high Reynolds' numbers and dropping to 1.18 when VL/v is 300,000 (L, of course, being taken as the diameter of the plate) and to 1.06 at 100,000.

Particular values of the coefficients hold only for a particular shape of plate. The resistance of a square plate is some 5 per cent higher than that of one of circular shape under like conditions, and Eiffel[4] found that the drag coefficient of a rectangular

[1] "Recherches expérimentales sur la résistance de l'air executées à la Tour Eiffel," by G. Eiffel, Paris, 1907; "Experiments in Aerodynamics," by S. P. Langley, *Smithsonian Contributions to Knowledge*, No. 801, Washington, 1891.

[2] "La résistance de l'air et l'aviation," by G. Eiffel, pp. 39–43, Paris, 1911.

[3] "Critical Speeds for Flat Discs in a Normal Wind," by J. C. Hunsaker, from "Reports on Wind Tunnel Experiments in Aerodynamics," *Smithsonian Misc. Coll.*, vol. 62, No. 4, p. 84, Washington, 1916; "The Effect of Turbulence on the Drag of Flat Plates," by G. B. Schubauer and H. L. Dryden, *Rept.* 546.

[4] "La résistance de l'air et l'aviation," *cit. supra.*

plate increases steadily with increasing ratio of breadth to height. The coefficient rises from a value of 1.06 for a square plate, tests being made at a low Reynolds' number, to 1.22 for an elongation ratio of 10, 1.41 for 20, and 1.53 for 50. This increase appeared notwithstanding the fact that the areas of all the plates were the same and that the Reynolds' number referred to the narrowest dimension, the most logical base of comparison, was therefore decreasing as the elongation increased. The reason for the rise of resistance obviously is that as the disproportion between the dimensions of the plate increases the flow becomes more and more nearly two-dimensional, and the escape of the air around the edges of the plate progressively less free.

For ordinary calculations, and as a base of comparison for the resistances of other shapes, the coefficient of the normal flat plate may be taken as 1.25, which is very close to the true value at all Reynolds' numbers above 1,000,000 and for shapes approximating the circular.

Cylinders and Spheres.—There are certain geometrical forms which, although they seldom appear directly in aircraft design, have some interest in the theory of air resistance and also furnish a useful transition between the flat plate and the streamline body. Cylinders and spheres are the best examples.

Both have been treated in an earlier chapter,[1] and a typical curve has been given for the cylinder in Fig. 85. They share the characteristic of possessing a critical regime, or range of Reynolds' numbers within which the flow changes rapidly and the drag coefficient falls sharply as N increases; they are alike in that the location of the critical range depends primarily on the degree of turbulence existing in the air and on the degree of roughness of the surface of the model; they are almost alike in the location of the critical range under given conditions of turbulence and surface condition, its central point falling at a Reynolds' number of about 300,000 in a stream as nearly as possible turbulence free, dropping off to as low as 50,000 when the surface of the object is made very rough and the stream is given a maximum of turbulence. Above the critical region, a long cylinder with its axis at right angles to the wind shows a minimum-drag coefficient of about 0.3, while the value for a

[1] Chap. VI, *supra.*

sphere falls to about 0.12; one-quarter and one-tenth, respectively, of the resistance of a flat plate.

The sphere is of direct interest only to the student of theory and to the worker in the laboratory, who find in its sensitiveness to flow conditions a very useful scale for calibrating the wind tunnels for turbulence. The cylinder is of more practical import, for its case is that of a round wire or of an unfaired pipe or circular tube. The cylindrical forms making an appearance in aircraft design may range in diameter from $\frac{1}{2}$ to 3 in. (in the case of an exhaust manifold) and may operate at a Reynolds'

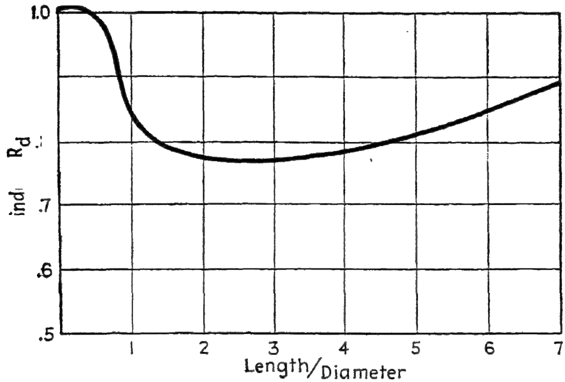


FIG. 226.—Variation of drag of a cylinder with length.

number of from 6,000 to 400,000. The range of practical interest then completely spans the critical region, and the exact point at which the drag coefficient begins to fall may have a distinct influence on the airplane's performance. Leave it at that for the moment. Comparison of the drag characteristics of various types of wires and tubes will be taken up in more detail shortly.[1]

Cylinder drag has been measured with the cylinder axis parallel, as well as perpendicular, to the wind direction. In that position the circular boundary of the cylinder acts as a fairing behind the flat plate that is its base, permitting some reestablishment of smooth flow after the disturbance where the air abruptly meets the cylinder and before the shock of

[1] P. 428, *et seq.*, *infra*.

leaving the object at its rear base. Though it is impossible to restore a true streamline flow after it has once been badly broken down by such an irregularity of form as the base of a cylinder occasions, the total resistance proves to be less when the shock to the air flow of meeting the object and the shock of leaving it are separated by a perceptible interval than when the one follows immediately on the other. That is evident from Fig. 226,[1] where there is plotted the ratio of the resistance of a cylinder to that of a flat disk similar to the base of the cylinder.

The ratio falls to a minimum for a cylinder with a length of 2.5 times its diameter. Thereafter, as the length is further increased, the resistance also slowly increases. The rule that there is some optimum ratio of length to diameter, for which the resistance is least, is a general one. It holds for good streamline forms as well as for cylinders, and the manifest reason is that, while the eddy-making resistance tends to decrease with increase of fineness ratio, the skin friction goes steadily up with increasing ratio of surface exposed to the flow of air to cross-sectional area. Even on the cylinder, skin friction begins to be a very appreciable part of total resistance for large length/diameter ratios. The skin friction on the cylinder having a length of four diameters, for example, amounts to 12 per cent of the total resistance. With the length increased to seven diameters the calculated skin friction rises to 15 per cent of the total, although the total resistance is about 12 per cent higher when the length is seven diameters than when it is only four.

Hemispheres, Ellipsoids, and Cones.—The testing of a hemisphere is a natural accompaniment to the work that has been done on spheres, and the tests show[2] a mean coefficient of resistance of 0.42 for a hemispherical cup with its convex side toward the wind, one of 1.36 for the same cup with the concave side facing the wind direction. It is on the difference of these two figures, of course, that the operation of the familiar Robinson cup anemometer, illustrated in Fig. 227, is based. It is not

[1] From "La résistance de l'air et l'aviation," pp. 73–75. See also "Note on the Resistance of Polished Cylinders and Cylindrical Wires with Generatrices Perpendicular to the Airstream," by A. Toussaint, *L'Aéronautique,* April, 1920, reproduced in *Tech. Note* 43.

[2] "Experimental Investigation of the Robinson-type Cup Anemometer," by M. J. Brevoort and U. T. Joyner, *Rept.* 513.

surprising to find that the hemispheres show but little scale effect, nor is it surprising that the resistance of the hemisphere with its convex side upwind should be virtually the same as that of a sphere below its critical Reynolds' number, for if the lines of flow break away from the surface and become turbulent at a point forward of the diametral plane perpendicular to the wind direction in any case the removal of the part of the solid which is behind that plane and which would lie in a region of comparative "dead water" is not likely to have very much effect on the flow pattern.

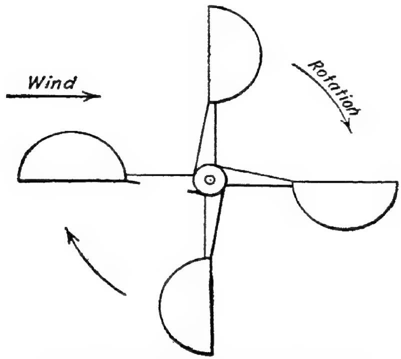


FIG. 227.—Robinson cup anemometer.

Transition stages between a geometrical form as simple as the sphere and the final streamline body are furnished by the testing of ellipsoids and combinations of spherical, cylindrical, and conical shapes. Riabouchinsky shows[1] that an ellipsoid of revolution having a fineness ratio, or ratio of length parallel to the wind direction to diameter (a term very much used in the discussion of streamline bodies, struts, and airship forms), of one-half has a resistance at a Reynolds' number of about 150,000 (the sphere having there completed its transition into the low-resistance region in his tests, his tunnel apparently showing more than usual turbulence) approximately seven times that of the sphere, while an increase of fineness ratio to 1.2 cuts the figure

[1] "On the Resistance of Spheres and Ellipsoids in Wind Tunnels," by D. P. Riabouchinsky, *Bull.* 5, Aerodynamical Inst. Koutchino, p. 73, 1921, translated in *Tech. Note*, 44.

down to two-thirds of that for the sphere. Unfortunately, no tests on ellipsoids of greater fineness ratio have been undertaken.

A simple form of low-resistance body is secured by capping the ends of a cylinder with hemispheres. Such a model was included among those tested by Eiffel[1] (its overall length being eight times the diameter), and the resistance was found to be only one-fifth that of a plain cylinder of like proportion with its axis parallel to the wind, and only about 15 per cent of that of a flat disk. The coefficient was almost equal to that for a sphere, but a lower figure would probably have been secured with a shorter cylinder. For a length so great the skin friction became dominant, the frictional resistance calculated by the formula on page 148 being more than two-thirds of the total.

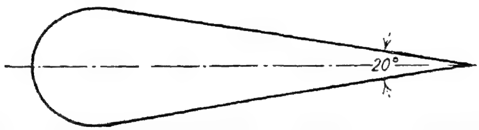


FIG. 228.—Rudimentary geometrical approach to streamline form.

Finally, the same series of Eiffel tests of geometric forms included measurements on a hemisphere with its base resting on the base of a cone of 20-deg. vertex angle, as shown in Fig. 228. With the point of the cone facing the wind direction the drag was about 9 per cent below that of the sphere, but with the rounded end foremost the figure dropped to just half that for the spherical form. The relative results give a general confirmation of the rule, long accepted by naval architects and illustrated also in the shapes of the bodies of fish and birds and in other streamline forms in nature, that a smoothly curved and pointed stern is of more importance in the reduction of resistance than a very elongated and pointed nose. It is a rule which must be interpreted with some caution, however, for there comes a point beyond which it does not hold true. Once the streamline flow around a body has been broken down by extreme bluntness or irregularity of form of the nose or by a sharp discontinuity of the surface, it is very difficult to reestablish it, and the shape of the forward part of the streamline body is by no means of secondary importance. Although there are no full and directly compara-

[1] "La résistance de l'air et l'aviation," p. 77.

tive tests, there is little doubt that the resistances of a cone without hemispherical cap would be less when the point was upwind than when the base faced the wind direction.

Streamline Bodies.—Of the testing of low-resistance or "streamline" bodies there is no end. Hundreds of them, some of forms expressible by simple analytic equations and others purely empirical and designed by eye, have been studied in the quest for a still further reduction of drag coefficients.[1]

As in the case of the airfoil, it will be helpful to give some attention to the geometry of the problem and to adopt certain conventions of representation before attacking a great mass of data. The essential geometrical qualities of a streamline form are (1) the fineness ratio, or ratio of length to diameter; (2) the location of the point of maximum diameter, measured by the ratio of its distance from the nose to the total length of the form; (3) the cylindrical coefficient, or ratio of the volume included within the form to the volume of the circumscribing cylinder; (4) the actual form of the bounding curve. The first three can be numerically specified; the last must be drawn.

Of the first two points nothing need be said at the moment. Their effect on resistance will be studied in due course. Of the third it need only be remarked that it would obviously be unity for a cylinder and that it approaches that value for a body with a long cylindrical mid-section and bluff ends, that its value depends

[1] The number of tests have been so great that only a few of the most important and the general compilations need be mentioned. "Characteristics of Stream-line Forms and Design Data for Airship Hulls," by T. L. Blakemore, and O. L. Lewis, Airplane Eng. Div., *Information Circ.* 39, U. S. Air Service, Washington, 1922. "Experiments in a Wind Channel on Elongated Bodies of Approximately Stream-line Form, Pt. II, The Effect of Form on Resistance," by J. R. Pannell and R. Jones, *R. and M.* 607; "The Drag of C-Class Airship Hulls with Varying Length of Cylindric Midships," by A. F. Zahm, R. H. Smith, and G. C. Hill, *Rept.* 138; "The Resistance of Certain Stream-line-shaped Bodies," by J. R. Pannell and N. R. Campbell, *R. and M.* 311; "The Drag of Airships," by Clinton H. Havill, *Tech. Note* 248; "Drag of C-Class Airship Hulls of Various Fineness Ratios," by A. F. Zahm, R. H. Smith, and F. A. Louden, *Rept.* 291; "The Resistance of the Airship Models Measured in the Wind Tunnels of Japan," *Rept.* 15, Aero. Research Inst., Tokyo Imper. Univ.; "Resistance of R. 101," *R. and M.* 1168; "Airship Model Tests in the Variable-density Wind Tunnel," by Ira H. Abbott, *Rept.* 394; "The Drag of Two Streamline Bodies as Affected by Protuberances and Appendages," by Ira H. Abbott, *Rept.* 451.

primarily on the degree of fineness of the nose and tail, and that in practice it proves to lie between 0.56 and ⅔ (which would be the value for an ellipsoid) for all reasonably good forms and between 0.56 and 0.6 for most of the best of them.

The bounding curve is customarily specified by the simple device of drawing it directly, but where nice discriminations and very minute comparisons of forms and the effect of their differences are sought that is hardly good enough. It is better to take renewed advantage of the device used in comparing airfoils,[1] to adopt as a base of reference a standard curve of which the perfect smoothness is assured by its being of analytic form, and to describe all other bounding curves by plotting the differences of their ordinates at each point from the corresponding standard figure of the reference curve.

For the streamline form it proves most convenient to use two curves joining each other at the point of maximum diameter in place of a single one, and the standard used in this volume will be a combination of the two conic sections

$$\left(\frac{x}{L}\right)^2 + 0.16y^2 = 0.16 \qquad (71)$$

forward of the maximum diameter (which is at 40 per cent of the length from the nose) and

$$\left(\frac{x}{L}\right)^2 + 0.0679y^2 + 0.2921y = 0.3600 \qquad (72)$$

from that point back to the tail. The origin, for both equations, is on the axis of the form immediately below the point of maxi-

FIG. 229.—"Standard" streamline form.

mum diameter. The resultant basic outline is plotted in Fig. 229, and its ordinates are tabulated on page 349.

[1] P. 169, *supra*.

TABLE VIII.—PROPORTIONS OF "STANDARD" STREAMLINE FORM

Distance from nose of form, Percentage of total length	$\frac{\text{Diameter at point}}{\text{Maximum diameter}}$
0	0
0.0125	0.2478
0.025	0.3480
0.05	0.4841
0.10	0.6615
0.20	0.8660
0.30	0.9682
0.40	1.0000
0.50	0.9765
0.60	0.9049
0.70	0.7818
0.80	0.6001
0.90	0.3469
0.95	0.1869
1.00	0

The resistance of such forms being primarily frictional, their total surface area is an important factor in their performance. It is given, with an error which practically never exceeds 2 per cent, by the formula

$$S_s = 2.55\sqrt{vL} + 1.22\frac{v}{D} \tag{73}$$

where S_s is the form's total superficial area and v, L, and D its volume, length, and diameter, respectively. Transformations of an obvious nature produce the form

$$\frac{S_s}{S} = (2.87\sqrt{k_c} + 1.22)R \tag{74}$$

S being the cross-sectional area, R the fineness ratio, and k_c the cylindrical coefficient, and the substitution of a mean value for k_c brings this in turn down to the elemental form

$$\frac{S_s}{S} = 3.43R \tag{75}$$

a formula correct within 4 per cent for virtually all low-resistance shapes. The coefficient of skin friction, necessarily given

originally in terms of superficial area, can then be converted into a drag coefficient of the ordinary form by multiplying it by $3.43R$.

Scale effect on streamline forms depends primarily, since separation is a very minor factor on a good form, upon the laws of change of the frictional coefficients. They have already been reviewed at length,[1] but the curve that was plotted in Fig. 92 will not suffice here. Transition from laminar to turbulent flow in the boundary layer first manifests itself around a streamline form at the tail, where the area affected is exceedingly small, and only at much higher Reynolds' numbers does it approach the

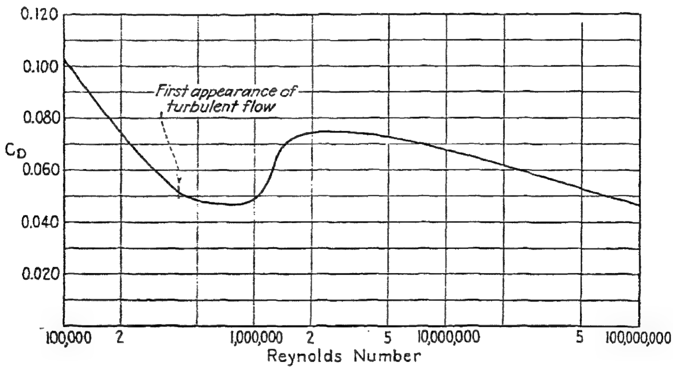


FIG. 230.—Variation with Reynolds' number of skin friction on a streamline form.

region of maximum diameter. The transition curve of frictional coefficient accordingly breaks away from the laminar curve very gradually, much more so than in the case of the flat plate plotted in Fig. 92. Dryden has calculated the case of a typical airship form,[2] and the resultant curve, converted to coefficients in terms of cross-sectional area, is reproduced in Fig. 230. The particular form for which the calculation was made had a fineness ratio of 8, but the shape of the curve would be substantially independent of fineness ratio and its ordinates would be simply directly proportional to fineness ratio. The values of the coefficients shown in Fig. 230 can be corrected to give approximate results for any other form by multiplying through by $R/8$. It must, of course,

[1] P. 148 *et seq.*, *supra.*

[2] "Effect of Turbulence in Wind Tunnel Measurements," by H. L. Dryden and A. M. Kuethe, *Rept.* 342.

always be remembered that, for reasons explained in Chap. VI,[1] L must always be taken from the extreme length of the body along the wind direction in calculating Reynolds' number. If the diameter is used, as has sometimes been done in plotting the results of tests, the resultant value of N will have no significance in determining the friction.

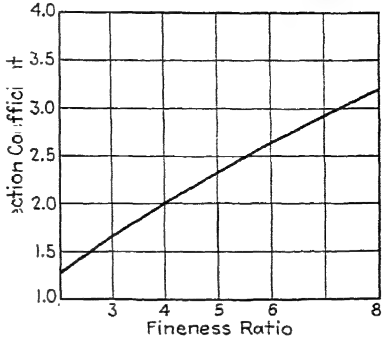


FIG. 231.—Correction factor connecting drag coefficients in terms of volume and in terms of area.

One more digression onto conventions of representation is demanded. Since pure streamline forms seldom appear in airplane design but are, on the other hand, of very direct practical utility as hull shapes for airships, the results of tests are commonly reported in the terms most useful to the airship specialist. His interest is in discovering the form of minimum resistance for a given contained volume of gas, and the cross-sectional area is of scarcely even incidental interest to him. Hence the common practice of expressing streamline form resistance as

$$D \quad C_v \frac{\rho}{2} (v)^{2/3} V^2 \tag{76}$$

where v is the volume of the form and C_v the experimentally determined coefficient. The factor of conversion from C_v to the familiar C_D in terms of cross-sectional area is

$$\frac{C_D}{C_v} = 1.08 k_c \tag{77}$$

where k_c and R are again the cylindrical coefficient and the fineness ratio, respectively. Since k_c seldom wanders far from 0.6, the ratio of the two drag coefficients can be approximated as a function of fineness ratio alone, as in Fig. 231. To get the drag coefficient for a streamline form with a maximum probable error of some 7 per cent, it is sufficient to multiply the volumetric drag

[1] P. 150, *supra.*

coefficient as reported by the appropriate factor selected from Fig. 231. If greater accuracy is required, the cylindrical coefficient must be determined and (77) used.

Scale Effect on Streamline Forms.—Since separation plays a minor part on a good form, the curve of drag coefficient against Reynolds' number ought to adhere approximately to the form of the computed skin-friction curve given in Fig. 230. Not by any means perfectly, but to a fair approximation, it does so. In Fig. 232[1] are plotted experimental curves taken in several differ-

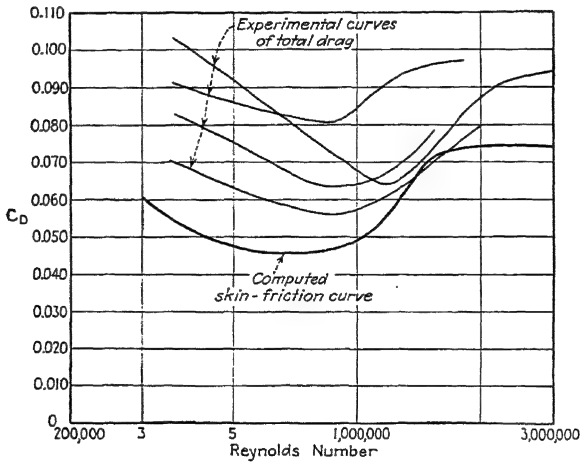


FIG. 232.—Effect of turbulence on scale effect on a streamline shape.

ent tunnels and at different degrees of turbulence for the model for which Fig. 231 was computed, and the pure friction curve is superposed. A multiplicity of friction curves could, of course, have been derived by assuming different values of the Reynolds' number at which turbulence first appears in the boundary layer—the true value, it will be recalled, depending on stream conditions. In this particular case N_x for the beginning of the transition was taken as 400,000. Comparison of the computed and the experimental curves indicates (1) that the assumption that the laws of friction control scale effect on good forms is essentially valid; (2) that the drag coefficient of a streamline form conventionally

[1] *Rept.* 342, *cit. supra.*

drops to a minimum in the neighborhood (very roughly indeed) of $N = 1{,}000{,}000$, then rises again to about 1.5 times the minimum value, and then falls off once more but does not regain the original minimum level until N is around 100,000,000; (3) that the appearance of turbulence in the boundary layer may be delayed to considerably beyond the farthest point that studies on friction in two-dimensional motion would lead one to anticipate, experiment indicating the retardation of the transition until N_x is almost 1,000,000 in some cases; (4) that streamline forms could be best compared in terms of their drag coefficients at a Reynolds' number of 10,000,000 or more, but that since

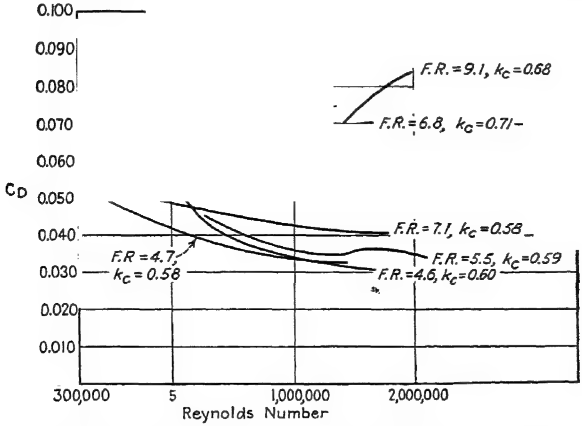


FIG. 233.—Scale-effect curves for some streamline shapes.

experiment is practically never carried that far the next best alternative is to base the comparison on the minimum of C_D at somewhere between 500,000 and 2,500,000, using data from the least turbulent stream available; (5) that the drag of this particular form, a good one but far from the best that is known, was about 75 per cent frictional.

All of which is loosely confirmed by Fig. 233,[1] where the scale curves are plotted for several different forms and the fineness ratio and the cylindrical coefficient of each are noted on the curve. The highest drags, and apparently the earliest appear-

[1] Based on data from "Experiments in a Wind Channel on Elongated Bodies of Approximately Streamline Form," by J. R. Pannell and R. Jones, *R. and M.* 607.

ance of turbulence in the boundary layer as well, are found for the forms which have the highest cylindrical coefficients—which in turn are due to the interpolation of a straight cylindrical portion of substantial length between the ends.

Form and Drag.—The attempt to draw final conclusions on the form of an ideal streamline body is considerably hampered by the incomparability of the data from various tunnels. Accurate measurement is difficult in the extreme, and the combined influence of stream turbulence and of differences of operating

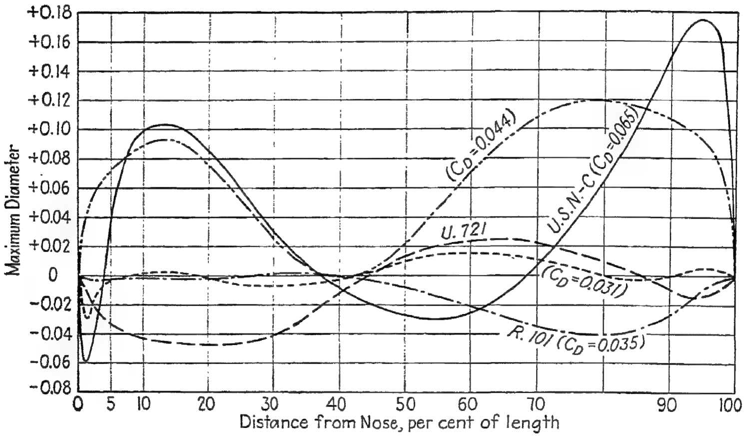


FIG. 234.—Comparison, by difference curves, of the geometry of well-known streamline forms.

technique may change the apparent minimum-drag coefficient of a given model by as much as 20 per cent from one tunnel to another. Figure 232 has already shown it, though all the tunnels entering into that study were under a common control and operated by the same staff.

If the average of the most reliable data indicates a frictional source for 75 per cent of the total resistance or even more, it would be natural to expect but little effect from small changes in form. A change of 10 per cent in eddy-making resistance would change the total drag but 2 or 3 per cent. The anticipation of insensitiveness to minute variations seems to square with experience. The standard curve of form plotted in Fig. 229 seems to be about as near to an ideal as could be found, as

appears from Fig. 234, where the difference curves are plotted for three forms[1] showing very low minimum drag coefficients, between 0.031 and 0.035, and for two others that are not nearly so good. They oscillate back and forth around the standard contour, and what appears to be the best form of all shows least deviation of all from the standard curve. To show what happens when the departures from ideal form are really large, there has been included in the same figure a difference-of-dimension curve for the hull form of the Class C airship,[2] a non-rigid ship and therefore forbidden by structural considerations the fine lines at the tail that are feasible in a rigid airship hull and necessary to an ideal low-resistance body. The drag coefficient for the Class C form is approximately 0.065, or just about double the minimum for a really good streamline shape. An intermediate form, with a coefficient of 0.044, is also included. Fully as bluff in the nose as the Class C, and with an even higher cylindrical coefficient, it lacks the Class C's extreme sharpness of curvature at the extreme tail, presumably reduces the seriousness of separation thereby, and attains its 25 per cent reduction of drag as compared with the Class C.

Though more work on the exact effect of particular variations would be of interest, the best approximation available for the moment is that any smooth form which at every point lies within 4 per cent of the maximum diameter from the dimension of the standard form at that same point should have a drag coefficient within 10 per cent of the minimum possible for a form of that fineness ratio.

The standard form has its maximum diameter at 40 per cent of the length from the nose, as opposed to the 30 per cent of the chord from the leading edge that is the most usual location for an airfoil's point of maximum thickness. It can be moved even farther back without damage, sharpening of the nose and its consequent prolongation having no harmful effect beyond the slight increase in superficial area and resultant increase in skin friction. Popular ideas on the importance of making the nose blunt and the tail exceedingly sharp and fine have been

[1] All of British origin, and tested in British official laboratories.

[2] "Drag of C-Class Airship Hulls of Various Fineness Ratios." by A. F. Zahm, R. H. Smith, and F. A. Louden, *Rept.* 291.

much exaggerated. The two forms in Fig. 235, for example, have almost exactly the same drag—though both have a third more than the best forms now known.

Wherever the maximum diameter may be, it is a fundamental requirement that there shall be no sudden breaks in the contour

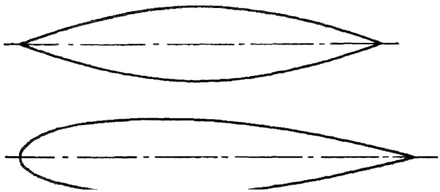


FIG. 235.—Effect of position of maximum diameter; two forms of equal drag.

and no sudden changes in the radius of curvature. Nothing is more surely destructive of the smooth flow upon which low drag depends.

Fineness Ratio and Drag.—Every analytical approach to the drag of low-resistance bodies leads straight back to the over-

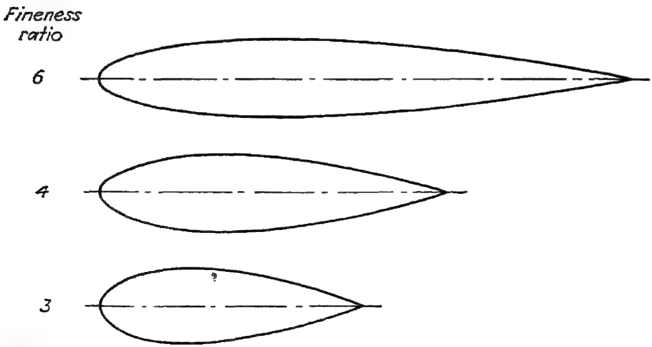


FIG. 236.—Variation of form with change of fineness ratio in a family of shapes otherwise similar.

shadowing importance of fineness ratio, and it is to the careful determination of the optimum fineness ratio that there must be special address if the lowest possible drag is to be found.

The effect can be studied by the comparison of forms elongated and contracted as a whole, the general shape remaining unchanged as in the series covered by Fig. 236, or the variation

may be made by keeping the form of the nose and tail constant, separating them at the section of maximum diameter and inserting cylindrical portions of varying lengths in between. Both systems of procedure have been covered by tests.[1] The

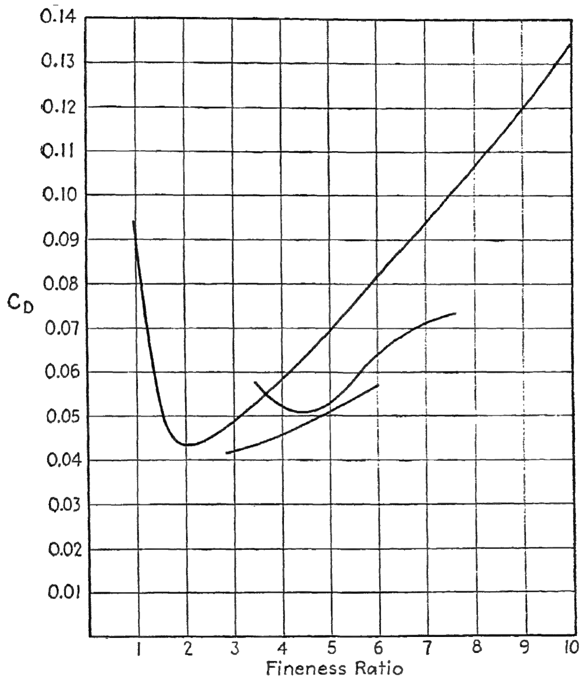


FIG. 237.—Variation of drag with fineness ratio, for several different basic shapes.

resistance coefficients for three series of the first type, no one of them unfortunately the very last word in low-resistance contour, are plotted against fineness ratio in Fig. 237. There would appear to be no general rule, but in two cases C_D seems to reach its minimum for a fineness ratio of about 2. The smoother the form, the lower the fineness ratio can be pressed without separation and sharp rise of resistance, and it seems likely that the minimum-resistance body has not yet been even closely

[1] For the first, see *R. and M.* 311 and *Rept.* 291, both *cit. supra.* For the second, *R. and M.* 607 and *Rept.* 138.

approached. There is a strong presumption that a form with the general shape of Fig. 229 and a fineness ratio of from 2 (as in Fig. 238) to 2.7 would have a drag coefficient no higher than 0.026, as against the 0.031 that is the minimum so far measured.

It was natural for the early students of aerodynamics to suppose that the minimum resistance would be secured with a very long and slender form. Realization of the importance of friction and of the extent to which it is practicable to reduce fineness ratio without increase of air resistance for a given cross-sectional area has come only very gradually. The process of working down into the lower end of the range of fineness ratios

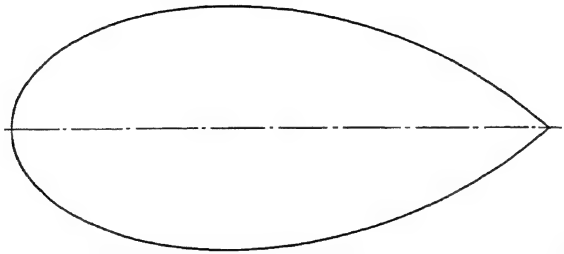


FIG. 238.—Good streamline form, with fineness ratio reduced to 2.

within which minimum drags are found has been especially slow because of that concentration upon their utility as airship hulls to which, as already noted, streamline forms have been subject. Comparison on a basis of volume of course favors the larger fineness ratios, for which the volume per unit of projected area is larger than for the short shapes.

Experiments on the effect of adding parallel middle body between the nose and tail again point to the desirability of small fineness ratio and a smooth and continuous change of radius of curvature, for such additions always increase the resistance for a given cross-sectional area if the original body even remotely resembled a good streamline form. Adding a short cylindrical section between the front and rear halves of a sphere would decrease the resistance somewhat, but a corresponding insertion in the middle of a body originally of fineness ratio of 2 or more appears never to do so except in cases where the forward and rear parts of the basic form did not go well together and where there was a sharp change in curvature at the maximum diameter.

British experience has shown one or two such cases, but they are decidedly the exception.

Pressure Distribution on Streamline Forms.—From the discussions of air flow that have gone before, and assuming a true streamline flow without separation, it should be easy to forecast the pressure distribution on a good low-resistance body. At the extreme nose there must be, as at some point on every body immersed in a moving fluid, a stagnation point where the pres-

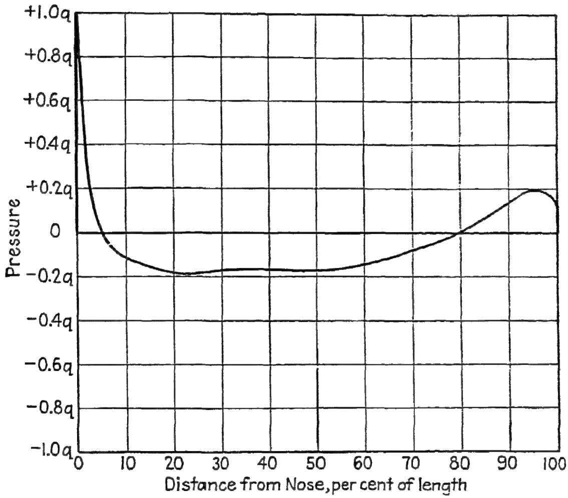


FIG. 239.—Pressure distribution on a streamline form.

sure is $\frac{1}{2}\rho V^2$. Immediately behind the stagnation point the streamlines must be sharply convex toward the hull as they bend outward to accommodate themselves to its contour. Having once established a flow along the curvature of the hull they ought to show a continuously negative pressure, the amount of its depression below atmospheric substantially proportional to the radius of curvature of the hull contour at each point. At the very tail, another forced reversal of curvature ought to produce a renewed rise of pressure to above atmospheric.

That is what it should be. Figures 239 and 240 are what it is, on a form of fineness ratio 4.6 and drag coefficient 0.031. The neat agreement of prophecy and experiment is so obvious

that Fig. 239 needs no further analysis or comment. Its companion, Fig. 240, is plotted on a system analogous to that used[1] for determining the longitudinal force on an airfoil by graphical integration. Pressures are plotted against the square of the diameter, instead of against the distance along the axis (the square, rather than the first power as in the case of the airfoil, to take account of the three-dimensional nature of the figure),

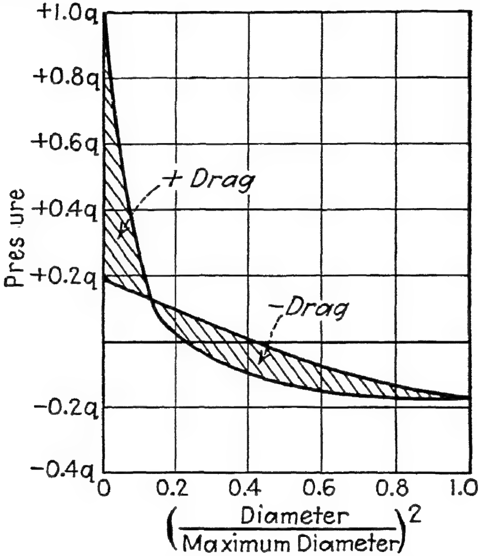


FIG. 240.—Pressure distribution on a streamline form, plotted to allow graphical integration to determine drag.

and the net area within the resultant curve gives the pressure drag, or what has heretofore been called the eddy-making drag, of the form. It is plain from the figure that the positive and negative loops are very nearly of equal area; that the pressure drag must therefore be exceedingly small, the suction forward of the mid-section and the pressure behind it combining to push the form forward almost as vigorously as the pressure on the nose and the suction behind the belt of maximum diameter act to retard it; that the total drag must therefore be almost entirely frictional. This is in corroboration of conclusions already drawn.

[1] P. 55, *supra*.

Protuberances on Streamline Forms.—We have already dwelt at some length upon the possible magnitude of the effect upon streamline flow of a relatively small disturbing influence, and upon the difficulty of reestablishing a smooth flow once disrupted. An extreme example of the harm that may be done has been furnished by tests of forms with one or more fine wires wrapped around the circumference to produce minute irregularities of surface.[1] With such a wire, having a diameter one six-hundredth that of the model, placed 10 per cent of the axial length back from the nose, the drag was increased by 60 per cent over that of the bare model. With the diameter increased to one one-hundredth of the maximum diameter of the streamline form the resistance went up about 90 per cent from the figure for the bare model. With the wire 45 per cent of the way back instead of 10 per cent, the effect was much decreased (to 20 per cent for the finer wire and 70 per cent for the larger one), although the length of the circumferential wire was, of course, greater near the mid-section. It is the breakdown of streamline flow that counts, and the effects are progressively more serious as the point of the breakdown's occurrence is moved progressively nearer to the nose. At low Reynolds' numbers, of course,[2] the drag of a poor form might be reduced by the introduction of the wire, but above the point of transition in the boundary layer the effects of a disturbance are always bad.

With a very small obstruction, on the other hand, the flow may reestablish quite readily, and the increase of drag due to the obstruction in such a case may be less than the drag of the obstruction tested solo. Thus, with a flat protuberance rising normal to the surface, having a width of one-eighth the diameter of the streamline form and a height of one-twentieth that diameter,[3] the effect on drag is again greatest when the obstruction is near the nose, but even at its worst it is but little more than the drag of the protuberant element alone. It is interesting to

[1] "The Resistance of a Spheroid with Special Reference to the Effect of Wires Placed around the Model in a Plane at Right Angles to the Axis," by R. Jones and A. H. Bell, *R. and M.* 858; "The Effect of Surface Texture and of Obstructions on the Surface of a Model of Streamline Form," by Felix B. Stump, an unpublished thesis at Mass. Inst. Tech., 1924.

[2] See p. 141, *supra*.

[3] "The Drag of Two Streamline Bodies as Affected by Protuberances and Appendages," by Ira H. Abbott, *Rept.* 451.

observe, as evidence that the stability of flow continues to increase with increase of Reynolds' number up to far beyond the ordinary limits of wind-tunnel testing, that the effect of such a protuberance set 8 per cent back on the length of the form, for example, is substantially independent of Reynolds' number above $N = 7{,}000{,}000$, but that below that point it increases rapidly with decrease of N, so that the total effect at $N = 1{,}500{,}000$ is double that at 7,000,000. The critical figure is significant, for the transition of the boundary-layer flow from the laminar to the turbulent form would be expected to make its way up to within 8 per cent of the nose at just about $N = 7{,}000{,}000$. In the same way, and with an equally fair qualitative agreement with anticipation from boundary-layer theory, the effect of a protuberance 30 per cent of the way back proves to be constant for all values of N above 3,500,000. The evidence is plain that the streamline flow is essentially undisturbed by so small a protuberance when the boundary layer is turbulent, but that it breaks down and reestablishes itself with some difficulty when the regime in the immediate neighborhood of the obstruction is laminar.

Fuselages.—The closest parallel to the streamline body in airplane design—and unhappily the parallel is usually not a very close one—is the fuselage. The disk ratio, or ratio of drag to the drag of a flat plate of equal cross-sectional area, for a good streamline body may fall as low as $\frac{1}{40}$ or, with a suitably reduced fineness ratio, even lower. For a fuselage to show a ratio below $\frac{1}{15}$ is exceptional; for it to run above $\frac{1}{8}$ is by no means unusual. For a fuselage must have its form, both longitudinally and in cross section, determined by its contents; it must provide a suitable mooring for the wings and tail surfaces, and often for the landing gear and for various bracing members; its smooth contour must be broken by cockpits or by windshields rising at an angle to the general curve of the surface to insure the pilot's vision; it must, in a single-engined or three-engined machine, be formed to have an engine hung on its foremost end. It need not be surprising to find the ideal resistance tripled, quadrupled, or even worse.

Fuselage data are much more confusing, and much more difficult to arrange, than those for the streamline forms. The geometrical complexities are infinite, and lend themselves to no

easy plotting of boundary curves and specification of essential elements. It is the excrescences and irregularities that cause most of the drag, and in their size, nature, and position they present literally millions of different alternatives having no definable relation to each other. The prediction of fuselage resistance, where no specific wind-tunnel test is made on the particular fuselage under consideration, must proceed largely by analogy; by looking through the records until a fuselage is discovered that looks as though its irregularities of form are of approximately the same gravity as those of the new design under examination and then adopting its reported disk ratio for the new calculation, modifying it slightly upward or downward if the parallelism in aerodynamic smoothness appears to be inexact.

There will be no attempt here to recite the complete roll of fuselage data. For those who require them, compilations have been published.[1] The purpose here will be rather to select a few instances especially helpful in formulating general conclusions and to exploit them to the limit. Unfortunately the existing store of data, approached with that object, proves to offer no embarrassment of riches.

The first step away from the streamline form is recorded in Fig. 241.[2] Such a fuselage might be used on a twin-engined plane or on a machine with a small liquid-cooled engine and an external radiator (though it would need a break in form at a windshield). Its contours make a very fair approximation to those of Fig. 229, except that the maximum diameter is a little too far forward, the nose a little too bluff in form, and the body closes on a vertical knife-edge instead of on a pointed tail. If the square root of the maximum cross-sectional area is taken as equivalent to the

[1] A number of widely various cases are covered in "Aviation Handbook," by Edward P. Warner and S. Paul Johnston, p. 139, New York, 1931. Individual tests are reported in "Wind Tunnel Test of Eight Model Fuselages," by A. L. Morse, *Information Circ.* 477, Airplane Eng. Div., U. S. Air Service, Washington; "Wind Tunnel Tests of Fuselages and Windshields," by Edward P. Warner, *Tech. Note* 226; "Tests of Quarter-scale Models of High-speed Seaplanes for the Schneider Trophy Contest of 1927," by W. L. Cowley and R. Warden, *R. and M.* 1296, 1297, 1298, published in special compilation of "Reports on British High-speed Aircraft," London, 1931; "Drag and Cooling with Various Forms of Cowling for a Whirlwind Radial Air-cooled Engine, Pts. I and II," by Fred E. Weick, *Repts.* 313, 314.

[2] From *Rept.* 313, *cit. supra.*

diameter, the fineness ratio is 5.5. The best of forms of that ratio has a drag coefficient of about 0.036. The fuselage in

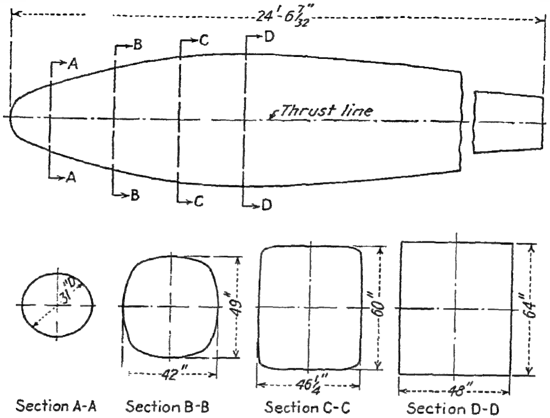


FIG. 241.—Basic fuselage form, without provision for engine.

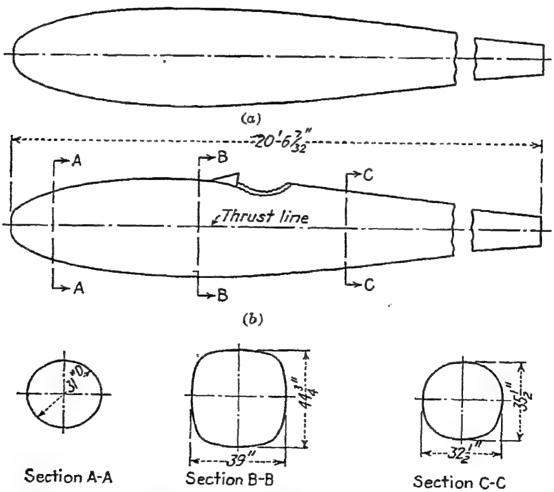


FIG. 242.—Basic fuselage forms, including (b) open cockpit and windshield, but without provision for engine.

Fig. 241 has a C_D of 0.080 at a Reynolds' number of 17,000,000 (tested full scale). An increase of drag of 120 per cent is the

price that has to be paid for a rectangular cross section and a rudder post and for the abrupt curvature at the nose.

Another fuselage of the same sort, but this time with a fineness ratio of 6.4 and with a more nearly elliptical cross section, appears in Fig. 242*a*.[1] Its drag coefficient is 40 per cent above that of its predecessor. In *b* it reappears, with a cockpit and windshield supplied and the drag increased another 50 per cent thereby, the disk ratio now down to 1/7.5.

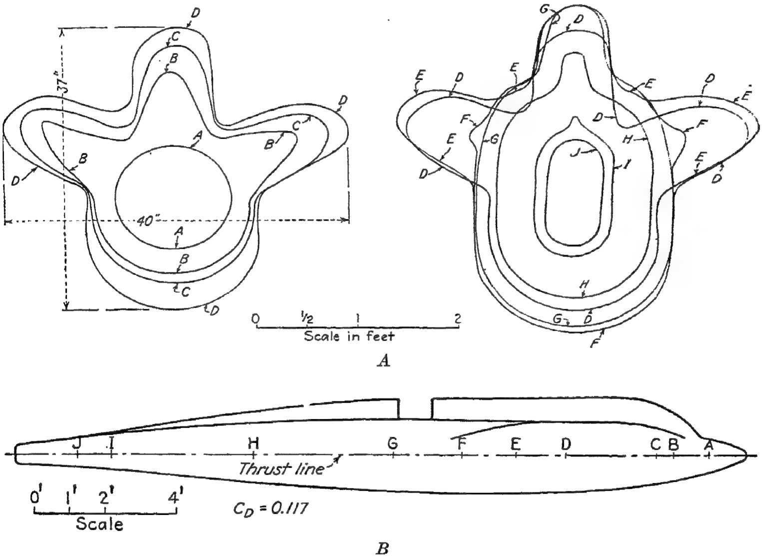


Fig. 243.—Fuselage of a 400-mile-an-hour seaplane.

The discrepancy between the performance of these two fuselages and that of a good streamline body of the same fineness ratio seems excessive. It would be an interesting and a valuable piece of research to start with the form of minimum resistance and to develop it into a fuselage by gradual modification, a step at a time, to discover just what it is in the geometry of the fuselage that loads on the major share of this alarming accrual of drag, but it is a research not yet performed.

[1] From *Rept.* 314, *cit. supra.*

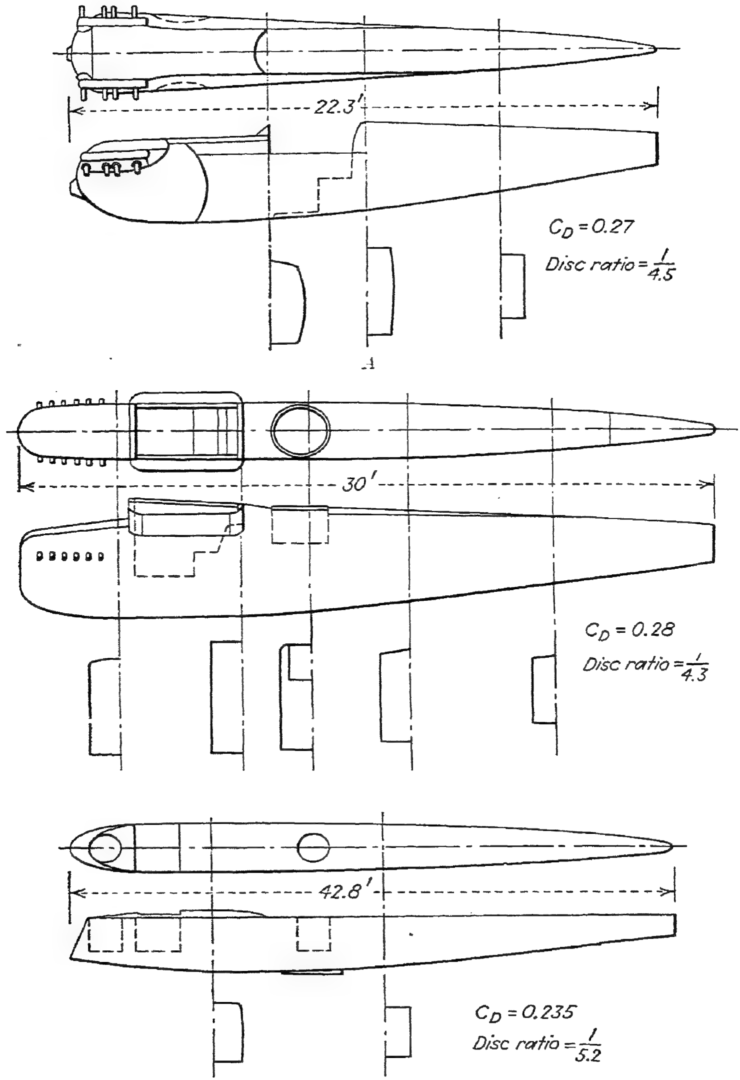


FIG. 244.—Early (1919–1922) fuselage forms.

A third form appears in Fig. 243[1] and is of interest as belonging to what was only a few years ago the fastest aircraft in the world (the exact aerodynamic details of its successors in that title not yet having been released to the public). It is the fuselage of the Supermarine S.5 seaplane that won the Schneider Trophy for Great Britain in 1927. It owes its peculiar cross section near the nose of the installation of a W, or "broad-arrow," engine, with three banks of cylinders. The effective fineness ratio is just under 10; that the drag coefficient is just a shade higher than that of the form shown in Fig. 242*a*, despite the clumsiness caused by the protruding cylinder banks just behind the nose, tells of an extraordinarily careful fairing of the contour in detail and once again suggests the possibility of doing much better than was done with the fuselages shown in Figs. 241 and 242, or than is done with fuselages in general, if equal care in detail were to be applied there.

By way of contrast (one might almost say of comic relief) three comparatively early forms are shown, together with their drag coefficients and disk ratios, in Fig. 244.[2] Their inferiority to the fuselages previously displayed is obvious. So are the reasons for it.

One specific point of variance among fuselages is the form of the cross section. At one extreme is the perfectly square section, at the other the circular. Both extremes are encountered in practice, though forms ranging from the almost elliptical to the rectangle with slightly bulged sides, top, and bottom are more common than either.

The circular section has, of course, the advantage of presenting a minimum superficial area for a given cross-sectional area and fineness ratio, and the further advantage that every possible streamline along the surface is of exactly the same form. With any section other than the circular the streamlines differ in curvature around the section, and the fuselage differs in effective fineness ratio (as judged by the sharpness of curvature of the streamlines immediately adjacent to the surface) around its periphery.

[1] From *R. and M.* 1296, from "Reports on British High-speed Aircraft," *cit. supra.*

[2] U. S. Air Service *Information Circ.* 477, *cit. supra.*

The effect of all this on drag is a little uncertain. The drag of a square-sectioned fuselage as commonly tested in the wind tunnel commonly exceeds that of one having a circular section and otherwise identical by from 10 to 25 per cent. With a propeller turning in front of the nose, however, the inferiority of the square section is likely to become more marked, for the propeller imposes a general rotation on the flow of air passing through it and the subsequent motion of the streamlines is not straight back along the body, but helically around it. The helix is, to be sure, one of extremely large pitch, but the twist in the stream is sufficiently pronounced to accentuate the bad effect of corners. Unfortunately there are no tests specifically covering this point, but it may be estimated that under maximum-speed conditions the drag of the square fuselage exceeds that of the circular one, other things being equal, by from 25 to 30 per cent on the average, while a section with bulged sides and with the corners rounded on a radius of one-fifth the mean cross-sectional dimension ought to show about one-half that increase.

Radial Engines and Fuselage Drag.—Without exception, the fuselages so far shown have had either liquid-cooled engines or no engines at all. The case of the air-cooled engine, which must have direct access to an air stream for cooling and which generally protrudes its cylinders in all directions like the rays of a starfish, is a special problem. It has been reserved for special handling.

As the first approach, return to Figs. 241 and 242. Those were the basic forms of the first chapter of the great National Advisory Committee research on cowling and drag,[1] in its entirety perhaps the most important single contribution that an aerodynamic laboratory has ever made to the efficiency of aircraft.

Within the nose of the form first shown in Fig. 241 was placed a nine-cylinder engine, its cylinders projecting about 9 in. from the fuselage but still lying well within the imaginary cylindrical tube bounded by the largest cross section and therefore being without effect on the cross-sectional area. Nevertheless, the drag was almost exactly tripled, increasing C_D to 0.25. The installation is shown in Fig. 245. To leave the engine as completely exposed as in Fig. 245, hanging it on the nose as an afterthought, produced only another 6 per cent of increase. To fit

[1] Embodied (so far) in seven parts.

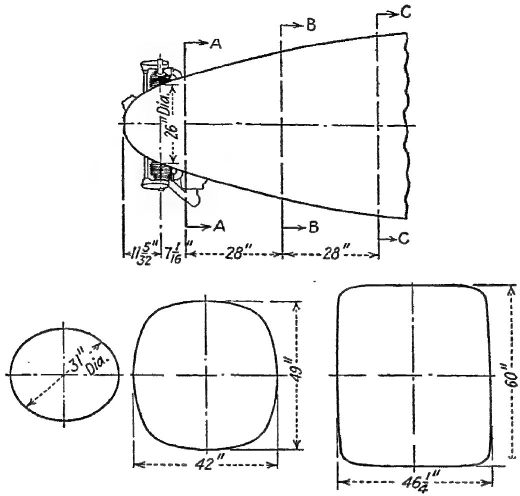


FIG. 245.—Installation of radial engine.

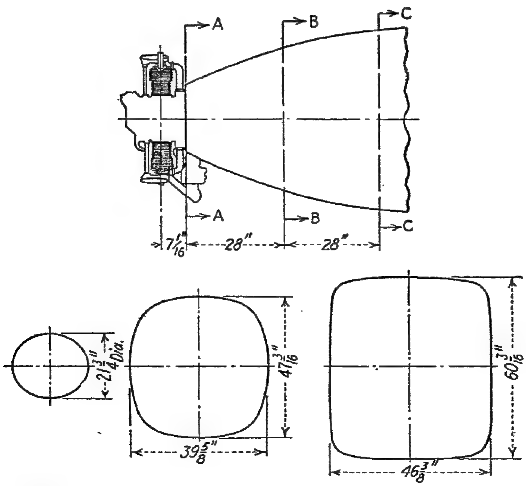


FIG. 246.—Installation of radial engine, completely uncowled.

an extension as in Figs. 247 and 248 so that the smooth outline would be extended to a point ahead of the propeller reduced the

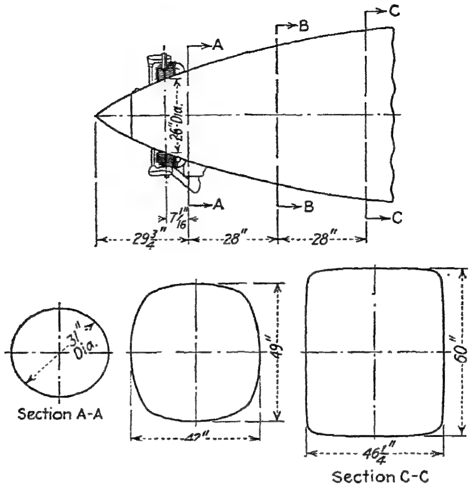


Fig. 247.—Installation of radial engine, with spinner for propeller.

drag only 2 per cent. To change the curvature so that only the extreme heads of the cylinders would project beyond the fuselage,

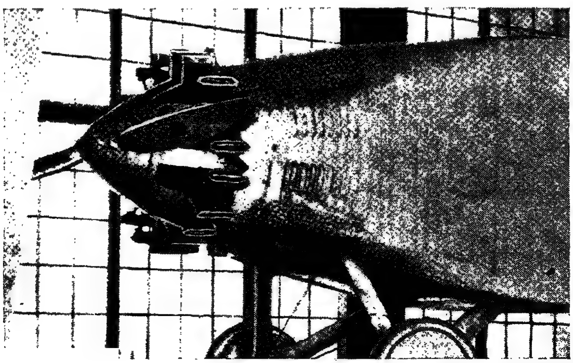


Fig. 248.—Installation of radial engine, with spinner for propeller.

the major part of the cooling air gaining entrance through holes in the nose and leaving through stamped slits or louvres in the

side of the fuselage behind the engine, as in Fig. 249, reduced it by 11 per cent as compared with the basic form with the cylinders fully exposed. The disk ratio was still 1/5.3, the drag 2.8 times that of the basic fuselage without an engine.

Savings of 10 per cent in drag are never unimportant, but they are discouragingly small when they come as the immediate sequel to a 200 per cent increase. The net of all this experience is a confirmation of the lesson already learned: that when an obstruction on the surface of a body has once occasioned separa-

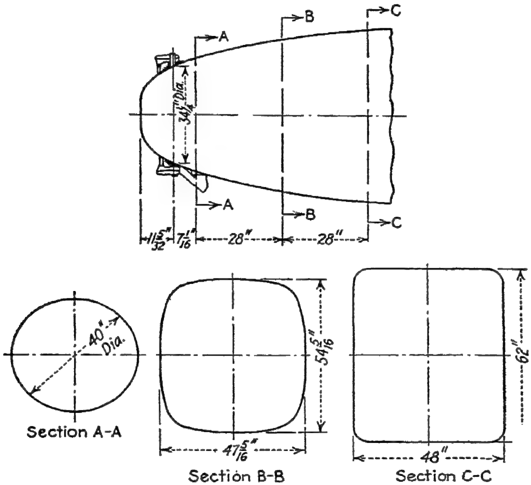


Fig. 249.—Installation of radial engine, with cowling around cylinder barrels.

tion it is extremely difficult to reestablish streamline flow, and that resistance is likely to go up as a result quite out of proportion to the size of the offending excrescence.

The remedy lies not in a partial screening of the obstruction but in a forcible prevention of separation by guidance of the flow along the boundary of the body. The problem of cowling for minimum resistance finds its solution not in an expanded fairing for the central part of a radial engine but in an annulus around it. The end product of the first series of N. A. C. A. tests was the annular cowl shown in Figs. 250 to 252.

The analogy with a slotted wing is obvious. The area of the annular exit passage at the rear of the cowl is less (usually by

about a half) than that of the circular opening at the front. Therefore the air that flows through the cowl and over the

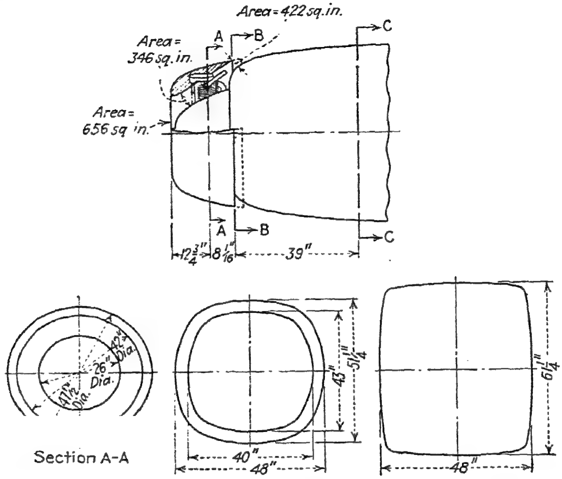


FIG. 250.—NACA cowl around radial engine.

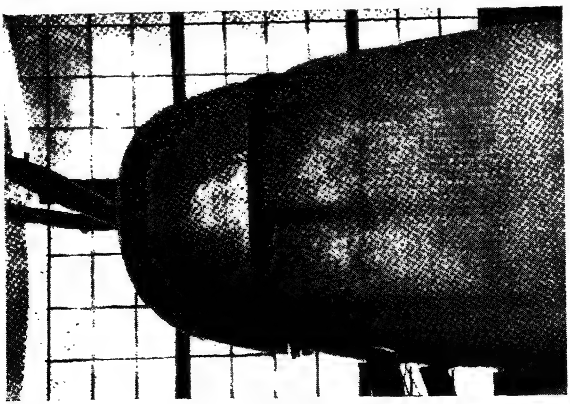


FIG. 251.—Fuselage with NACA cowl.

cylinder heads accelerates as it leaves; therefore it serves to scour the boundary layer and to accelerate it; at the same time it leaves the constrained passage through the cowl in a direction

that carries it straight back along the sides of the fuselage; the aggregate of all this reestablishes a smooth flow and suppresses separation.

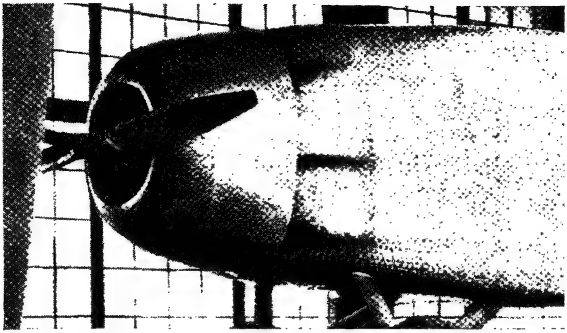


FIG. 252.—Fuselage with NACA cowl.

The magnitude of the service to aviation that the NACA cowl, as it is commonly known, represents can be read in the figures.

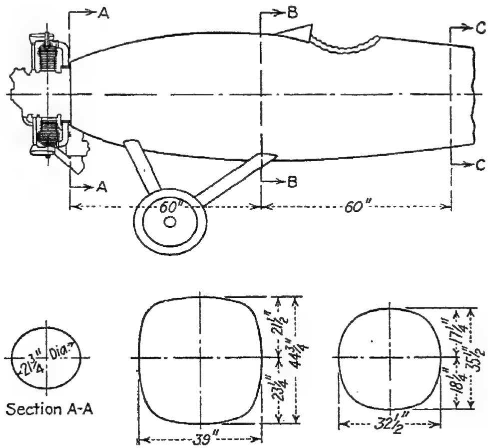


FIG. 253.—Installation of radial engine in open-cockpit fuselage.

The most effective cowl so far discussed permitted an increase of drag, due to the engine, of about 180 per cent over the drag of the bare fuselage. With the NACA cowl the percentage of

increase was reduced to 90. As compared with the best alternative arrangement, the effect of the engine in increasing drag

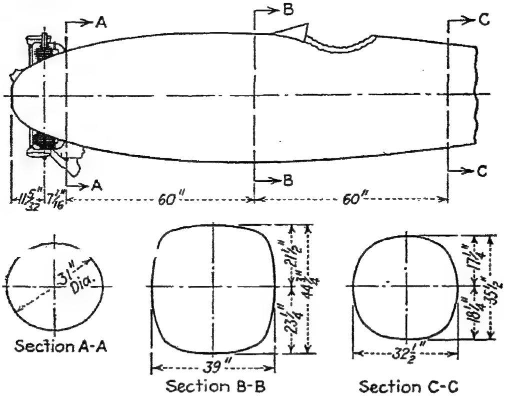


FIG. 254.—Installation of radial engine in open-cockpit fuselage.

was exactly halved. The disk ratio of the fuselage with the engine installed stood at 1/7.8.

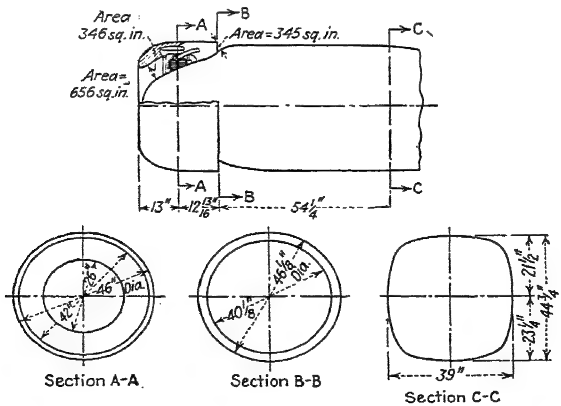


FIG. 255.—Open-cockpit fuselage with NACA cowl.

Experience with the basic form shown in Fig. 242 was essentially identical. With an engine hung on the nose, as in Fig. 253, the drag was increased 235 per cent. (In this case the engine diameter, still 44 in., exceeded the fuselage width by 5 in. and

increased the projected area by about 6 per cent.) The best form of cowling of the internal-fairing-and-spinner type, shown in Fig. 254, reduced that by only 6 per cent. The annular, or venturi, or NACA, cowl, with its passages proportioned by trial to allow the passage of enough air to insure the proper cooling of the engine, cut it down to only 73 per cent above the drag of the bare fuselage without an engine. In this case the cowl reduced the additional drag due to the engine by over two-thirds.

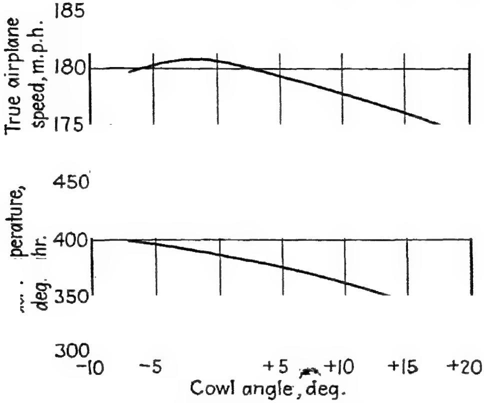


FIG. 256.—Variation of flight speed and cylinder temperature with variation of cowl's exit opening by operation of flaps.

Figure 255 illustrates the best form of NACA cowl for this fuselage.

Though there is no rigorous code of rules for cowl design, experience and experiment[1] have led to certain general conclusions. The passage of air through the cowl must be free enough to cool the engine properly, and it is engine cooling that sets a minimum limit on the size of the cowl's openings. Where the airplane must operate over a wide range of speeds and at all air temperatures the effectiveness of the cooling may

[1] By far the most important of it is embodied in two papers recording work done by the staff of the United Aircraft Corporation; "The Cowling and Cooling of Radial Air-cooled Aircraft Engines," by Rex B. Beisel, A. Lewis MacClain, and F. M. Thomas, *Jnl. Soc. Automotive Engrs.*, May, 1934; and "Further Progress in Controlled Cooling of Radial Aircraft Engines," by J. M. Shoemaker, T. B. Rhines, and H. H. Sargent, Jr., *Jnl. Soc. Automotive Engrs.*, October, 1935.

be varied by the use of hinged flaps attached around the trailing edge of the cowl and capable of being swung through an angle of some 30 deg. to vary the area of the annular exit opening and the amount of pressure drop that exists on the fuselage surface adjacent to that opening and that serves to pull the air through the cowl. Figure 256[1] shows the effect of flap angle, with such a variable-angle cowl, upon the cylinder head temperature and the maximum speed of flight.

Flap angles up to 5 deg., it will be observed, have very little effect on performance and a pronounced effect on temperature, but after the angle passes 5 deg. the speed begins to fall off rapidly.

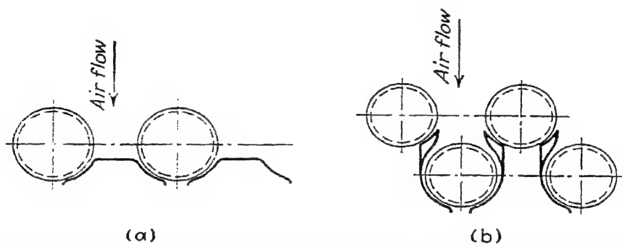


FIG. 257.—Arrangements of baffles for cooling single-row and twin-row engines inside an NACA cowl.

In present practice, cowl flaps are little used except with fourteen-cylinder twin-row engines, and little used except in taxiing and take-off. Even in steady climbing the air speed is usually high enough to cool the engine without turning the flaps outward. The flaps are most useful on flying boats, often subjected to taxiing for protracted periods at high power output and comparatively low speeds.

Engine cooling can be materially helped, and the mass flow of air required through the cowling reduced, by the use of baffles to guide all the air that enters the cowl into the closest possible contact with the cylinders. Without baffles, a considerable part of the energy expended in taking air into the cowling and out again is wasted. The best form of baffle surrounds a considerable part of the cylinder, just outside the tips of the cooling fins, and blocks off as completely as possible any leakage of air through

[1] From Beisel, MacClain, and Thomas, *loc. cit., supra.*

the space between the cylinders. The total mass flow through the cowling is very much reduced, but cooling and parasite drag are both improved. Figure 257[1] shows typical systems of baffling, developed for a single-row and a twin-row engine, respectively.

Where the cowl lacks adjustable flaps, the ratio of exit area to inlet area ought to lie between 0.40 and 0.50, the lower values being practicable only where baffles are used between the cylinders. Without baffles, the inlet area at the nose ought to run about 3.5 sq. in. for every engine horsepower, on an airplane having its best rate of climb at 80 m. p. h., down to as low as 2.8 sq. in. per hp. when the best climb is found at 110 m. p. h. With a good set of baffles, the inlet area per horsepower can be cut about 25 per cent from those figures, bringing it down to 2 sq. in. per hp. on a machine of high speed. In general, the calculation of the inlet area in accordance with those constants will be found to call for an opening in the nose of the cowl having about three-quarters the diameter of the engine itself.

The section of the cowl ought to be roughly that of the upper surface of an airfoil, but with the maximum camber not more than 40 per cent of the way back on its chord, and with a camber 10 per cent of the way back from the nose which should be at least 60 per cent of the maximum. The curvature at the leading edge is, in other words, unusually sharp, accommodating itself to the radial motion that the air acquires in flowing past the propeller spinner, through the propeller, and outward as much as necessary to clear the crankcase of the engine. The maximum camber, referred to the chord of the cowl's section, should be at least 12 and preferably 16 per cent of the chord.

The diameter of the fuselage immediately behind the cowl ought to be at least equal to the maximum diameter of the cowl itself; the fuselage contour ought to curve inward quite abruptly as it is brought forward toward the cowl, contracting the fuselage diameter to allow the necessary width of annulus for the exit of the air; and the contour of the after part of the cowl ought to fair smoothly into the fuselage contour behind

[1] From Beisel, MacClain, and Thomas, *loc. cit.*, *supra*. See also "The Effect of Baffles on the Temperature Distribution and Heat-transfer Coefficients of Finned Cylinders," by Oscar W. Schey and Vern G. Rollin, *Rept.* 511.

the region of abrupt change. A good form of cowl (in solid line) and two bad ones (dotted lines) are shown in Fig. 258.

At the same time (1928) that the NACA cowl was under development in the Advisory Committee laboratories, a some-

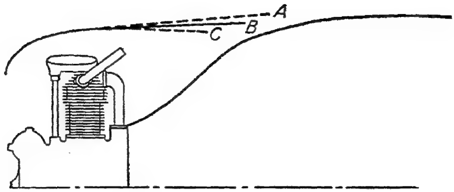


FIG. 258.—A good cowl form (in solid lines) and two inferior ones (dotted lines).

what analogous line of study was under way in England. There resulted the Townend ring,[1] an annulus of comparatively short chord and approximately of airfoil section running around the periphery of the engine as shown in Fig. 259. In controlling the

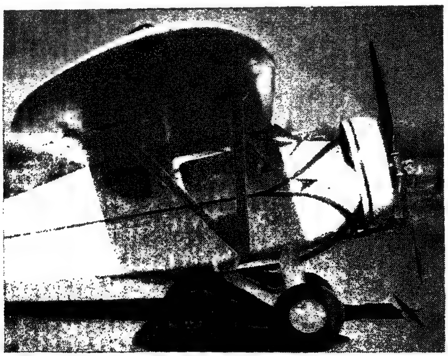


FIG. 259.—Townend ring around radial engine.

air direction, steering it back into its proper course, so to speak, after passing the disturbance of the engine, the ring and the full annular cowl work much alike; the ring, however, does not have its exit passage so shaped and so restricted in area as to gain any

[1] "Reduction of Drag of Radial Engines by the Attachment of Rings of Aerofoil Section, Including Interference Experiments of an Allied Nature, with Some Further Applications," by H. C. H. Townend, *R. and M.* 1267 (almost a book in itself; a study of over 100 pages).

substantial benefit from the acceleration of the air by the venturi action. The ring even proves effective in controlling the flow around a normal flap plate. The lines of flow around such a plate are mapped in Fig. 260 both for the simple case and for that with a ring in position, and direct measurement has shown a combined drag for circular disk and encircling ring actually 62 per cent less than the drag of the disk alone—the best arrangement for reducing the drag of a disk being with a ring diameter of 1.5 times, and with a chord of one-third, the disk's diameter. Mr. Townend suggests the feasibility of using an airfoil above a windshield to prevent separation there and to reduce its effect on drag.

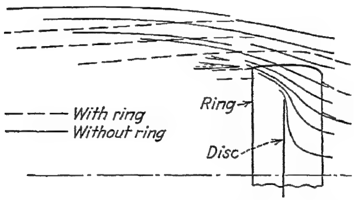


FIG. 260.—Influence of Townend ring on flow around a normal disk.

The Townend tests with narrow fuselages and radial engines show in general a reduction of about a half the total fuselage resistance, or about two-thirds of the drag due to the engine, by the use of the best available ring. Just what is the best ring depends very largely on the basic fuselage form, but it seems to be universal that a thin-plate section of roughly parabolic curvature is very much superior to a biconvex or even a medium-thickness cambered-airfoil section for the ring; that the chord of the ring should be at least equal to the outside diameter of a cylinder; that a ring of polygonal section, with the vertices of the polygon above the cylinder heads and chords bridging the gaps between the cylinders, is quite as good as and perhaps a little better than a circular one; that the ring should be set a little ahead of the cylinder, with the mid-point of its chord about at the cylinder's forward edge; and that its chord should be inclined inward at the leading edge, as if to parallel the contour of the nose of the fuselage, but by not more than one-third the angle of the fuselage contour's inclination. The installation found to be best for a particular open-cockpit fighting plane is shown in Fig. 261.

The choice between a simple ring and a full annular cowl is often a difficult one, but no longer can there be any doubt about the necessity of using one or the other in every case. As a general rule, a good ring gives results about equal to those of the full NACA cowl on open-cockpit machines having fuselages of which the width is appreciably less than the engine diameter, while on cabin planes with fuselages distinctly wider than the engine the full cowl is distinctly superior.[1] For the nacelles of multiengined airplanes, also, as will appear in due course, the NACA cowl has an easy triumph over all competition.[2]

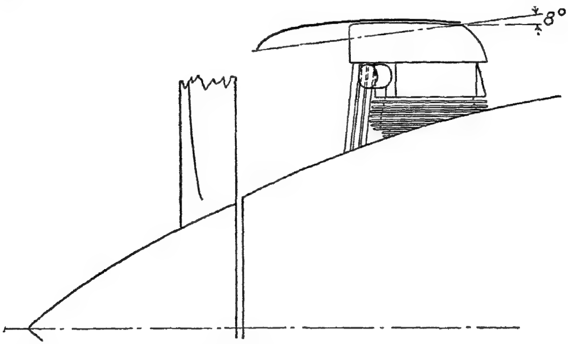


FIG. 261.—Ideal arrangement of Townend ring on a high-speed open-cockpit biplane.

Where the engine is a nine-cylinder or a double-banked fourteen-cylinder one, the cylinders form so continuous a *chevaux-de-frise* around the fuselage's circumference that there is little temptation to interrupt the circumferential continuity of the cowling. Let the number of cylinders be reduced to seven or less, however, and the idea of reducing enclosed area by giving the cowling a wavy form, drawing it in radially between each two cylinders, naturally suggests itself. Carried far enough, that comes to putting a separate helmet over each cylinder as in Fig. 262,[3] and with that idea the British were experimenting

[1] "The Effect on Airplane Performance of the Factors That Must Be Considered in Applying Low-drag Cowling to Radial Engines," by William H. McAvoy, Oscar W. Schey, and Alfred W. Young, *Rept.* 414.

[2] P. 398, *infra.*

[3] From *Rept.* 314, *cit. supra.*

long before the first ring or annular cowl had appeared. British[1] studies extended even to the fitting of individual helmets on nine-cylinder engines.

Direct comparison with the best ring developed in such a case shows the helmets effecting a drag reduction about a third of that attainable with the ring. The theory of the helmet's operation is very much the same as that of the ring, the air being discharged through a narrow slit and guided by the form of the helmet to leave in a direction approximately parallel with, or inclined slightly inward relatively to, the sides of the fuselage.

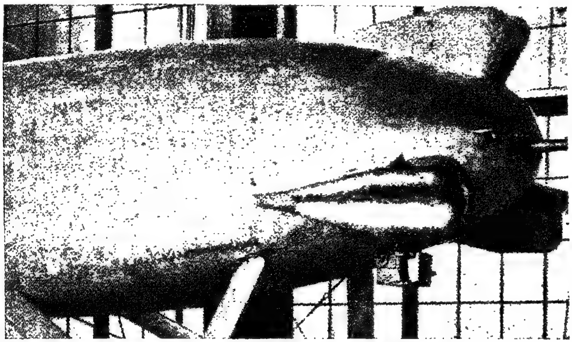


FIG. 262.—Helmet cowling over individual cylinders.

In the American experiments, limited to three-cylinder engines, three cylinders projecting nakedly from the nose of the fuselage were found to increase the resistance of the bare fuselage by 65 per cent (as against the 200 per cent increase due to the nine-cylinder engine, or virtually in direct proportion to the number of cylinders). The helmets used reduced the added drag due to the engine by only a third. With a nine-cylinder engine their relative performance would presumably have been much more unfavorable, and the discrepancy between the American and the British data suggests that the helmets used in the N. A. C. A. tests probably were not the best that could have been found. It seems quite certain that when a three-cylinder engine

[1] "Tests on the Crusader Models for the Schneider Trophy Contest of 1927," by W. L. Cowley and R. Warden, *R. and M.* 1300, published in special collection of "Reports on British High-speed Aircraft," London, 1931; *R. and M.* 1267, *cit. supra.*

is to be used a carefully designed form of individual cowl or helmet will give a lower total drag than any all-enveloping system of cowling. It seems equally certain that with seven or more cylinders the ring or the NACA cowl will give the superior result. The case of the five-cylinder engine is beyond the reach of generalized conclusion on the basis of data so far available, and the best system of cowling may depend upon the proportions of the particular fuselage and the particular cylinder heads that have to be adapted to each other. The difference between the best general-cowling and the best individual-cowling system probably would not exceed 10 per cent in total drag in any five-cylinder case.

Effects of Inclination of Fuselages.—Fuselages are sure to be yawed to the wind direction at times, particularly during circling flight or cornering on a race course. Furthermore, as the angle of attack of the airplane changes with changing speed of flight, the angle between the fuselage axis and the direction of the relative wind must change at the same rate. There is for any particular airplane only one speed of flight at which the axis of the fuselage coincides with the relative wind direction, and deviations from such coincidence of course increase the resistance.

For a streamline form without appendages, an angle of yaw or pitch of 5 deg. increases the resistance by from 4 to 10 per cent, and an angle of 10 deg. by from 25 to 40 per cent.[1] For fuselages, as might be expected, the effects are in general somewhat less, although a flat-sided fuselage may show an increase of drag of as much as 15 per cent for 5 deg. of yaw and 55 per cent for 10 deg. Substituting an elliptical for a flat-sided section reduces very sharply the effect of yaw on resistance, as would be anticipated. The direct importance of flat sides is shown by tests by Wieselsberger,[2] in which bodies of round and square cross section, but otherwise similar, were tested at angles of yaw. A 10-deg. angle

[1] See, in addition to reports already cited, "Air Forces, Moments, and Damping on Model of Fleet Airship Shenandoah," by A. F. Zahm, R. H. Smith, and F. A. Louden, *Rept.* 215.

[2] "Air Forces Exerted on Stream Lined Bodies with Round or Square Cross Sections, When Placed Obliquely to the Airstream," by C. Wieselsberger, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. II, republished as *Tech. Memo.* 267.

from the position of minimum drag increased the drag 33 per cent with the circular cross section, 235 per cent with the square. The latter figure is, of course, exceptional, as bodies of square cross section without appendages or cockpit openings are seldom used, although that very high increase of relative drag might be approximated in some wide fuselages with enclosed cabins. The rate of change of lift with changing angle was similarly about four times as large with the flat sides as with the circular sections. It is in line with reasonable prediction, also, that the pitching of the fuselage, corresponding to changes of angle of attack, has less effect than the yawing, since the width of a fuselage is in most cases only from a half to two-thirds of the depth. The typical fuselage form, which has its upper surface more nearly flat than the lower one as seen in side view, gives its minimum resistance at a small negative angle of pitch, and a deviation of 10 deg. from that setting at minimum resistance is unlikely to increase the drag more than 40 per cent, except for a very wide fuselage and especially one with upper and lower surfaces flat transversely (as seen in section). The range of angles of attack of the wings within which performances are of most interest (*i.e.*, the range from the angle of attack corresponding to a steep dive to that corresponding to the steepest angle of climb) is about 12 deg. The fuselage axis need then never be inclined more than 8 deg. to its best attitude except when landing, at which time the reduction of drag is of no importance, and the resistance should seldom be more than 20 per cent in excess of the minimum.

A fuselage that is set at an angle of pitch gives a lift as well as a drag. The lift can be specified very roughly, as it is so very small in magnitude compared with the lift furnished by the wings, but it is not entirely negligible. Zero lift, like minimum drag, is generally found at a negative angle of pitch, and the lift increases at such a rate that at an angle of 10 deg. above that of zero lift it is a little less than the drag for a typical fuselage. The lift coefficient based on the maximum cross-sectional area of the fuselage then averages about 0.12 at that angle. The maximum lift of a fuselage is then much less than that of an additional wing area equal to the cross-sectional area of the body.

Windshields.—Among the factors responsible for breaks in the smoothness of a fuselage contour, the most constant is the

windshield. Even the engine is missing in the case of a twin-engined or four-engined plane, where the fuselage carries no power plant, but the fuselage invariably provides accommodation for the pilot and the accommodation must invariably include a transparent surface so set as to allow him satisfactory vision.

As a general rule, the windshield must be inclined at not more than 45 deg. to the normal line of vision, and in no case must the inclination be less than 35 deg. On an open-cockpit fuselage, where the windshield is a pure excrescence, it adds an amount of

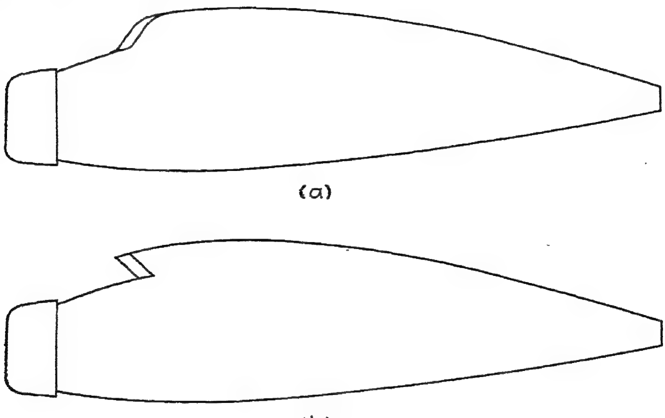


Fig. 263.—Alternative windshield forms.

drag ranging from 24 lb. at 100 m. p. h., when the shield is a large one and set at an angle of 75 deg. to the fuselage contour, down to 9 lb. when the area is minimum and the angle 45 deg. On a cabin fuselage it is unfortunately more difficult to generalize, both because of a scarcity of data[1] and because the windshield effect is enormously sensitive to the basic form, but it may be assumed on the average that the type of shield shown in Fig. 263*a* adds about 0.12 to the total drag coefficient[2] of a fuselage

[1] Interesting studies of the air flow around windshields of various types, though unfortunately without directly measured drag data, are contained in "Windscreens with Openings," by F. B. Bradfield and B. Lockspeiser, *R. and M.* 1613, and in "Improved Airplane Windshields to Provide Vision in Stormy Weather," by William C. Clay, *Rept.* 498.

[2] Product of C_o by cross-sectional area.

without a break in contour corresponding to the shield, or about 30 lb. increase of total drag at 100 m. p. h., while an undercut shield like that in Fig. 263*b* has about twice that effect. If these figures seem large compared with the ones given for open-cockpit machines, it must be remembered that the windshield through which the pilot's compartment looks out from a transport plane is typically about twice as high and fully twice as wide as the typical shield in front of an open cockpit.

Interference between Fuselages and Wings.—It is a commonplace, and no doubt obvious at sight, that the aerodynamic properties of a combination of elements are not obtainable by simple summation. Where the normal flow patterns around two or more objects interfere with each other the drag rises in consequence, precisely as the resistance of the hull of a ship is determined primarily by the interrelation of the wave patterns originating at the bow and stern, respectively. Furthermore, wherever one of the elements is carrying a lift there is induced drag, and wherever there is induced drag its magnitude is highly sensitive to the form of the curve that represents the distribution of lift along the span.[1] The particular case of the interaction of a fuselage and a wing abutting it gets the full benefit of both direct and induced interference, but especially of the second.

The simplest of all possible "fuselages" is a flat plate, parallel to the wind direction and perpendicular to the wing, inserted through the wing at its central point. Even that rudimentary interruption of the wing's continuity affects the lift distribution enough so that the drag of the combination exceeds the total drags for the wing and the plate tested separately by an amount equal to an increase of about 0.001 in the airfoil's drag coefficient.[2] Furthermore, it decreases the maximum lift of the wing by an appreciable amount, likely to range from 1.5 to 4 per cent, with the largest loss of lift on the thickest airfoils.

From the flat plate to the flat-sided rectangular-sectioned fuselage is but a short step, but it involves the new problem of vertical dimension. The plate was presumed to extend far

[1] Pp. 89, 100, 104, *supra*.

[2] "Investigation of the Effect of the Fuselage on the Wing of a Low-wing Monoplane," by H. Muttray, *Luftfahrtforschung*, June 11, 1928, translated as *Tech. Memo.* 517; "The Interference Effects on an Airfoil of a Flat Plate at Mid-span Position," by Kenneth E. Ward, *Tech. Note* 403.

enough beyond the wing in all directions so that it would have made no difference how much farther it might have been extended. The fuselage is very definitely limited in height, and whether its height projects entirely above the wing, entirely below, or a little of each—whether, in other words, the monoplane is low-wing, high-wing, or mid-wing—may be expected to make a difference.

Tests show[1] that with a rectangular fuselage the difference is small. As a general rule the interference is most favorable in the high-wing or mid-wing arrangement, grows much worse as the wing is lowered, but even in a low-wing type (lower surface of the wing flush with the bottom of the fuselage) the total minimum drag of the combination is only equal to the sum of the minimum drags of the wing and fuselage taken separately. With a mid-wing position the combination drag is less than the sum of the minimum elemental drags by about 5 per cent in a typical case, or by an amount just about equal to the product of the bare wing drag by the ratio of area screened within the fuselage to total area. The apparent negative interference then represents in fact nothing more than a reduction of the wing drag in proportion as its exposed area is reduced. It is, however, customary to refer the interference to a summation of the drag of the fuselage and that of a wing having the full area as defined on page 16, including the area contained within the fuselage. So referred, the drag interference with a rectangular fuselage ranges from -5 per cent in a high-wing or mid-wing combination to zero in a low-wing arrangement or $+10$ per cent in one with the wing actually crossing in the clear below the fuselage.

Interference also has an effect on maximum lift, and in this case it is most convenient to define it as the difference between the maximum for the combination and the maximum for the wing alone—again taking the wing forces without deduction for the screening of a part of the area. Interference in that sense is

[1] "Effects of Varying the Relative Vertical Position of Wing and Fuselage," by L. Prandtl, *Tech. Note* 75; Muttray, *Tech. Memo.* 517, *cit. supra;* "The Interference between the Body and Wings of Aircraft," by J. H. Parkin and G. J. Klein, *Jour.* of the Roy. Aero. Soc., p. 1, January, 1930; "Some Aspects of the Mutual Interference between Parts of Aircraft," by E. Ower, *R. and M.* 1480.

likely to be around −10 per cent for a low-wing case, −5 per cent with the wing at mid-height of the fuselage. Since a typical fuselage covers about 8 per cent of the wing area, the reduction is equal on the average to the reduction in effective area, and the fuselage itself appears to contribute nothing to the lift. Its actual lift is offset by the fuselages effect on the downwash from the wing.

The N. A. C. A. has recently undertaken a research on interference that promises to be as complete, in a field with an even greater profusion of independent variables, as was the N. A. C. A.

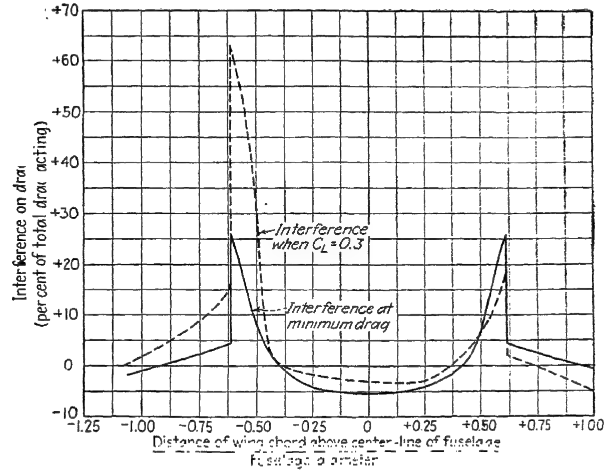


FIG. 264.—Variation of interference drag with vertical position of the wing on a circular fuselage.

study of airfoils.[1] The first results to be published[2] have made it possible, for the first time, to plot definite curves of the effect of wing position on interference with a circular fuselage.

The curves are in Fig. 264. They are plotted for interference at minimum drag (necessarily symmetrical about the mid-wing position, since the airfoil was biconvex and set with its chord parallel to the axis of a symmetrical fuselage, so that a high-wing

[1] P. 181, *supra*.

[2] "Interference of Wing and Fuselage, from Tests of 209 Combinations in the N. A. C. A. Variable-density Tunnel," by Eastman N. Jacobs and Kenneth E. Ward, *Rept.* 540.

combination at zero lift was simply a low-wing inverted, and vice versa) and at a lift coefficient roughly corresponding to cruising conditions or to a gentle climb.

Obviously, the drag is tremendously sensitive to the vertical position of the wing when the wing is very near to being tangent to either the upper or the lower surface of the fuselage—especially the latter. A low-wing monoplane in cruising attitude may have a drag exceeding by as much as 60 per cent the total drags of the wing and fuselage taken separately. That increase, due to interference, can be reduced by as much as three-quarters

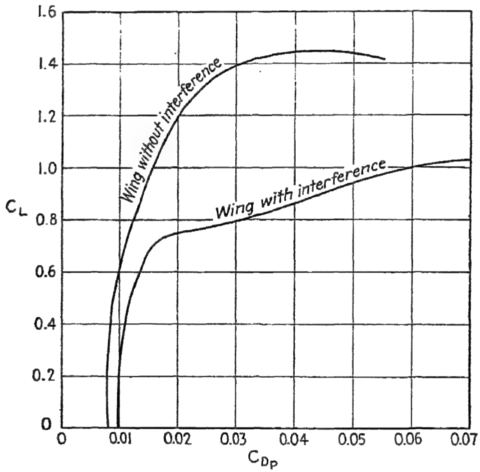


FIG. 265.—Airfoil polar showing an interference burble.

by lowering the wing by as little as 2 in. on a medium-sized transport, leaving 2 in. of open air space between the bottom of the fuselage and the upper surface of the wing.

The case plotted, to be sure, happens to be a particularly bad one. If a cambered wing be used instead of a symmetrical one with zero mean camber, the worst increase of drag due to interference at the cruising attitude is appreciably less serious, though there are not enough data as yet available to plot a curve for that case. A tapered wing, also, shows considerably less interference than a rectangular one. Nevertheless it remains true that a position very near the bottom of the fuselage is an exceed-

ingly critical one, whatever the shape of the wing, and that only the most careful shaping of the two elements in the neighborhood of their juncture[1] will avoid alarming increases in drag.

Accelerated burbling immediately adjacent to the fuselage, due to a redistribution of downwash and a consequent modification of true angles of attack such as has already been shown[2] to occur where the wing chord is locally reduced, is interference's most serious aspect. It produces profile-drag curves like that of Fig. 265. It not only increases the landing speed by reducing the maximum lift; in an extreme case it may extend to angles of attack so small as to have a very bad effect on the coefficient of minimum power consumption and on the performance of the

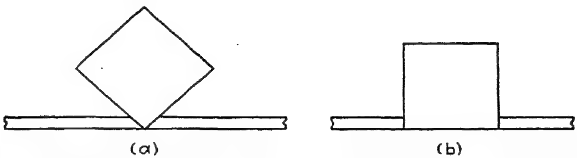


Fig. 266.—Wing-fuselage combinations of maximum (*a*) and minimum (*b*) interference.

airplane in climb. In any case where it appears it has the worst possible influence on stability and control, imparting both short-period turbulence and long-period irregularities to the flow over the tail surfaces, preventing proper response to the controls and initiating the violent shaking of the tail surfaces that is known as *buffeting*. The chance of all these phenomena is bad enough with a rectangular fuselage. It is far worse with one of circular cross section, which is coming to be the more common case in an endeavor at once to reduce the bare fuselage resistance and to serve the convenience of all-metal construction.

Muttray has shown, by way of explaining the increased severity of interference with a circular form, that a combination of a square fuselage and a wing as in Fig. 266*a* has a minimum drag 75 per cent higher, and a drag at $C_L = 1.0$ that is 50 per cent higher, than the corresponding figures for the more conventional arrangement of Fig. 266*b*. With the diamond-shaped fuselage the air flowing over the wing near the fuselage finds itself pocketed, its velocity reduced by the friction of two convergent

[1] See pp. 393–394, *infra*.
[2] P. 237, *supra*.

surfaces. Furthermore, and more important, the fact that the cross-sectional area of the pocket so formed increases as the trailing edge is approached gives it something of the characteristics of the exit portion, or diffuser, of a venturi tube, where speed is decreasing and pressure rising. Separation takes place in the exit portion of a venturi unless the angle of the walls to the axis is exceedingly fine, and similar factors produce a similar effect at the juncture of the fuselage and the wing. Stagnation in the boundary layer and separation ensue, even at low values of C_L. Muttray found separation, by direct investigation of the flow, at angles of attack of less than 2 deg. The rough geometrical analogy between the diamond-shaped section and a circular one, and the existence of a similar expanding pocket and a similar diffuser action in the latter case, is obvious.

The interference with the wing anywhere well below the center line of the fuselage, unless it be set clear of the fuselage by at least 10 per cent of the fuselage's depth, appears to be disastrous. The corresponding position at the top of the fuselage, though not nearly so bad, still shows undesirable characteristics. A study of interference on maximum lift for the low-wing machine shows surprisingly little effect, the lift dropping off but very little more than in proportion to the area of wing that the circular fuselage cuts off. The apparent evil tendencies of the low-wing monoplane are then less manifest in an increase of landing speed than in bad control characteristics and proneness to buffeting and in increased drag through the range corresponding to the best climbing conditions. At the normal cruising attitude, interference may represent a difference of 30 per cent in total power consumption, or of 10 per cent in speed for a given power.

On the face of the record to this point, low-wing monoplanes in combination with circular fuselages would seem to deserve abandonment. But their structural advantages are great, they are convenient machines to handle and to work on mechanically, and they provide a most convenient space (the lower wing) within which to withdraw a retractable landing gear. A cure for their defects becomes exceedingly desirable, and as it was Muttray who first called attention to the fundamental ailment of the type so it was Muttray who provided the remedy.[1]

[1] *Tech. Memo.* 517, *cit. supra.*

It seems obvious enough, in looking at Fig. 267, that the reentrant angle that makes the trouble should be rounded off with a fillet, but if a fillet be built up by eye, approximately symmetrical about the mid-chord of the wing, the burbling at the wing root is only slightly delayed. Muttray's particular contribution was the concept that since separation is due to the diffuser action of the pocket between the wing and fuselage, expanding as it does toward the trailing edge, separation should be suppressed by filling in the diffuser space. Looking back to

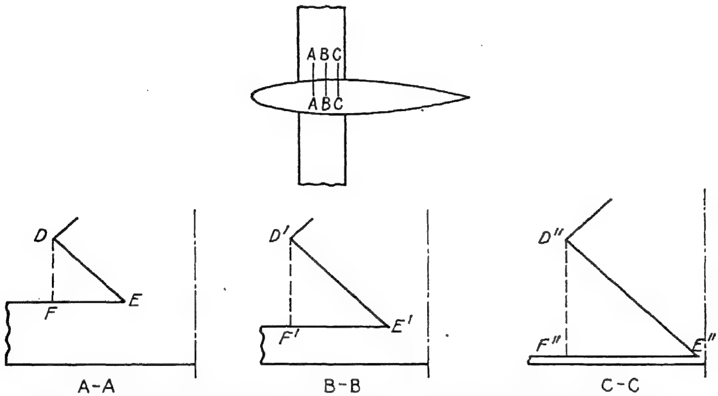


FIG. 267.—Pocketing of air under the fuselage of a low-wing monoplane.

Fig. 266, the most straightforward plan would be to fill in the space *DEF* as shown in Fig. 267. Equally effective in canceling the expansion of cross-sectional area of the pocket, and simpler in that the discontinuity to be faired into the basic fuselage contour behind the trailing edge would be reduced in breadth, would be one of the modifications illustrated in Fig. 268. An obvious extension from Fig. 268, as conforming to the general aim of replacing sharp corners with smooth curves wherever possible, is Fig. 269. It was Fig. 269 or something of that order on still more generous radii of curvature, an expanding fillet, that Muttray recommended. It is this form that has triumphed by test and practical experience, until it has become a necessary part of the design of every low-wing monoplane with a round fuselage or even with one with slightly bulging sides. Though the effect is much

less important on a biplane, it is of distinct advantage even there to provide a wing root fairing the lower wing into the curving side of the fuselage on a radius expanding toward the rear. Even

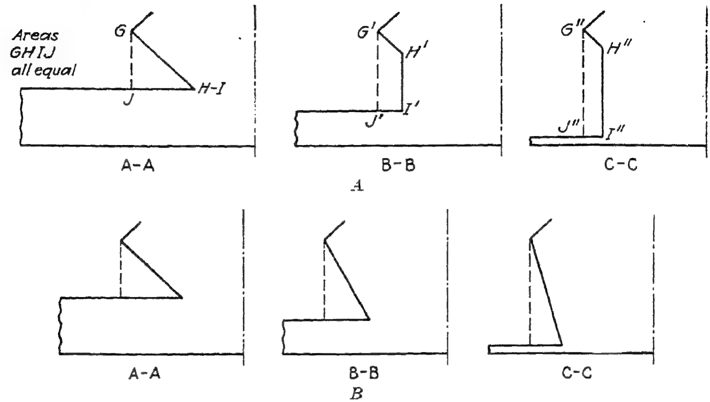


FIG. 268.—Methods of preventing pocketing of air under the fuselage.

with a fuselage with virtually flat sides merging into a semi-elliptical top and bottom, one test[1] has revealed a saving of about 3 per cent in minimum drag and an improvement of 5 per cent in maximum lift by the use of an expanding fillet.

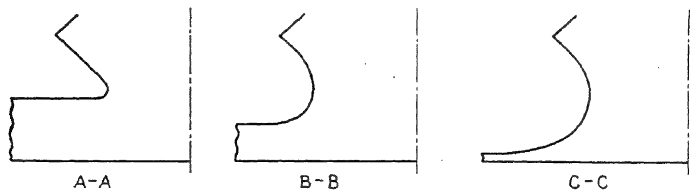


FIG. 269.—Filleting of a low-wing monoplane, developed from Figs. 267 and 268.

As a general rule the effect of fillets on drag at low angles is not large, though it is too large to neglect. It seldom exceeds 10 per cent of the combined drag of wing and fuselage at lift coefficients

[1] "Effect of Fillets on Wing-fuselage Interference," by A. L. Klein, *Trans. Am. Soc. Mech. Eng.*, p. 1, January, 1934 (an excellent review of the whole subject, with many examples of fillets good and bad, followed by a discussion to which a number of the leaders of American and British experimental aerodynamics contributed).

of 0.2 or less, but above that point the effect begins to mount. In Fig. 270[1] are plotted the curves of profile drag for a mid-wing combination and for low-wing monoplanes without a fillet and

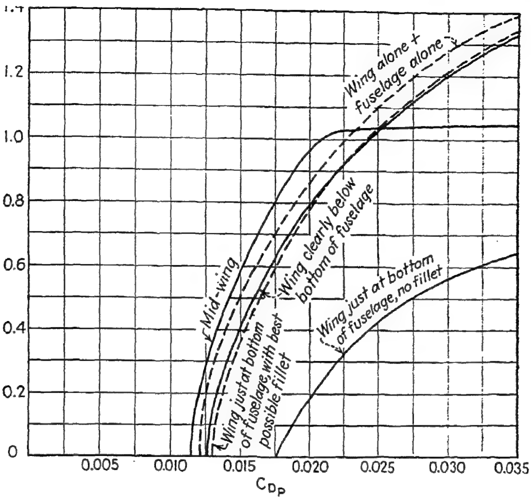


FIG. 270.—Variation of drag characteristics of a monoplane with wing location and filleting.

with the best form of expanding fillet, the three cases being similar in every other respect and the fuselage being circular. The combined drag is compared with the sum of the drags of the

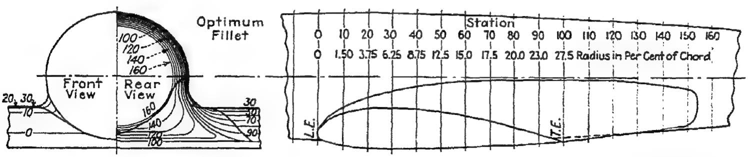


FIG. 271.—Lines of the ideal fillet for a particular low-wing monoplane.

fuselage and the wing, tested separately. Between mid-wing and low-wing positions there is now, upon aerodynamic grounds alone, little to choose. The apparent advantage of the mid-wing arrangement at low angles is due to the screening of a substantial

[1] Based on *Rept.* 540, *cit. supra.*

portion of the wing within the fuselage, and has to be paid for by a corresponding loss of lift.

To emphasize the matter still further, and to show just how the fillet has to be faired off into the fuselage behind the trailing edge of the wing, the lines of a particular fillet are shown in detail in Fig. 271[1] and a photograph of another is reproduced in Fig. 272.[2]

When the wing chord is dropped below the bottom of the fuselage, by anything up to a bit over the maximum thickness

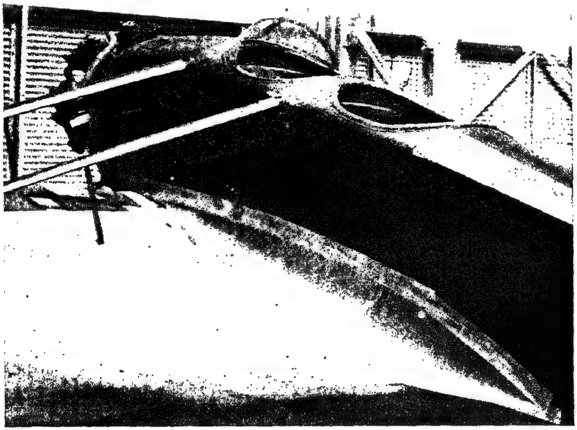


FIG. 272.—Low-wing monoplane, with expanding fillet to wing.

of the wing, the two elements should be welded into one by a system of filleting essentially (as nearly as the change of position permits) like that shown in Fig. 271. A dropped position generally proves, however, to be aerodynamically inferior to the regular low-wing arrangement. It is justified as a rule only where it seems very desirable to keep the fuselage and wing structures separate and continuous, making the wing in a single piece to be passed through below the fuselage and secured to its bottom surface. The fillet then becomes a separate fairing, to be added after the structure is otherwise complete.

[1] From Klein, *op. cit.*

[2] From "Wing-fuselage Interference, Tail Buffeting, and Air Flow about the Tail of a Low-wing Monoplane," by James A. White and Manley J. Hood, *Rept.* 482.

Compared with the effect of vertical position of the wing and of correct filleting, all other aspects of interference are unimportant. Fore-and-aft position on the fuselage has very little effect within the customary range, though if the nose of the fuselage sticks out very far ahead of the leading edge, as it has been especially prone to do on large British airplanes, the liability to premature burbling due to interference is somewhat increased. The angle at which the wing chord is set with respect to the fuselage axis, too, proves unimportant.

The effect of the fuselage on pitching moment is but little influenced by interference. Moment on the combination can in almost every case, except at lift coefficients above 0.6, be predicted within 0.015 of coefficient by simple addition of the moments of wing and fuselage—first, of course, transferring them to a common axis. At very large values of C_L the error of prediction is, similarly, unlikely to exceed 0.030. So far as there is a correction it is usually positive, the downwash on the rear part of the fuselage introducing a stalling tendency. The moment on the fuselage alone is hard to discuss in general terms, being immensely sensitive to changes both of cross-sectional form and of profile, and most especially to sharp curvature of profile near the nose and to radical dissymmetry about the longitudinal axis. The broadest generalization available is that the moments on a fuselage are likely to be large compared with the lift, so that the center of pressure is far behind the tail or (more commonly) ahead of the nose, and that the dissymmetry necessitated by cockpit or windshield in practical airplanes generally produces a positive moment at zero angle.

Nacelles on Wings.—Though a nacelle and a fuselage are closely allied in a part at least of their purpose, and although the choice of form is influenced by many of the same considerations in each case, the intimate contiguity of the nacelle and the wing and the necessity of planning the nacelle installation with primary reference to the interference characteristics of the combination have made it seem advisable to hold off any mention of the nacelle until wing-body interference should have been discussed in general terms.

Nevertheless, the first step can well be to consider the drag of the nacelle alone. In that connection, since the nacelle's sole function is to support and to fair an engine and since its cross-

sectional area is not a primary variable but can be set at whatever figure the designer thinks will give best results, the convention of measurement will be changed from the one that seemed best suited for fuselages. Instead of determining the maximum sectional area and from it computing C_D, the figure used will be the products C_DS, the coefficient of total drag. It needs only to be multiplied by q ($\frac{1}{2}\rho V^2$) to give the total drag of the nacelle in pounds.

Comparison on this basis is facilitated by the fact that the world supply of full-scale nacelle research has been concentrated

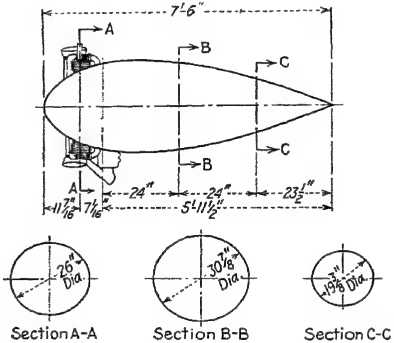


FIG. 273.—Uncowled nacelle.

in the N. A. C. A. laboratories[1] and that all the work has been done with reference to a single type of engine, the same type and size as were used in the fuselages illustrated in Figs. 241 and 242. It is nine-cylinder, 220 hp., 44 in. in overall diameter.

Set up by itself in a wind stream, with just enough structure of steel tubes to support it and with no fairing of any kind, the engine has a drag coefficient of 7.00. That, incidentally, gives it just a little more than half the drag of a flat normal disk having a diameter equal to the engine's and with no reductions of

[1] "Drag and Cooling with Various Forms of Cowling for a Whirlwind Radial Air-cooled Engine," Pt. II, by Fred E. Weick, *Rept.* 314; "Tests on Nacelle-propeller Combinations in Various Positions with Respect to Wings," Pts. I, II, III, by Donald H. Wood, *Repts.* 415, 436, 462; Pt. IV, by James G. McHugh, *Rept.* 505; Pt. V, by E. Floyd Valentine, *Rept.* 506; Pt. VI, by Donald H. Wood and Carlton Bioletti, *Rept.* 507; "Drag Tests of ⅕ Scale Model Engine Nacelles with Various Cowlings," by Ray Windler, *Tech. Note* 432.

area to allow for the open spaces between the engine's cylinders.

The first attempt at improvement is the fitting of a nacelle of the type that was standard on multiengined airplanes until about 1929—the type shown in Fig. 273. It proves almost as ineffectual as putting a spinner ahead of an uncowled engine on a fuselage. The air flow has been broken up in passing over and between the cylinders, and inserting a brief streamlined tail into the region of deadwater so created is of little avail. The drag coefficient comes down just 15 per cent, from 7.00 to 6.00. A simple nacelle of 15 per cent larger diameter, however, making

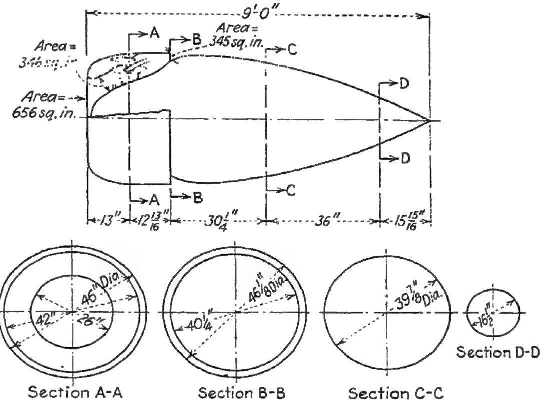


FIG. 274.—Nacelle with NACA cowl.

its overall diameter almost equal to that of the engine, showed a coefficient of but 4.50.

To surround the engine with the best type of ring that has been found, while keeping the nacelle already shown, is good for another 38 per cent of reduction, to a coefficient value of 3.73. The best ring found, incidentally, was of polygonal section, a nine-sided figure with a vertex directly over each cylinder-head, and had its chord set at an angle of −8 deg. to the nacelle's axis, roughly conformable to the rule already suggested that the ring angle should be one-third the angle between the axis and a tangent to the surface of the nacelle or fuselage at the plane of the ring. Though there was no trial of rings with the larger nacelle, the probability is that a well-designed ring applied there would bring the drag coefficient down to 3.0 or a trifle lower.

By no possibility, however, could the ring as now known bring the drag down to the level reached with an NACA cowl. The full venturi shroud shown in Fig. 274 gave a coefficient of 1.55, barely one-quarter the drag of the only type of nacelle that was known up to the time when Weick in America and Townend in England introduced the designer to redirection of streamlines and to separation control as a device for reducing parasite drag on bodies with protuberances near the nose.

With the NACA cowl the nacelle, since its cross-sectional area is 11.5 sq. ft., shows a C_D of 0.135—10 per cent lower than the

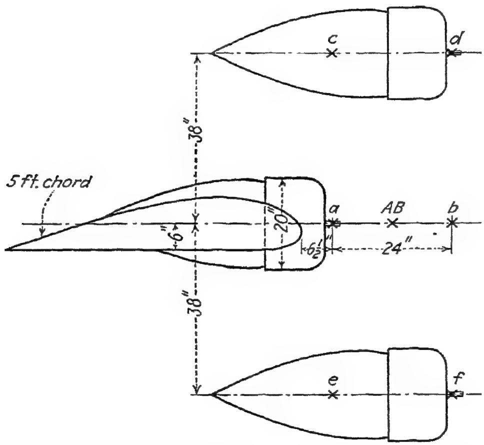


FIG. 275.—Variation of location of nacelle with respect to wing. (The letters denoting the positions are the same used in Figs. 276 to 280.)

lowest value attained with a similar engine installed in a fuselage like that in Fig. 241, and thereby another testimonial to the possibilities of low fineness ratio, for the fineness ratio of the nacelle with the cowl in place is but 2.3.

In the fashion characteristic of bodies of low fineness ratio, nacelles show a large scale effect. The data here given were taken at Reynolds' numbers of about 3,000,000, the overall length of the nacelle being used for L in the expression for N. They are thus well outside the range where a rapidly falling drag coefficient would be expected if it were only friction that had to be considered. That tests reveal a decrease of from 10 to 30 per

cent in the value of the coefficient in going from $N = 1{,}500{,}000$ to $N = 3{,}000{,}000$ suggests that separation must also be a factor, that eddy-making resistance must still be falling off with increase of N up to the point where the change from laminar to turbulent boundary-layer flow is taking place only one-sixth of the nacelle's length back from its nose. Nacelles are very unlikely to be used in actual design at Reynolds' numbers of less than 3,000,000, and for all values above that level the N. A. C. A. figures that have been given here can be used directly, with correction only for skin friction (which would reduce the drag 5 per cent if the Reynolds' number were raised to 6,000,000). There may be a more rapid decrease than that, at least in some cases, but there is no present evidence for it.

The performance of a nacelle *solo* is of somewhat academic interest. Nacelles appear in airplane design only in immediate conjunction with wings. Interference is paramount. So is the relation between the nacelle's form and position and the functioning of the propeller that turns immediately in front of it.

A study of nacelle performance offers the means of shedding light on so many aspects of interference and of the effect of the power plant upon the airplane and vice versa, and presents so many illustrations of the practical effect of the air-flow structure considered in Chaps. III and V, that the subject will be considered in exceptional detail. In Fig. 275 a cowled nacelle is successively located in seven different positions with respect to a thick wing. In Fig. 276 the curves of interference drag, or difference between the actual combination drag and the sum of the drags for wing alone and nacelle alone, are plotted against lift coefficient for each of the six. The units being fractions of the minimum drag of the nacelle alone, a reading of $+1.0$ means that the effective nacelle drag under that particular condition is double the isolated value of $1.55(\rho/2)V^2$. A reading of -1.0 indicates a complete cancellation of nacelle effect, a combined drag no larger than the drag of the wing alone.

The curves speak for themselves, but in general terms they can be summarized that, as might have been expected, the nacelle sunk directly into the wing shows the best average performance throughout the range of flight conditions, with a negative interference at every point; that a location below the wing (which was the common rule on landplanes until these researches

had been completed) gives fairly good results at values of C_L of 0.4 and over but is extremely bad in the range corresponding to maximum flight speed; that a nacelle above the wing (commonly used, prior to 1931, on flying boats, to keep the propellers well clear of the water) is bad under all conditions and even worse at

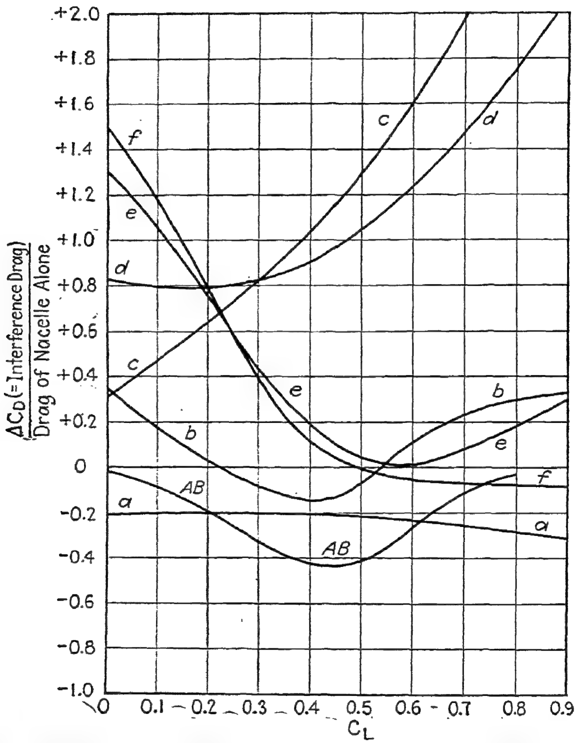


FIG. 276.—Variation of interference drag with nacelle location. (For the significance of the designations on the curves, see Fig. 275.)

high lift coefficients than at low. The longitudinal position is of minor significance as compared with the vertical coordinate, seldom changing the interference factor by more than 0.4; its effect at low values of C_L generally shows the most favorable results with the least advanced position of the nacelle; at high lift coefficients there is no general rule.

In Fig. 277 corresponding interference curves are drawn for the uncowled nacelle (as shown in Fig. 273) for positions level with the wing, above it, and below it, the fore-and-aft location in each case being about midway between those for the two stations on each level selected in plotting Fig. 276.[1] The interference ratios shown are much smaller than those in Fig. 276, but it must be remembered that the unit in each case is the drag of the nacelle alone, 6.00 for Fig. 277 and 1.55 in Fig. 276. The

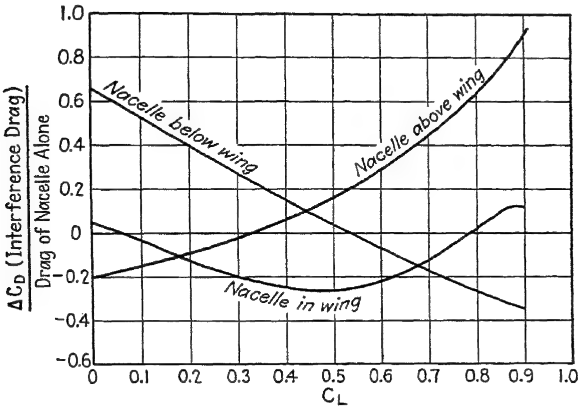


FIG. 277.—Variation of interference drag with nacelle location for an uncowled nacelle.

values plotted are purely relative, and an interference ratio of 0.10 in Fig. 277 is equal to one of 0.39 in Fig. 276. If that is taken into account the two sets of curves agree well in their general indications on the effect of position. The nature of the effects is not difficult to understand in view of what we now know about the direction and velocity of air flow about a wing.[2]

So far, the location in the leading edge of the wing appears to have a good lead over all competition. But the findings cannot

[1] The original reports cover twenty-one nacelle locations with the fully cowled nacelle, about a third that number each with a bare nacelle, and one with a Townend ring. Searchers after more exhaustive data than have been plotted here can supply their wants at the source.

[2] See Chaps. III, V.

be complete until the effect of nacelle location on the efficiency of the propeller has been taken into account.[1]

The mutual interaction is in two parts. In the first place, the nacelle has an influence upon the propeller, affecting both the power that it will absorb at a given rate of rotation and air speed and the degree of its efficiency as a converter of engine power into useful driving effort. Second, the propeller influences the nacelle, the increased speed of air flow that usually exists in the *slipstream* immediately behind the propeller acting to increase the parasite resistance of any object that may be placed there. Both influences depend in some degree upon the nacelle's location with respect to the wing.

The first subdivision of the first factor is unimportant, except in connection with the problem of picking the propeller best adapted to a particular machine. For that purpose, the extreme spread in power-consuming capacity among the twenty-one nacelle locations tested corresponds to a required spread of about 3 in. in propeller diameter or about 1.5 deg. in pitch angle. The best propeller for a given location once chosen, the value of its power coefficient has almost no further influence on the characteristics of the airplane.

Propeller efficiency is of greater importance, and it is plotted in a modified form in Fig. 278. The modification consists in considering the increase in nacelle drag due to propeller action as being a negative component of thrust, so that instead of being

$$TV$$

the power output at the propeller is taken as

$$(T - D')V$$

where T is the thrust, D' the increase in drag due to slipstream, and V the speed of flight. The propulsive efficiency is then

$$\eta_e = \frac{(T - D')V}{P} \qquad (78)$$

P being the engine power developed in ft.-lb. per sec.

[1] It is suggested that those who do not already possess a working knowledge of the methods of analyzing the characteristics of a propeller and the conventions that are commonly used for representing them should skip the next few pages until after they have read Chap. XIV, where these matters are fully explained.

The propulsive efficiency so defined is plotted in Fig. 278 for the seven locations already chosen for the cowled nacelle and for two positions of the uncowled one, and upon the further assumption that a controllable-pitch propeller is used and that it can therefore be kept operating at its maximum efficiency at

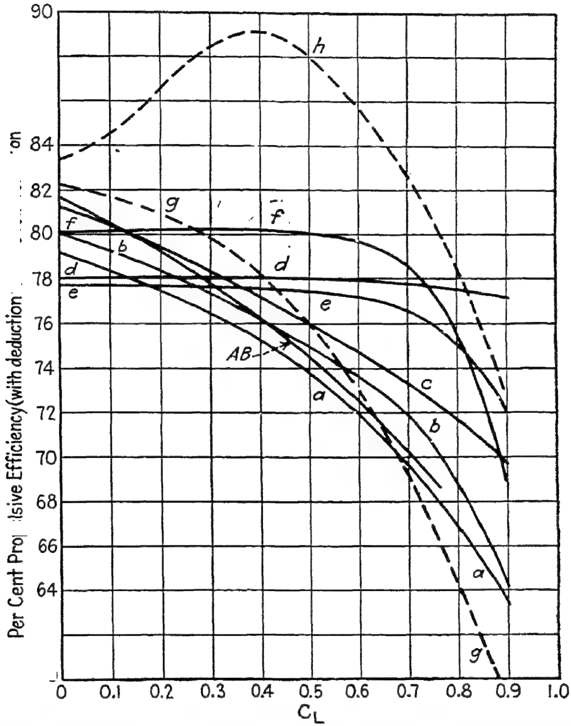


FIG. 278.—Variation of gross propulsive efficiency (including slip-stream effect on nacelle drag) with nacelle type and location. (For the significance of the designations on the curves, see Fig. 275 for *a* to *f*, inclusive; *g*, nacelle located in leading edge, at same point as *ab*, in Fig. 275, for case of cowled nacelle; *h*, below wing, at point midway between *e* and *f*, in Fig. 275.)

all values of C_L. Calculations on this basis must be approximate, for in fact a single rigid propeller was used throughout the tests and their only rigorous application would be to a machine with a propeller identical with the one so used. The efficiency

curves would then come, as propeller-efficiency curves habitually do, to a peak at a particular value of V and so of C_L, and they would fall off rapidly to either side of their maxima. In view of the increasingly widespread use of controllable-pitch propellers, however, and also of the desirability of getting a set of curves that will not be limited in their application to a particular propeller pitch and wing loading, it has seemed permissible to make the assumptions necessary in constructing Fig. 278.[1] They are (1) that a controllable-pitch propeller, if always properly adjusted for the prevailing flight condition, will give the same efficiency at all values of V/ND; (2) that the ratio of increase of nacelle drag due to slip stream to the propeller thrust, D'/T, will be the same for all propeller pitches and flight speeds.

Neither assumption is accurate, but neither one is seriously enough in error to invalidate the general conclusions, particularly as it happens that the errors that they introduce into the final result are of opposite sign. It then becomes possible to plot propulsive efficiency as a function of C_L alone. The curves of Fig. 278 should be correct within a very small margin of error in their relative position and separation from each other along any particular ordinate, and so in indicating the relative merits of the various nacelle locations in respect of propeller performance. Their absolute values are likely to run a little high at high lift coefficients, perhaps by as much as 8 per cent at a maximum, and may also be below the truth at the other end of the scale, though the error in that case should be smaller.

It proves, as might have been expected,[2] that the apparent propulsive efficiency is highest with the propeller axis below the wing, where the velocity of the air is least; and well forward of the leading edge. It may be a little surprising at first sight that the propeller shows a better efficiency in front of the uncowled nacelle than with the fully cowled one, but the reason for that relationship, too, becomes clear from the discussion of slipstream effects in Chap. XIV.

Figure 279 illustrates another aspect of slipstream effect—this one favorable. The increased velocity of the air flow across the wing with the propeller running serves in general to increase

[1] The alternative method of calculation, strictly limited to the propeller used and the data obtained in the tests, was used in the original reports. Its results can be found there.

[2] See Chap. XIV, *infra*.

the lift. It then becomes possible to reduce the angle of attack somewhat, while the same average lift coefficient is kept, and so to reduce the basic wing drag. Also, the increased velocity in the slipstream increases the mass of air flowing over the wing in unit time, and so reduces the induced drag.

There is no wholly satisfactory way of expressing the effect of changes in lift upon the drag in general terms. A wind-

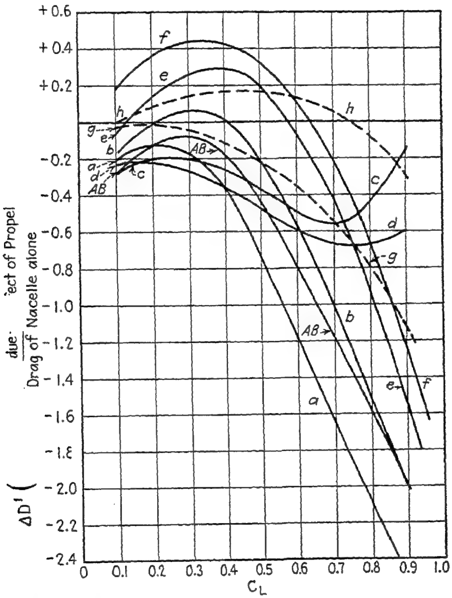


FIG. 279.—Apparent change in drag due to slipstream effect on lift, and its variation with nacelle type and location. (For the significance of the designations on the curves, see Figs. 275 and 278.)

tunnel test really applies only to the precise combination of wing and propeller, and to the precise angle of attack and propeller slip, for which it was made. But an approximation is possible by assuming in every case a speed of the fixed-pitch propeller used in the tests which would produce the same ratio of slipstream velocity to speed of flight that would actually exist in full-throttle operation of the controllable-pitch propeller at

the angle of attack for which the analysis is being made; and then crediting the nacelles with the difference between the actual drag of the combination, with propeller running, at the angle of attack in question and its drag at the slightly different angle at which its lift would be equal to the lift of the bare wing, without a propeller, at the angle of attack specified for analysis. By that method, the interference factors plotted in Fig. 279 were determined. They are given in the same units as were used in Figs. 276 and 277. This factor, of course, would become still more important if the ratio of propeller diameter to wing span were to be increased. That the effect in general increases at high angles is due not only to the increased effect of the slipstream on the apparent average lift coefficient there but also to the more rapid change of drag with lift in those attitudes.

The next step is to determine the power required to overcome the basic drag of the nacelle, including the interference between it and the wing as plotted in Figs. 276 and 277, and to subtract it out from the power delivered by the propeller. Since the nacelle is so integrally connected with the power plant, it seems appropriate to eliminate immediately the total power used in driving it through the air, and to treat the net propulsive efficiency as the ratio of the power left for useful work, after taking out what is lost in the propeller and what goes to overcome nacelle resistance, to the power originally supplied at the crankshaft.[1] The nacelle-drag power consumption is

$$P_n = \frac{C_{D_n}S_n(1 + k_i + k_i')\frac{\rho}{2}V^3}{550} \tag{79}$$

where $C_{D_n}S_n$ is the coefficient of total drag, k_i the interference factor as plotted in Figs. 276 and 277, k_i' the change in drag due to the effect of the propeller on lift, from Fig. 279, V the speed in ft. per sec., and P_n is in horsepower. Since the factor of present interest is the proportion of the power output that has to be used at the nacelle, this is to be divided by the engine

[1] The basic concept of subtracting out the power consumed in overcoming nacelle drag was used in the original N. A. C. A. reports. The method of reduction and presentation of the data there employed was, however, quite different from the present one, and it was applied only to two sets of flight conditions.

power. Substituting for P_n, changing the units so that V will be in m. p. h., and inserting the sea-level value for the air density, we have

$$\Delta\eta = 6.8\frac{C_{D_n}S_n(1 + k_i + k_i')}{P}\left(\frac{V}{100}\right)^3 \tag{80}$$

In the case of the cowled nacelle used in these tests, which was designed to accommodate a 220-hp. engine of the vintage of 1929 but would have been well able to take one of 400 hp. of 1935 model (the power output for a given diameter having been almost doubled in six years), $C_{D_n}S_n$ was 1.55, the overall diameter of the engine 44½ in. Substituting for $C_{D_n}S_n$, and putting 400 for P,

$$\Delta\eta(\text{in }\%) = 2.64(1 + k_i + k_i')\left(\frac{V}{100}\right) \tag{81}$$

$C_{D_n}S_n$, of course, varies, for a given type of nacelle, as the square of the nacelle diameter and so of the engine diameter. Since multiengined airplanes are generally of very high power, and since an 800-hp. radial can now be built with a diameter only about 20 per cent greater than that of one of 400 hp., the numerical coefficient in (81) has been further arbitrarily reduced to 2.50.

Upon that basis the calculations for Fig. 280 have been made, and upon the further assumption (necessary to tie the speed and the lift coefficient together and so to allow the transfer of data from Figs. 276, 278, and 279 to Fig. 280) that the wing loading is 20 lb. per sq. ft., Fig. 280 is derived simply by determining the value of C_L corresponding to each speed, determining from (81) and Figs. 276 and 279 the percentage of power that goes into driving the nacelle at each speed and for each type of nacelle and location, and subtracting the corrections so obtained from the gross propulsive efficiencies for the same conditions, read off from Fig. 278. Though the resultant curves strictly correspond only to a particular structure, wing loading, and so on, changes likely to be encountered in practice will make relatively little difference except that a reduction of wing loading will improve the relative standing of the positions below the wing (curves *e* and *f*) and that a material change of D_e^2/P from the value 4.7 (D_e being the engine diameter and P the engine power,

and the nacelle being assumed to be fitted as closely as possible about the engine in every case) will roughly slide every curve

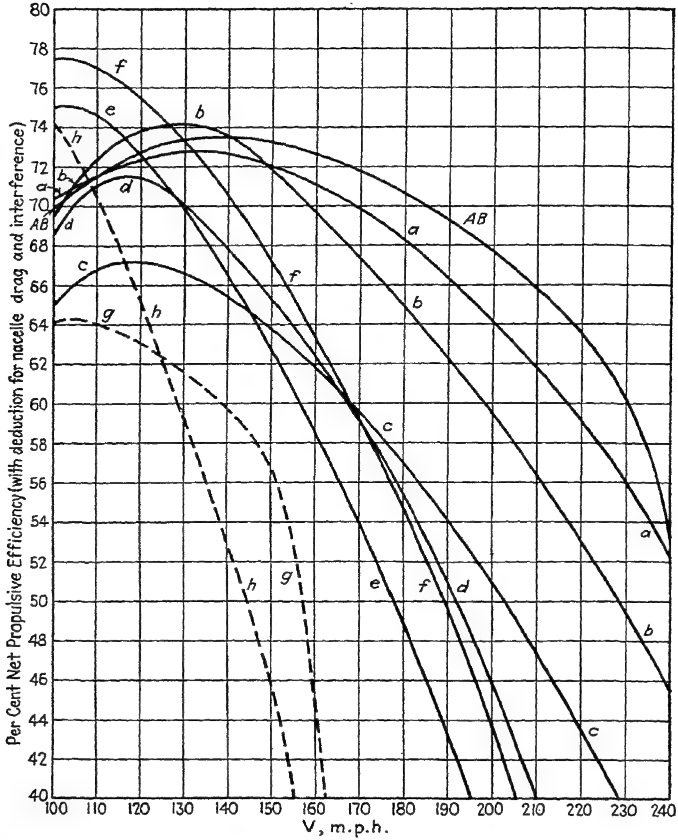


FIG. 280.—Variation of net propulsive efficiency with nacelle type and location. (For the significance of the designations on the curves, see Figs. 275 and 277.)

horizontally, changing the abscissa of every point in the ratio $\sqrt[3]{4.7P/D_e^2}$.

It must be borne in mind that the scale of abscissas in Fig. 280 is reversed from that used in Figs. 276 to 279. Since low speed goes with a high lift coefficient, points at the left of Fig. 280

correspond to those near the right of the curves in the previous figures, and *vice versa*.

The final curves show, too clearly to require explanation, the wide superiority of the cowled nacelle sunk in the wing over all other arrangements at high speeds; its gradually diminishing relative merit as the speed falls off, until at a speed of less than 125 m. p. h. (as in climbing) the location below and well in advance of the wing actually appears to take the lead; the low standing of a nacelle above the wing under all conditions; the hopelessness of attaining a high speed with an uncowled nacelle, since at 160 m. p. h. more than half the available power would have to be spent in propelling the nacelle. At 220 m. p. h. at sea level, approximately the top performance of the fastest multiengined planes now built, the substitution of a nacelle either above or below the wing for the best in-the-wing position would be equivalent to reducing the engine power by at least 30 per cent, without at the same time effecting any reduction of engine weight or fuel consumption.

The indications of Fig. 280 read directly only upon full-power sea-level operation, either at maximum speed or in take-off or climb. In cruising with partially closed throttle the effect is the same as that of reducing the engine power while keeping the engine diameter unchanged. To find the relative virtues of the various nacelle locations for cruising at 75 per cent of rated engine power, at sea level, and at a speed of 175 m. p. h., then, the readings should be taken along the ordinate at $175\sqrt[3]{1/0.75}$, or 192 m. p. h. Since the ratio of cruising speed to maximum speed is generally within 5 per cent of equalling the cube root of the cruising power ratio, this is equivalent to saying that the conclusions drawn for a particular airplane at maximum speed are equally valid under cruising conditions, whatever the cruising power. The effect of altitude upon the conclusions is small when the engine used is unsupercharged. If a supercharger capable of maintaining constant power is used, net propulsive efficiencies should be read off at an abscissa equal to the true speed times the cube root of the ratio of the air density at altitude to the standard sea-level density. The reason will be obvious from a reexamination of (79) and (81).

The curves so far plotted have exploited only a fraction of the material available in the first two segments of the N. A. C. A.

nacelle research. In particular, they have covered only seven of the twenty-one locations in which the cowled nacelle was tested, but the method here described can be used to extend the study at will.

The general conclusion that it is better to bury a nacelle, as far as possible, in the wing instead of leaving it fully exposed may appear one that should have been obvious without waiting for a special research. In a sense it was, but all practical experiments in that direction before 1930 had been discouraging in the extreme. The uncowled type of nacelle set into the wing disturbs the flow over the wing, and subsequently over the tail surfaces, to such an extent as to react very unfavorably on control. Unless the wing is exceedingly thick, furthermore, there is likely to be a bad effect on maximum lift; and, as the N. A. C. A. experiments as plotted in Figs. 278 and 280 show, the interaction of the wing and the uncowled nacelle reacts in turn upon the propeller so that the gross propulsive efficiency falls so much below its value for a nacelle position below the wing that the net efficiency is also lower at all speeds up to 125 m. p. h. Only with the coming of the ring and the NACA cowl, which smooth the flow out after its passage around the engine cylinders, did an installation in, or on a level with, the leading edge become practically feasible.

The effect of a nacelle on the maximum lift of the wing with which it is mounted depends, of course, very largely upon the aspect ratio and the airfoil thickness. In general terms it can be said that with the engine stopped or idling a cowled nacelle actually increases the maximum lift, usually by from 1 to 3 per cent, owing to the concentrated scouring effect of the jet of air emerging at high speed from the annular exit of the cowl; that the uncowled engine, on the other hand, decreases the maximum lift by from 3 to 10 per cent and that it is at its worst when it is installed in the wing. Moments escape even more nearly unscathed by nacelle influence. The combined effect of the nacelle, in anything approaching its best location, and the propeller is generally to produce a little nose heaviness with power on, a little tail heaviness with power off, but its magnitude seldom amounts to a change of more than 0.010 in moment coefficient on the wing.

Figure 281 shows how very small is the effect of airfoil section and of relative size of nacelle and wing. The same nacelle, mounted in the same relative attitude (the best position available) in two wings of wholly different characteristics, gives net propulsive efficiency curves of essentially the same form and lying within 3 per cent of each other throughout—further encouragement to take not only the general conclusions drawn from Fig. 280, but the values plotted there, as generally applicable.

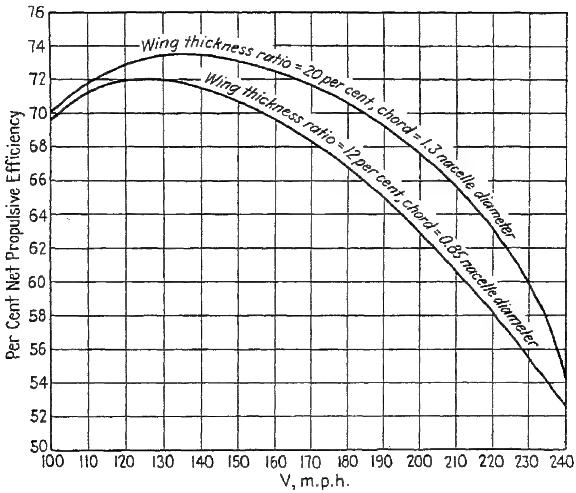


FIG. 281.—Change of net propulsive efficiency with airfoil thickness and chord.

With tandem propellers the best system of cowling has proved[1] to be a full NACA cowl over the forward engine, a ring with its chord set at a small positive angle over the rear one. Nothing else approaches it in efficiency. For location, there is little to choose between the positions shown in Fig. 282, for their peak net propulsive efficiencies lay within 1 per cent of each other. Positions above the wing were again, as when the single tractor airscrew was used, bad. The best net efficiencies in the best positions fell about 10 per cent below the best values for the single-engined installations. Tandem propellers always lose efficiency by mutual interference, and a tandem combination in

[1] *Rept.* 505, *cit. supra.*

the presence of a wing seems to be even worse than a tandem combination in isolation.

Only rarely do circumstances develop that give a designer the impulse to use tandem propellers, and when the efficiency

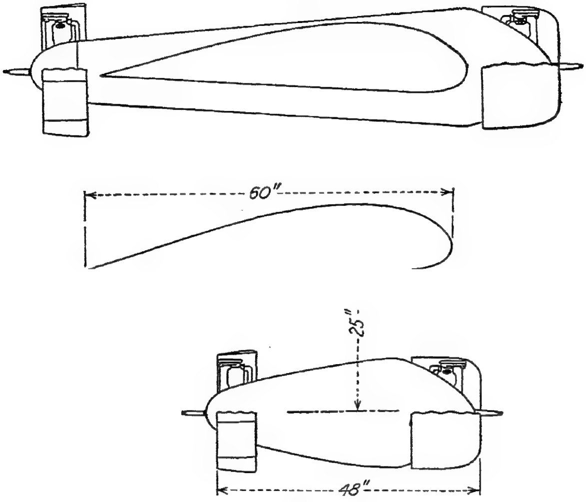


FIG. 282.—Alternative (and approximately equally desirable) installations for tandem propellers.

figures are seen the impulse is likely to be suppressed.[1] Of more frequent, though of rapidly diminishing, practical interest is the case of the multiengined biplane. Though it has been losing ground rapidly before the inroads of the monoplane in the heavy-bomber and large-transport field the biplane still finds a certain

[1] Exception to this must be made only in the case (most often encountered in racing planes and exemplified in the present holder of the world's speed record) where enormous power is to be stored in a fuselage of minimum dimensions and where a combination of structural and mechanical factors and of the difficulties of control resulting from so large an unbalanced torque make it undesirable to take all the power off through a single shaft, or in that (very rare in the light of modern knowledge) where a thrust applied far off the center line of the plane is thought likely to have so bad an effect that four or more engines must be arranged in tandem pairs in nacelles rather than spread out over a wider span by spacing them out transversely as individual units.

place, particularly in British military practice. Again research proves[1] that the NACA cowl is the best of drag-reduction devices. Again it proves that the nacelle ought to be set on a level with the wing and, so far as its thickness permits, buried in it. Which wing is used makes little difference. A nacelle location in the upper wing shows a net efficiency about 2 per cent higher than the corresponding position in the lower one, but the practical convenience of a lower-wing mounting would be likely to tip the scales in its favor except in the case of a seaplane, where water clearance is a major factor.

Whatever may be the disadvantages of the biplane for large aircraft, low propulsive efficiency is not among them. Its net efficiency, for the best nacelle location, ranges from 2 to 4 per cent higher than the best values for the same nacelle and propeller combined with a single wing.

Floats and Hulls.—Seaplane floats are like fuselages in their general resistance characteristics, and flying-boat hulls, of course, serve the function of a fuselage as well as that of a float. A number of tests of seaplane-float resistances have been made[2] in the course of ordinary wind-tunnel routine, and some of the results are shown in Fig. 283. The tests having been run at a Reynolds' number of only 600,000, and separation undoubtedly being a factor on the bottom of the float behind the step if not elsewhere, the measured values of C_D have been reduced by 15 per cent. So reduced, as presented in Fig. 283, they can be applied directly in full-scale calculations. The drag coefficient for the average float is about equal to that for the average fuselage with a properly cowled radial engine in the nose, the gain due to the absence of excrescences at the nose being counterbalanced by the loss due to the large fineness ratio of the floats. The coefficient is lowest, fineness ratio being equal, for those floats of which both nose and tail appear as sharply pointed in at least one of the two

[1] *Rept.* 506, *cit. supra.*

[2] "Wind Tunnel Data for Eight Seaplane Floats," by R. M. Bear, *Aero. Rept.* 281, Bur. Construction and Repair, Washington Navy Yard U. S. Navy, May, 1925; "Nouvelles recherches sur la résistance de l'air et l'aviation," by G. Eiffel, p. 257, Paris, 1915; "Untersuchung von fünf Flugzeugschwimmern," by Dr. L. von Prandtl, C. Wieselsberger, and A. Betz, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," vol. I, p. 130, Munich, 1921; "Tests of a Model Seaplane," by L. W. Bryant, H. B. Irving, and W. L. Crowley, *R. and M.* 199.

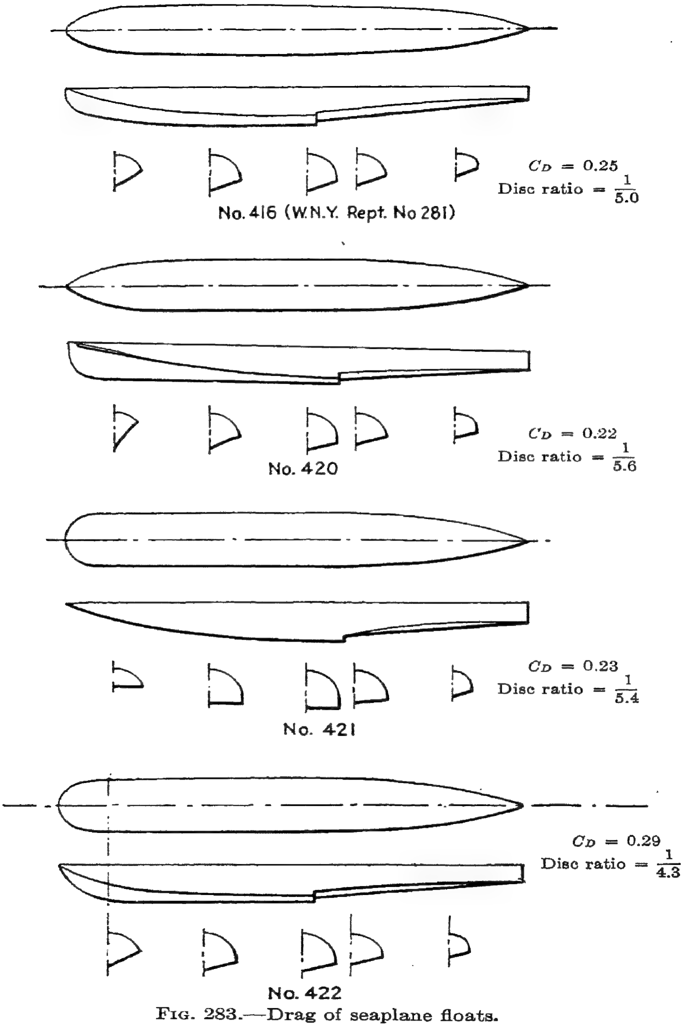
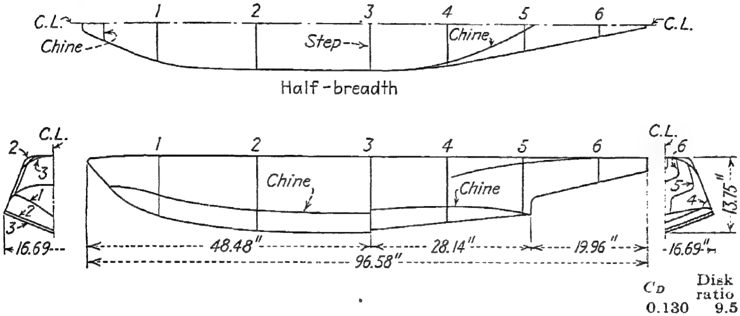


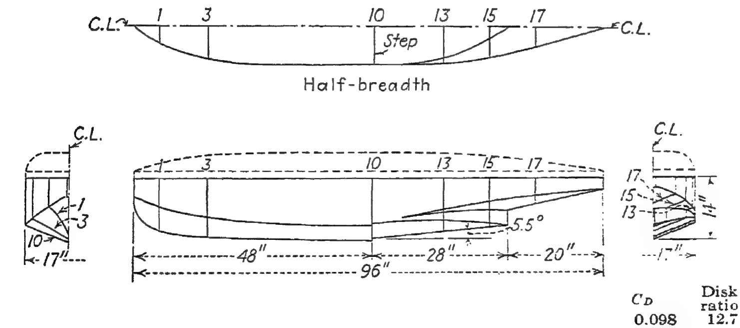
FIG. 283.—Drag of seaplane floats.

views and in which the discontinuity of form of profile at the step is a minimum.

The nearly flat bottom of a float is of course capable, too, of producing a considerable lift. With their upper surfaces at an angle of attack of +6 deg., the forms shown in Fig. 283 give a lift averaging a little over 1.5 times their minimum drag.



A



B

Fig. 284.—Drag of flying-boat hulls.

Flying-boat hulls having a large superstructure can be made of very fair form except for the interruptions at the cockpits. Four forms tested in a full-scale wind-tunnel at a Reynolds' number of about 7,500,000 are covered in Figs. 284 and 285,[1]

[1] "The Aerodynamic Drag of Flying-boat Hull Models as Measured in the N. A. C. A. 20-foot Wind Tunnel," by Edwin P. Hartman, *Tech. Note* 525. See also "Air Force for Hull of Curtiss *NC*-1 Flying Boat," *Rept.* 64, Bur. Construction and Repair, Washington Navy Yard, U. S. Navy, March,

and the drag coefficients and disk ratios are tabulated there. The smoothly curved form and rounded sections of a hull give excellent aerodynamic results, despite the discontinuity on the lower surface formed by the step or steps. To a remarkable degree, drag proves to be a function of fineness ratio. When that quantity, taken as the ratio of overall length to the square

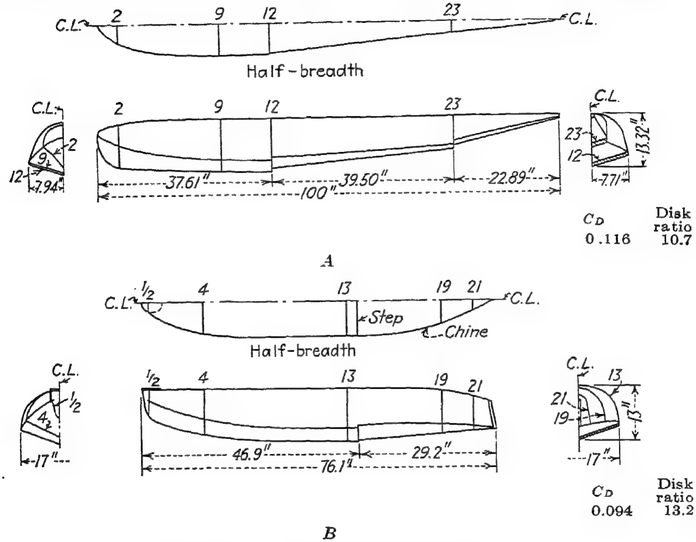


FIG. 285.—Drag of flying-boat hulls.

root of the cross-sectional area, is around 8, C_D runs around 0.12. Lower the fineness ratio to 6, and C_D comes down to 0.10. The general use of long tails and high fineness ratios (9 or more) in British hulls accounts for their generally high drag coefficients (0.11 to 0.14)[1] as compared with the American figures.

Hulls are somewhat more responsive to inclination in pitch, on the average, than are fuselages. On the average, they show a 60 per cent increase in drag for a 10-deg. inclination from the

1918; "Air Force for Hull of Navy *F-5L* Flying Boat," *Rept.* 83, Bur. Construction and Repair, Washington Navy Yard, U. S. Navy, August, 1918; "The Air Resistance of Flying Boat Hulls," by R. Jones and G. N. Pell, *R. and M.* 461.

[1] "Handbook of Aeronautics," vol. I, pp. 141–142, London, 1934.

attitude of minimum resistance, and a parabolic variation for other angles. The corresponding average coefficient of increase for the float forms shown in Fig. 283 is 100 per cent for 10 deg.

Two open cockpits, one near the nose of the hull and the other behind the wings, may increase the drag as much as 40 per cent. The two windshield arrangements shown in Fig. 286 produced increases of 4 and 25 per cent, respectively—a fresh reminder of the vital importance of superficially minor details of form, especially near the nose of a low-resistance body. Another inter-

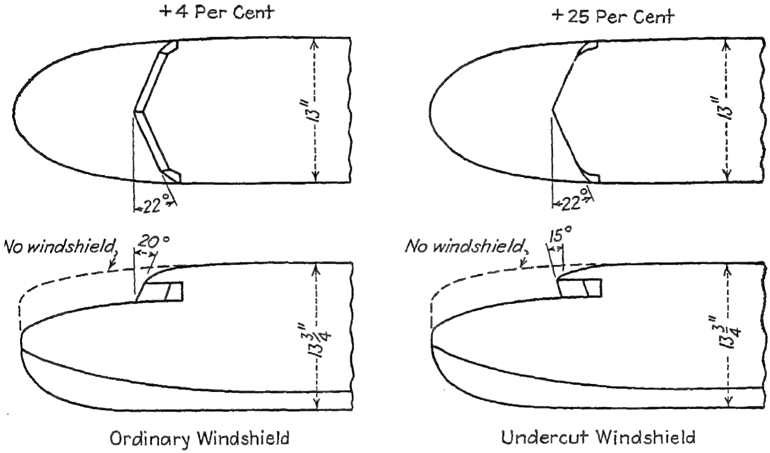


FIG. 286.—Windshield arrangements for flying-boat hulls.

esting essay in modification, made on the basic form of Fig. 284(*b*), showed a 10 per cent reduction of drag when the step (which had a height of 0.4 in. on the model) was doubled in height, a 10 per cent reduction when it was completely eliminated. The last figure is lower than might have been anticipated, but the sudden break in contour that still existed at the step location evidently had much of the effect of an actual jog in the surface in breaking up the streamline flow.

Struts.—Struts have much in common with streamline bodies, and they differ from most other airplane parts in that they need no apertures or excrescences to break up the smoothness of their form. The principal difference between the body of low resistance and the strut, which makes it unsafe to assume that the

section that is aerodynamically best for one will be best for the other as well, is that the flow is two-dimensional in one case, three-dimensional in the other. The difference is like that between the cylinder and the sphere and, as in the case of cylinders, the resistances of struts are commonly measured in the laboratory in such a way as to represent characteristics of a strut of infinite length, "end effect" being eliminated. The results of strut tests from different laboratories are more directly comparable than for the radially symmetrical forms, as the problem of the interference of the support used to carry the model in the wind tunnel does not enter with anything like the same importance that it has in the other case. It is still unsafe, however, to assume interchangeability of data from different sources.

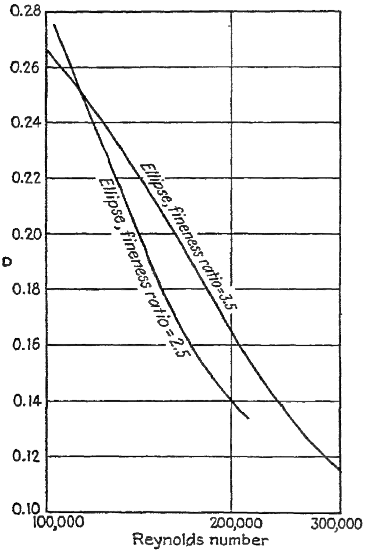


FIG. 287.—Drag of elliptical struts.

The basic strut form is the circular cylinder. The first approximation to a fairer shape is the strut of elliptical section, and curves of C_D against Reynolds' number for two such struts (of different fineness ratios) are given in Fig. 287.[1] Within the range covered by these tests, C_D varies a little more rapidly than in proportion to the square root of N, and more rapidly for the low fineness ratio than for the high one—so establishing fair agreement with the laminar-boundary-layer rule and with general experience.

From the ellipse we proceed at a single stride to the best forms now known. To permit a suitably intimate analysis of the relation of contour to drag, the device of plotting a curve of differences between the ratio of thickness at each point to maxi-

[1] "Forces on Elliptic Cylinders in Uniform Air Stream," by A. F. Zahm, R. H. Smith, and F. A. Louden, *Rept.* 289.

mum thickness for the strut under investigation and the corresponding ratio at the corresponding points for an arbitrarily

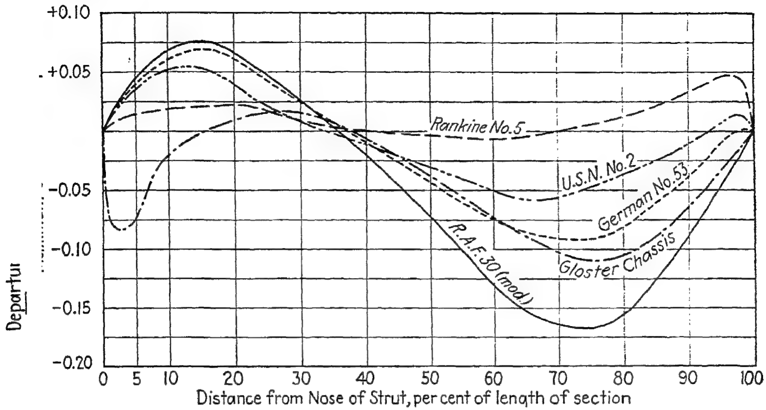


FIG. 288.—Geometrical form by comparison with "standard" form, of strut forms.

chosen "standard" contour will again be used. The form described on pages 348 to 350 and illustrated in Fig. 229 will

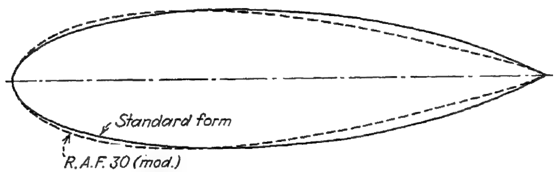


FIG. 289.—"Standard" strut form and one form in current use.

again serve the purpose. In Fig. 288 are plotted the difference-of-thickness curves for several of the best struts of which there is record.[1] To establish a basis of comparison between these

[1] From "Wind Tunnel Tests of Seven Struts," by A. S. Hartshorn, *R. and M.* 1327; "Resistance of Certain Strut Forms," by R. Warden, *R. and M.* 1599; "Aerodynamic Theory and Test of Strut Forms," by R. H. Smith, *Repts.* 311, 335. See also, among many others, "Experiments on the Forces Acting on an Aeroplane Strut," by J. R. Pannell and T. Lavender, *R. and M.* 183; "The Resistance of Struts," by C. H. Powell, *R. and M.* 416, (a general summary and compilation); "Characteristics of Stream-line Strut Sections," by R. B. Beisel and W. S. Diehl, *Tech. Note* 117, Bur. of Aero., U. S. Navy; "Point Drag and Total Drag of Navy Struts No. 1 Modified," by A. F. Zahm, R. H. Smith, and G. C. Hill, *Rept.* 137.

difference curves and the actual appearance of a section, too, the standard curve and the curve for that one of the five sections illustrated in Fig. 288 that differs most widely from the standard are superposed in Fig. 289.

In Fig. 290 are drawn the C_D curves for the five. At first sight two of the sections appear very much superior to the other

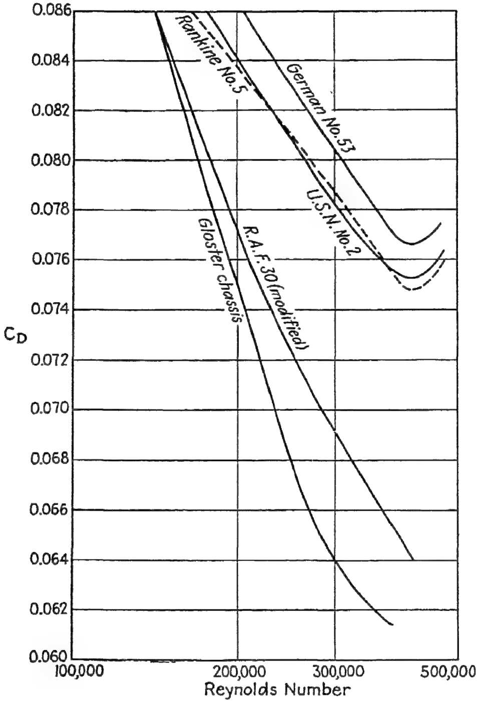


FIG. 290.—Drag characteristics of struts (see Fig. 288 for forms).

three, but the pair showing the lowest values were tested in England, the other three at the Washington Navy Yard, and the indications of lack of comparability are strong. In particular, the German section (No. 53) that was tested at Washington has a nose form almost exactly like one of the two British struts, a tail form almost exactly paralleling that of the other, and it is quite impossible to believe that its resistance is 20 per cent above

that of either of the forms from which its components were, substantially, drawn. Failing further evidence to the contrary, the five forms may be considered of the same order of merit, with the Rankine 5, the U. S. Navy No. 2, and the "Gloster chassis" (developed for use on one of the British machines in the Schneider Trophy race) slightly superior to the others. There are, of course, an immense number of available tests on sections that have been found far inferior to any of these. Figure 290 skims the cream of experience. Its indication is that with a carefully chosen form C_D can be kept down to about 0.070, double the value for the best streamline forms (the fee of two-dimensionality), roughly two-thirds of the minimum for the best elliptical strut, a quarter of that for the circular section.

It is apparent, too, that the standard form plotted in Fig. 289 and again, to a different fineness ratio, in Fig. 238, comes very near indeed to the ideal in contour. It has an advantage over some of the low-resistance sections (notably the Gloster chassis) in that its lines are fuller, it is more nearly symmetrical about its mid-section, and it therefore offers better structural properties. The function of a strut, in practically every case, is to carry a compressive or a bending load or a combination of the two. The greater the moment of inertia for a given width of section, then, the better. And in that respect, of width of section required for a given minimum moment of inertia of cross section, either the standard form or the R. A. F. 30 with its very full lines at the nose has a 3 per cent advantage over the Gloster chassis strut.

A comparison of Figs. 288 and 290 suggests that unduly full lines near the nose (rising to more than 5 per cent above standard width at from 10 to 20 per cent of the way back from the nose) ought particularly to be avoided. Very fine lines near the tail, on the other hand, seem to be of no particular benefit.

Disk ratios can be calculated for strut forms if it is desired that all parasite resistance be transmuted into terms of the equivalent area of flat plate, and for the best forms at the value of N that gives minimum resistance the ratio will be found to be about 1/17. It is commoner practice, however, to deal with running length of strut rather than with projected area, and a convenient quantity is the resistance per 100 ft. of strut 1 in. wide at a speed of 100 m. p. h. It is numerically equal to 212 C_D, at sea-level, and for the best strut forms it drops to 15.

Although disk ratio is a concept not very often used with struts, it is of some interest to introduce it in modified form as a means of comparing strut and body resistance. A direct comparison would manifestly be unfair as a measure of the relative efficiency of streamlining a two-dimensional and a three-dimensional flow. When the performance of the best strut is compared with that of a normal flat plate around which the flow is two dimensional (infinite aspect ratio), the resistance ratio is found to be approximately 1:36,[1] almost exactly equal to the corresponding ratio between the best streamline bodies so far developed and a circular disk.

Singularly little work has been done on struts of good form and low fineness ratio. Most of the best forms so far tried have fineness ratios of between 3.25 and 3.6, but there is a strong probability that further research would develop the possibility of a further reduction of drag with a fineness ratio of 3 or a little less, at least at Reynolds' numbers of over 1,000,000. Scale effect is a serious matter, for, unlike bodies, struts usually have to work in the region of transition in the boundary layer. Fuselages, and even nacelles, almost never operate in practice at an N of less than 4,000,000, and values of 10,000,000 and up are the common rule. The length of a strut section, on the other hand, is usually from 4 to 12 in.—its Reynolds' number from 120,000 to 1,750,000. Extension of the experimental curves into that region ought to be an early obligation on some wind tunnel capable of reaching the required speeds. Pending the accumulation of more data, it must be assumed that the general form of the skin-friction curve (Fig. 230) holds good, and that C_D will rise gradually with increasing N to about 1.5 times its first minimum value[2] at a Reynolds' number of around 2,000,000. The rise being very slow at first, a value 10 per cent above the experimentally determined minimum can safely be used for all Reynolds' numbers from 500,000 to 1,250,000. It must always be remembered that N has to be computed in terms of the length of the strut section, not its width.

Where tubes of circular section are exposed to the air, N is unlikely to exceed 300,000. Since that lies below the critical

[1] Values for the flat plate of infinite aspect ratio taken from *Tech. Note* 121, *cit. supra*.

[2] As shown in the Washington Navy Yard curves in Fig. 290.

value for the change of the separation point,[1] C_D for such a tube ought to be taken at about 1.1 rather than at the minimum value of 0.3. Instead of having only four times the drag of the best strut, as a comparison of minima would suggest, the ratio is more than 15 to 1. Obviously circular sections are to be shunned.

Scale effect is of course interrelated with surface condition, and the experiences with cylinders that were reviewed in Chap. VI[2] have been essentially repeated for struts by Wieselsberger.[3] Since there are no serious separation problems on a good strut section, and since the object therefore is to delay as long as possible the appearance of turbulence in the boundary layer and the start of the migration from the low-friction toward the high-friction regime, the highest possible degree of surface polish is of more importance on struts than on any other part of an airplane.

Fairing of Tubes.—Where circular tubes are used, it should be possible to cut some 90 per cent of their drag by fairing them as

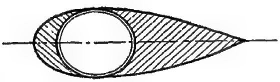


FIG. 291.—Fairing for circular tubes.

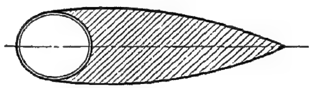


FIG. 292.—Simplified fairing for circular section.

indicated in Fig. 291. In practice, however, it is very undesirable to have to build a fairing up in two pieces to be fitted on either side of a tube, and the attempts at drag reduction are commonly limited to the application of a shaped piece of wood or metal behind the tube. The forward part of the strut section must then have a constant radius of curvature, and experiments in Great Britain[4] have borne on the determination of the best form for

[1] See p. 135.

[2] P. 139.

[3] "The Effect of the Nature of Surfaces on Resistances as Tested on Struts," by Dr. C. Wieselsberger, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Feb. 28, 1920, translated in *Tech. Note* 33; "Similitude Tests of Model Balloons and the Effect of the Nature of Surfaces," by Dr. C. Wieselsberger, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, p. 125, 1915.

[4] "An Investigation to Determine the Best Shape of Fairing Piece for a Cylindrical Strut," Pt. I, by W. L. Cowley, L. F. G. Simmons, and J. D. Coales, *R. and M.* 256; Pt. II, by L. F. G. Simmons and R. A. Frazer, *R. and M.* 381.

such a fairing. What appears to be the best simple shape available, though the tests were limited to low values of N, was made up of circular arcs having a radius approximately eleven times that of the tube to which they were fitted. The overall length of the resultant section is about three and a half times the tube diameter, as shown in Fig. 292. The drag is largely guesswork in view of the limited scope of the tests, but it would probably reach a minimum of about 0.10.

The relation of radius of curvature of the fairing to its length for the best form is such that the point of tangency between the tube and the fairing lies forward of the diameter of the tube, requiring either that the fairing be made in two pieces or that a gap at its leading edge be closed in with some other material or covered over with fabric. Those expedients are often undesirable, and there is therefore especial interest in the best fairing that can be made up out of parts tangent to a tube at the ends of its diameter or behind those points. No specific tests have been made to cover that point, but by extrapolation from existing results it appears that it would again be best to make the fineness ratio about 3.5, using on the sides of the fairing a radius of approximately eight times the diameter of the tube, and that the drag coefficient for a tube so faired should be about 0.15 at the minimum. Even that simple form of fairing, ideally made and applied, would suffice to reduce the resistance of a tube by approximately 85 per cent. In actual practice the gain is not likely to be quite so great as that, because of the surface irregularities almost sure to exist at the points where the fairing is attached to the tube.

Strut Interference.—Struts do not always stand alone. Occasionally they must be placed close together side by side (most often when a control member must run from a wing to the fuselage close alongside a strut). Occasionally they must be located in tandem, and near enough to influence each other. Very often, two or more struts must meet at an angle. Those cases too must be the subject of study.[1]

Placed side by side, their interference becomes measurable with a clear space between them of four times the width of a single strut, important with a gap of 2½ times the width. With

[1] "The Interference between Struts in Various Combinations," by David Biermann and William H. Herrnstein, Jr., *Rept.* 468.

an open gap of seven-eighths of the width, interference doubles the combined resistance of the two struts. What happens after that does not matter, for with that much information before him as a warning no sensible designer will allow any such condition to exist. Either struts will be kept more than three widths apart, or one will be made smaller than the other and stowed away inside its tubular companion.

Two struts in tandem have a trifle (never more than 50 per cent) more drag than the sum of the two operating independently. With a clear gap of five times the strut width, interference still increases the combined resistance by 10 per cent. When the

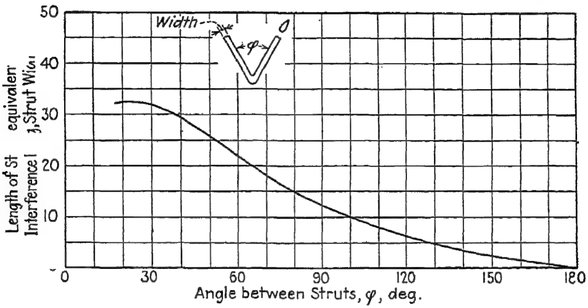


FIG. 293.—Interference drag where struts intersect.

gap is eight times the width or less it becomes profitable to enclose the struts within a single fairing either by placing a flat surface along either side of the pair of struts, to make an enclosure of constant thickness except at the nose and tail, or by bulging the surface outward to make a single streamline form of high aspect ratio. With a clear gap of three times the strut width the first arrangement reduces the combined drag to 1.75 times that of a single isolated strut; the second type of fairing to 1.1 times.[1]

In the very practical case of the intersection of two struts at an angle, the answer is given by Fig. 293. The interference is best expressed in terms of the length that must be imagined added to the total length of strut, were it operating without interference, to give a total drag equal to the actual figure includ-

[1] For the application of a similar process to round members, see p. 435, *infra*.

ing interference. How much more interference would have to be allowed for where two struts cross each other in an X or where two or more meet at an angle immediately adjacent to the surface of a wing there are no direct tests to show, but presumably the effect would be at least doubled. As the typical strut in a wing-bracing system has a length of about eighty times its width, the total effect of interference on a truss with two series of intersecting interplane struts may increase the total strut drag by about 20 per cent as compared with the drag of the same length of strut in complete innocence of all contiguous influences. To put a fillet between the struts at their meeting point or between a strut or a wing has a somewhat uncertain effect but generally a favorable one—especially in the latter of the two cases. A fillet of a radius about equal to the width of the strut section is recommended, and one carefully faired off to allow the air to flow on an easy curve over the fillet itself.

Effect of Inclination of Struts.—It is occasionally necessary to set a strut with the axis of its section inclined at a small angle to the wind direction, and any strut is likely to be so inclined momentarily when the airplane of which it is a part sideslips for a turn. An angle of yaw of 2 deg. generally increases the resistance of a strut by from 3 to 7 per cent, an angle of 6 deg. by from 30 to 45 per cent. The accurate alignment of a strut with the local relative wind is therefore of some importance where minimum resistance and maximum performance are sought.[1] The cross-wind force (or lift, if the strut is horizontal and the angle is one of pitch) that the strut contributes can be computed by considering it as an airfoil. The area must then be taken as the length of the strut times the length of its section, rather than the width, or as the area as ordinarily considered times the aspect ratio. On that basis, the cross-wind force coefficient changes at a rate of from 0.085 to 0.105 per degree, with an average of 0.090, about 20 per cent below the theoretical slope of the lift curve for a thin airfoil.[2]

In addition to having the section axis inclined to the relative wind, struts may be set with their longitudinal axes inclined at some angle other than 90 deg. to the wind direction. In a

[1] In addition to the strut reports previously cited, see "Wind Tunnel Tests of Five Strut Sections in Yaw," by Edward P. Warner, *Tech. Note* 167.

[2] See p. 72, *supra*.

staggered biplane, for example, the interplane struts are almost always set at a slope, and the same is true of landing-gear struts in general, the wind striking the member obliquely. The effect, of course, is to change the section around which the air flows, making it an oblique section through the strut and increasing the apparent fineness ratio (the inclined section of a circular tube, for example, becoming an ellipse), and also to make the flow a three-dimensional one, the air tending to be guided along the strut by the inclination.

It might be expected that the effect of inclination would be to reduce the resistance per unit of area projected on a plane per-

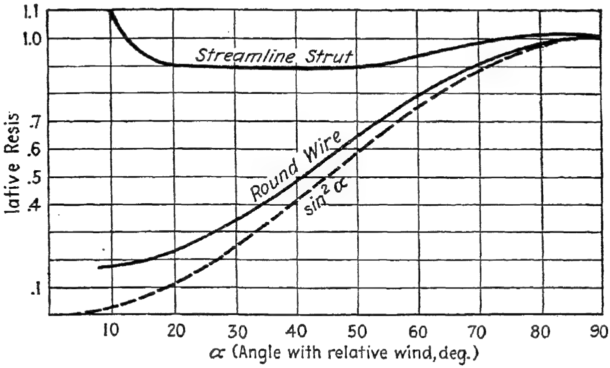


FIG. 294.—Effect of inclination on drag of struts and wires.

pendicular to the wind direction, and that such reduction would be much greater for a circular cylinder than for a true strut form, as the benefit from changing a circular section to an apparent elliptical one is much greater than the gain from increase of fineness ratio of a fairly good streamline shape. Experimental results[1] verify the expectation. Curves of relative resistance per unit of projected area are plotted in Fig. 294 for a good strut section and for a circular cylinder, and it will be observed that the figure for the streamline section is nearly constant, while that for the cylinder follows closely the curves of $\sin^2 \alpha$, also plotted for purposes of comparison. If the curve per unit length of strut,

[1] "The Resistance of Inclined Struts," by C. H. Powell, *R. and M.* 599; "Tests on Smooth and Stranded Wires Inclined to the Wind Direction," by E. F. Relf and C. H. Powell, *R. and M.* 307.

instead of per unit of projected area, were desired, it could of course be found by multiplying the ordinates in Fig. 294 by sin α.

The resistance for the circle falls off somewhat more rapidly than it should if the change of apparent section alone were taken into account, showing that the introduction of a component of flow along the axis of the strut must be of some importance. At a 45-deg. inclination, for example, the apparent section is an ellipse of eccentricity 1.41. Such tests as are available indicate that an ellipse of that form would have a resistance not less than two-thirds that of the circle, yet the actual resistance of the strut at a 45-deg. angle is almost exactly half that of a circular section of the same projected area normal to the wind.

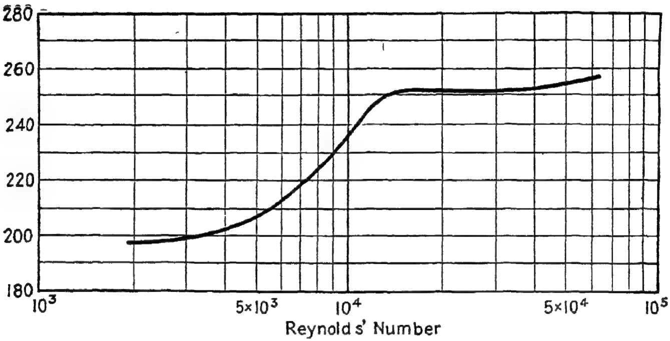


FIG. 295.—The drag of smooth round wires.

Wires and Cables.—The transition from struts to wires is an easy and in fact an imperceptible one, as a solid round wire is aerodynamically indistinguishable from a circular tube strut. The resistance coefficient of such a wire can therefore be read directly from the curve for cylinders. For wires as for struts, however, it is most convenient to use somewhat specialized units, and the part of the resistance curve corresponding to ordinary design practice (N ranging from 2,000 to 60,000) has been redrawn in Fig. 295 in terms of pounds of resistance per hundred feet of length per inch of width at 100 m. p. h. The left-hand extremity of the curve corresponds to a $\frac{1}{16}$-in. wire at 45 m. p. h.; the right-hand to a $\frac{3}{8}$-in. diameter at 200 m. p. h. For still higher values of N (up to 200,000) a uniform figure of 250 may be used without serious error.

The use of solid wire of round section in exposed positions on an airplane is comparatively rare in present-day practice. When round wire is used, it is much more likely to be stranded cable, and that, of course, has a different resistance coefficient. It is indeed hardly possible to treat the problem of stranded cables with aerodynamic rigor except by making tests over a considerable range of N on every size of cable that is likely to come into use, for it is seldom that two different sizes are exactly geometrically similar. Such tests as have been made,[1] including in addition to studies in wind tunnels some resistance measurements with the cables submerged in water and moving through it,[2] have, however, given resistance coefficients not very different from those for smooth cylinders, and there does not seem to be so much difference as might be expected between cables with different numbers of strands and different types of stranding. It must be borne in mind, however, that none of the wind-tunnel tests has been carried to Reynolds' numbers above 8,000. At high Reynolds' numbers (100,000 or over) the roughness of the surface ought to promote the transition of the type of flow in the boundary layer which leads to delayed separation and to the rapid drop of C_D.[3] In the range of N from 100,000 to 400,000, therefore, cables might be expected to show less resistance than smooth wires—less than half as much, perhaps, at around 200,000. The conclusions on the desirability of surface smoothness are radically different from those previously drawn with respect to struts[4] because separation, a negligible factor in the one case, exercises a dominant control over the magnitude of the drag in the other.

Streamline Wire.—The aerodynamics of round wires and cables in exposed positions need not engage us at length in any case, for they are now virtually obsolete. Wires of "streamline,"

[1] "Tests on Smooth and Stranded Wires Inclined to the Wind Direction and a Comparison of Results on Stranded Wires in Air and Water," by E. F. Relf and C. H. Powell, *R. and M.* 307; "Expériences sur les cables," by G. Eiffel, "Nouvelles recherches sur la résistance de l'air et l'aviation," p. 100, Paris, 1914; "Tests of Smooth Wires and Cables at Göttingen Laboratory," *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Oct. 29, 1910.

[2] "Tests by Sir John Thornycroft," worked up by G. S. Baker and included in *R. and M.* 307, *cit. supra*.

[3] See pp. 132–135, *supra*.

[4] P. 423, *supra*.

or more often of simple lenticular or elliptical, section have captured the field, except in certain cases where the individual cable is so large in diameter that it can conveniently be provided with an individual fairing such as was discussed on page 423.

The ideal streamline-wire form would, of course, be identical with the ideal for a strut. But practical considerations of what can readily be rolled from high-strength alloy steels, and particularly the certainty that a wire of which the section comes to a sharp point or a very small radius of curvature would crack under

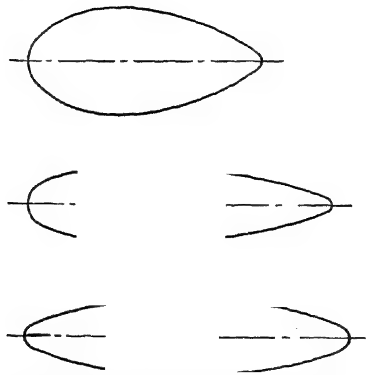


Fig. 296.—"Streamline"-wire sections.

the influence of vibration, impose limitations. In practice such forms as those shown in Fig. 296[1] are most often used, the doubly symmetrical or lenticular form (*C*) being the commonest of the lot. Their drag coefficients are plotted against Reynolds' number, the linear dimension in N again being the length of section, in Fig. 297. The standard form of streamline wire has a minimum disk ratio of about $\frac{1}{4}$, roughly four times the minimum drag of the best strut forms and a third that of a round section. The adoption of such a section as *B* would further reduce the resistance, by as much as 50 per cent on large wires or at very high speeds, but dissymmetry with respect to the lateral axis

[1] From "The Drag of Streamline Wires," by Eastman N. Jacobs, *Tech. Note* 480. See also, although the tests that it contains are limited to much lower Reynolds' numbers, "Wind Forces on Wires of Various Sections," by W. L. Cowley and J. D. Coales, *R. and M.* 256.

materially increases the mechanical difficulties of fabrication and use.

The drags of wires of various forms obviously ought to be specified in terms of sectional area rather than of width of section if they are to be directly indicative of the relative aerodynamic merits of the several forms, for it is sectional area that determines the size of wire that has to be used at any particular point in the structure. On that basis, the drag per 100 ft. of length per sq.

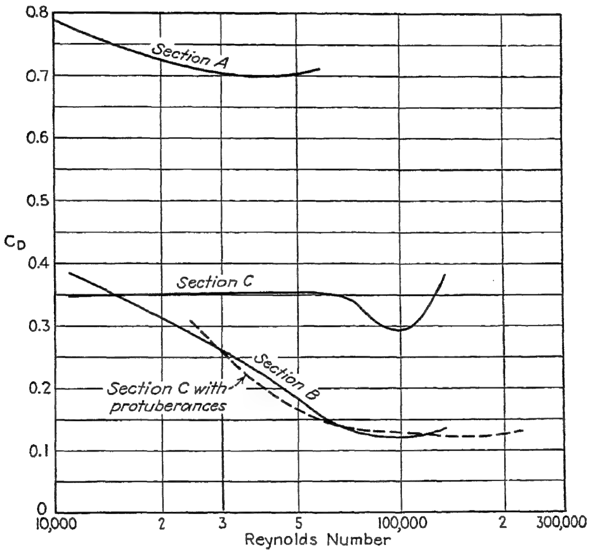


FIG. 297.—Drag coefficients of streamline wires.

in. of cross section at 100 m. p. h. (N being taken for a wire of 0.04 sq. in. sectional area and a speed of 200 m. p. h.) is 280 lb. for a round wire, and 125, 17, and 42 lb., respectively, for the three forms illustrated in Fig. 296.

In addition to the curves for the three typical wires in their normal forms, Fig. 297 contains a fourth, illustrative of the possibility of putting the effect of boundary-layer turbulence on air flow to practical use. It is the curve for a wire of section C (Fig. 296), with protuberances having a diameter of 0.004 times the length of the wire's cross section fixed as indicated in Fig.

298 and running the whole length of the wire. As in the case of the roughened circular cylinder,[1] the imposition of irregularities of surface proves actually to reduce the drag over a certain range of N, the suppression of separation by the forced introduction of turbulence into the boundary layer more than overbalancing the direct increase of frictional drag due to the increased roughness of the surface, and in the present case the range of N affected includes the entire range of practical utility. No doubt the addition of the protuberances of section would increase the drag at Reynolds' numbers above 400,000, but streamline wires never operate at such values.

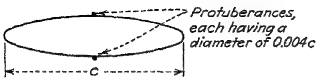


FIG. 298.—Lenticular wire with protuberances to induce turbulent flow.

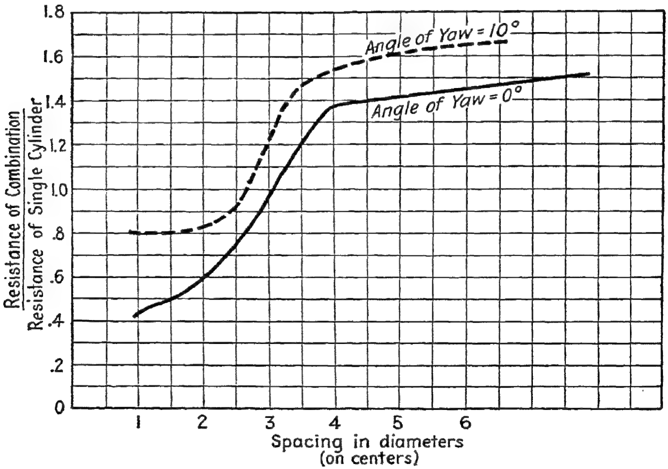


FIG. 299.—Drag characteristics of wires in tandem.

Shielding and Fairing of Round Wires.—It is very often the case, especially in the wing truss of an airplane where duplication of bracing is desired for safety, that one wire is placed close behind another. The resistance then is not equal to the sum of the two, but to a smaller force, as the rear wire is shielded by the front one.

[1] P. 140, *supra*. See also the deductions on the effect of roughness on struts (p. 423) and on round wires (p. 429).

The placing of two circular wires in tandem also facilitates fairing by a strip of wood placed to fill the gap between them.

The shielding effect with two circular wires in tandem is so important that if the gap between the two is small their combined resistance is actually considerably less than the resistance of a single wire. Only when the clear gap between the two wires exceeds two diameters does the resistance equal that of a single wire, and with a gap of five diameters the combined resistance is still 25 per cent less than it would be if the wires were entirely independent of each other. The curve of relative resistances in Fig. 299 is plotted from the average of two sets of tests in different laboratories.[1]

If the full saving in resistance is to be realized it is, of course, important that the wires remain directly in tandem with respect to the relative wind. A 10-deg. angle of yaw, equivalent to a lateral displacement of one of the wires by approximately one-fifth of the distance between them, increases the resistance ratios to the values shown by the dotted curve in Fig. 299.

As would be expected, the shielding effect of lenticular wires is of much less importance. When two such wires are placed in direct contact with one behind the other the combined resistance is almost exactly equal to that of a single wire, but with a clear separation between the two equal to the length of section of one of the wires the saving of combined resistance by the shielding action is but 13 per cent.

Drag can be still further reduced by the use of a fairing, filling the gap between the front and rear wires of a pair in tandem. The experiments of Wong and Yu[2] applied to the filling of the

[1] "Air Resistance of Cylinder Combinations," by T. Wong and T. Yu, a thesis in aeronautical engineering at the Mass. Inst. Tech., abstracted by T. H. Huff, *Aviation*, p. 395, June 1, 1917; "Experiments on the Interference between Pairs of Aeroplane Wires of Circular and Lenticular Cross-section," by J. R. Pannell, E. A. Griffiths, and J. D. Coales, *R. and M.* 206. Similar tests by the N. A. C. A., reported in "The Interference between Struts in Various Combinations," by David Biermann and William H. Herrnstein, Jr., *Rept.* 468, indicate resistances from 10 to 20 per cent lower than those given by the curve for all separations of from three diameters up. The N. A. C. A. tests included nothing on the effect of yaw, however, and the curve has been plotted from the earlier data for the sake of the yaw comparison.

[2] *Aviation*, June 1, 1917.

gap with a flat-sided strip, such as is shown in Fig. 300, and for all spacings of the wires from one diameter up to seven the resistance of the combinations so faired was almost exactly half that of a single wire. The work was done at Reynolds' numbers (based on length of the faired section) of from 40,000 to 150,000 and the result corresponds, using unit area as a basis of comparison, to 86 lb. per 100 ft. of length per sq. in. of area at 100 m. p. h.,

FIG. 300.—Fairing of the gap between wires in tandem.

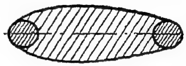


FIG. 301.—More elaborate fairing for tandem wires.

a figure only twice as large as that for a standard lenticular wire.

British experiments bearing on the same point[1] have shown the possibility of getting still better results with a fairing of the shape represented in Fig. 301. With that form a pair of wires separated by a clear gap of four diameters can be contained within a fairing of which the width is twice the diameter of one of the wires, with the result of reducing the resistance at a Reynolds' number (for the individual wire) of 13,000 to approximately 28 per cent of that for a single wire, so cutting down the resistance

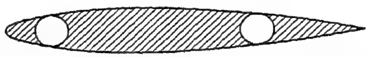


FIG. 302.—Complete enclosure for tandem wires.

factor in terms of sectional area to about 50 lb. This comes down into still closer competition with the rolled lenticular form. It is clearly apparent that some sort of fairing, even though a very simple one, is well worth while. Where simplicity is forsaken to the extent of building a completely enclosing structure around the wires, as in Fig. 302, substantial further savings can be made, bringing the areal resistance factor as low as 20 lb.[2] The complications of constructing and applying such a fairing are, however, obvious. As a general rule it will be found better either to be content with the simpler form of Fig. 300 or to replace round wires with streamline ones.

[1] "Tests on Struts Suitable for the Fairing of Duplicate Cables," by R. A. Frazer and L. F. G. Simmons, *R. and M.* 433.

[2] *Rept.* 468, *cit. supra.*

Wheels, and Landing-gear Interference.—Among the parts of the airplane not so far covered, the most important in its contribution to resistance is the landing gear. It has lost something of its significance by the growing adoption of retractable gears, but the airplanes on which the wheels and all their attachments completely disappear when in flight are still in a minority. Furthermore, even a retractable gear must be exposed when taking off and landing and during the first, and what is usually the most critical, portion of the climb to operating altitudes. Even with full retraction, then, landing-gear drag cannot be entirely neglected.

In the typical case, the landing gear is an assemblage of struts, wires, and wheels. Its drag cannot, however, be treated by simple summation. The effect of mutual interference of neighboring struts has already[1] been emphasized, and the typical landing gear offers the student of interference his example *par excellence*. Struts run in front of each other; they intersect each other at all manner of angles; they run close alongside the wheels; half a dozen members may enter the same fitting. Often no more than half the total of landing-gear drag can be accounted for by the summation method.

Studies have been numerous, both upon wheels as the principal separable part of the gear and upon landing gears as a whole.[2]

Wheels are of four classes: high pressure, medium pressure (the most commonly used in present practice), low pressure or airwheel, and streamline—all as shown in Fig. 303. The dimensions given there are for the wheels appropriate to an airplane of

[1] P. 425, *supra*.

[2] The best general reference, especially on interference and on the comparative performance of various types of landing gear, is "The Drag of Airplane Wheels, Wheel Fairings, and Landing Gears," by William H. Herrnstein, Jr., and David Biermann, *Rept.* 485; do., Pt. II, *Rept.* 518; do., Pt. III, *Rept.* 522. An important study from the British point of view, covering the practices in wheel and tire form that are common in Great Britain, is "Some Wind-tunnel Tests on Wheels, Fairings, and Mudguards," by F. B. Bradfield and G. F. Midwood, *R. and M.* 1479. Earlier data of some interest are contained in "The Wind Resistance of an Airplane Wheel," by E. A. Griffiths and J. D. Coales, *R. and M.* 207; "Experiments on the Resistance of Airplane Wheels and Radiators," *Repts.* from the Rijksstudiedienst voor Luchtvaart, Amsterdam, Pt. II, 1923, translated in *Tech. Memo.* 269; "Air Resistance Measurements on Actual Airplane Parts," by C. Wieselsberger, *Technische Berichte,* vol. III, p. 275, translated as *Tech. Memo.* 169.

3,000 lb. gross weight, and those sizes will be used throughout the present section for purposes of comparison. For larger or smaller planes, the drag of the wheels can be considered to vary in direct proportion to the gross weight.

The established low in aerodynamic quality is a wire wheel without fairing. Such a wheel, in the 30 by 5 size, has a drag of 15 lb. at 100 m. p. h. The basic drags of disk wheels and of low- and medium-pressure wheels are tabulated in Fig. 303, where it appears that there is no great difference among them and that in general the types currently used offer a drag reduction

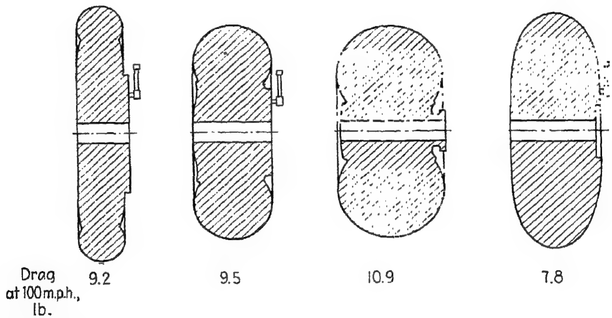


FIG. 303.—Drag of wheels, of load-carrying capacity equivalent to 30 × 5 high-pressure type.

of about a third below the old wire-wheel figure. The disk ratio, incidentally, ranges from 1/3.5 (for the high-pressure disk wheel) down to 1/6 (for the streamline type).

At 200 m. p. h., a pair of medium-pressure wheels such as are shown in Fig. 303 would consume just over 40 hp., or about 13 per cent of the total available power on the airplane likely to use such wheels and to make such a speed. No such amount can be spared, and the exposure of bare wheels has no part in modern design except for machines with a top speed of 135 m. p. h. or less or for those having to be operated from such muddy fields that any sort of fairing would clog. Elsewhere, a fairing such as Fig. 304 illustrates in two different forms is the rule. Wheel fairings go by the name of "pants" in America; in Great Britain, with a considerably more perfect sartorial analogy, by that of "spats"; under either name they serve, as the figures attached to the illustration indicate, to reduce the drag of the bare wheel

by approximately a half. The least enveloping of the forms illustrated in (*a*) approximates present design practice more

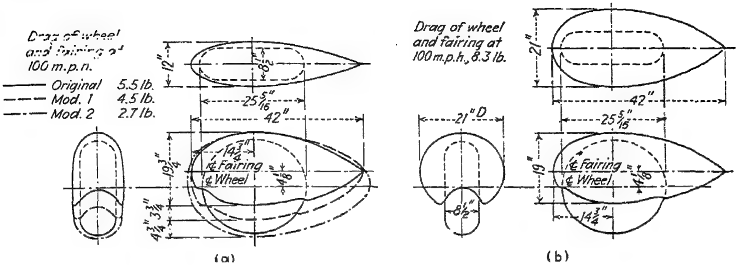
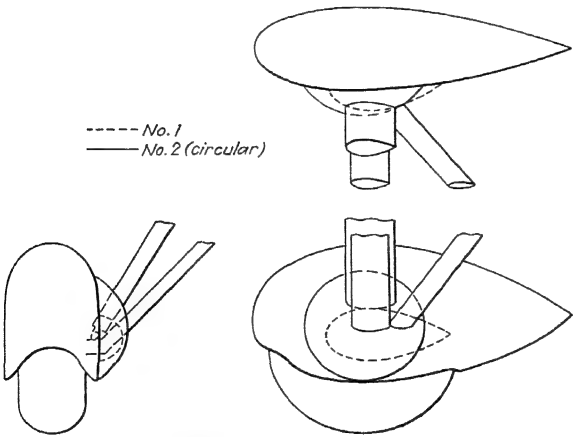


FIG. 304.—Fairings for wheels, and their effect on drag.

closely than the other, which would make trouble on any sort of soft field. On racing machines, however, the fairing may be extended down to leave a ground clearance of 5 in. or even less.



Drag of half of gear at 100 m.p.h.

Without wheel fairing	*18.0 lb.*
With wheel fairing	*15.9 lb.*
With wheel fairing, modification No. 1	*14.0 lb.*
With wheel fairing, modification No. 2	*16.5 lb.*

FIG. 305.—Drag of lower extremity of landing gear.

The next step in a study is to include the struts, as in Fig. 305. The total length of strut added in that particular case was 11 ft.;

its mean width of section was $1^{11}/_{16}$ in.; it was of good streamline form, and with a drag coefficient of 0.08 it should have added a total of 3.2 lb. of drag at 100 m. p. h. Actually, interference added another 5 lb., and the resistance of the bare wheel with the three struts attached exceeds that of the wheel alone by 8.5 lb.

With a fairing in place over the wheel the interference is still greater, the total drag now exceeding that of the faired wheel alone by 10.4 lb. A properly formed streamline protuberance on the fairing to enclose the fittings at the lower ends of the struts saves approximately 2 lb. but still leaves an interference drag

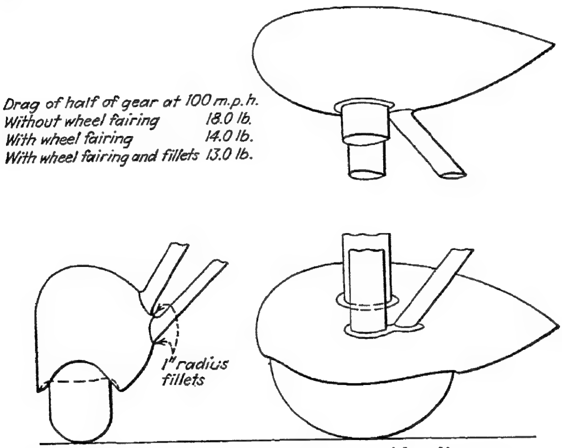
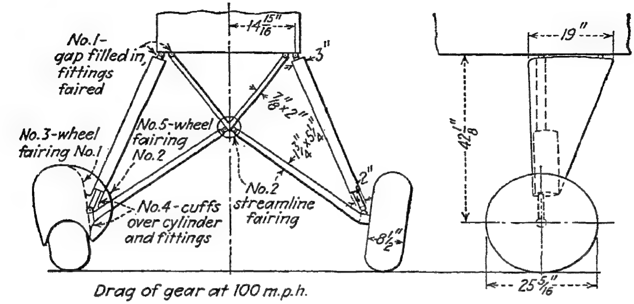


FIG. 306.—Drag of lower extremity of landing gear.

virtually as large as the drag of the faired wheel alone. Another example of the same sort is shown in Fig. 306, where the fairing has been bulged on one side to enclose the junction of the struts. Though the wheel in this type of fairing, when tested alone, has 30 per cent more drag than when faired as in Fig. 305, tested as a part of a landing gear it shows exactly the same drag as the alternative form and when proper fillets have been added around the struts there is another saving of 1 lb., reducing the apparent interference drag as compared with the sum of the elemental drags to 2.5 lb., or less than 20 per cent of the total.

Of complete landing gears there is an almost infinite variety of types, and more than a score were included in the N. A. C. A.

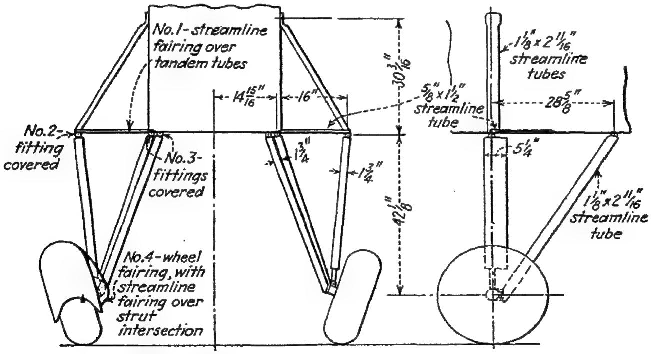
program already referred to. A few samples are shown in Figs. 307 to 310, with their respective total drags at 100 m. p. h.



Drag of gear at 100 m.p.h.

8.50-10 wheels,		44.0 lb.
" " " , modification No. 1		43.0 "
" " " , "	Nos. 1 and 2	41.0 "
" " " , "	Nos. 1, 2, and 3	40.0 "
" " " , "	Nos. 1, 2, 3 and 4	28.0 "
" " " , "	Nos. 1, 2 and 5	27.0 "

FIG. 307.—Drag of conventional V landing gear with various types and degrees of fairing.



Drag of gear at 100 m.p.h.

8.50-10 wheels,		44.0 lb.
" " " , modification No. 1		44.5 "
" " " , "	No. 1 and 2	43.0 "
" " " , "	No. 1, 2, and 3	41.0 "
" " " , "	No. 1, 2, 3 and 4	30.0 "

FIG. 308.—Drag of landing gear with widened track.

Figure 307 shows the progressive reduction of drag by fairing one fitting after another, the relatively slight gain that is made by fairing the wheels when nothing is done about the strut ends

(since they serve to break up the flow around the wheel fairing and to produce a separation otherwise unnecessary), and the major saving that results from filleting the strut ends and enclosing their fittings in a smooth sleeve. Of the 27 lb. of drag that still remain in the final form, 14.4 lb. can be charged directly to the faired wheels and 7.5 lb. to the struts, leaving 5 lb. (again just under 20 per cent of the total) as interference.

In Fig. 308, illustrative of a type of gear used on certain biplanes and high-wing monoplanes where the arrangement of Fig. 307 is felt to give too narrow a tread, the relation of the struts looks at first sight much more complex than in Fig. 307, yet the total drag is only 10 per cent higher. But when the strut

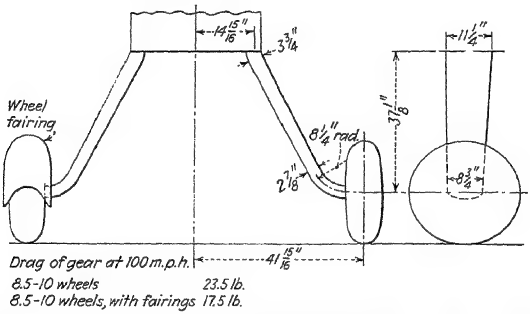


FIG. 309.—Drag of single-strut cantilever landing gear with inclined struts.

sizes are analyzed the interference proves to be 11.5 lb., or almost 40 per cent of the total.

These percentages of interference offer the best means of approximating the drag for other gears, since no two will ever be exactly alike. For a gear resembling that in Fig. 307, and assuming that reasonable care will be used in enclosing or fairing fittings, the drag of the struts can be computed as though they were isolated from every disturbing influence, the drag of the wheels determined on the same hypothesis can be added in, and the resultant total can be increased by 25 per cent. With a gear like that in Fig. 308 the process is the same, but the percentage of increase is 65. With Figs. 307 and 308 as a starting point, and a little experience, a fair estimate can be made of the relative degree of complexity of any other strut-and-wire-trussed gear from the point of view of interference and of the

percentage by which the total of the elementary drags should be increased.

Figures 309 and 310 illustrate gears supersimplified by the enclosure of all their structure inside one single heavy streamline member. For Fig. 309 the interference accounts for but 14 per cent of the total drag; for Fig. 310, in its first modification, it is zero, although without the fillet it would be over 30 per cent. The effect of an expanding fillet here is even more startling than in its application between a low-wing and a circular-section fuselage,[1] and the argument and the explanation are essentially the same in the two cases. The drag of either of the best arrange-

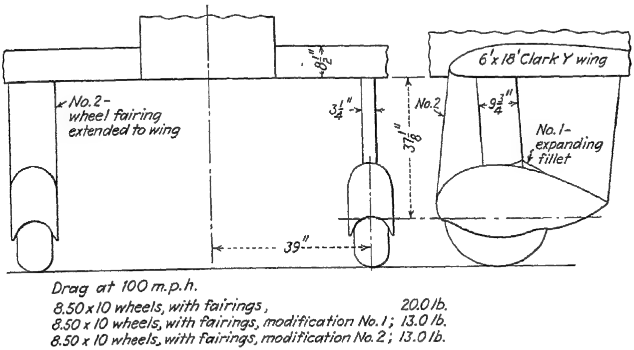


FIG. 310.—Drag of cantilever landing gears for low-wing monoplane.

ments shown in Fig. 310 is so low that the exposure of such a landing gear will decrease the maximum speed of a typical 200 m. p. h. plane by only 3½ per cent as compared with the performance with a landing gear fully retracted.

Radiators.—An appendage of great aerodynamic importance on those airplanes in which water-cooled engines are used is the radiator. In many instances it is a part of the fuselage, being fitted into the nose or (more commonly) into the lower surface a short distance behind the nose in such a way that the contributions to resistance by the radiator and by the remainder of the fuselage structure are inseparable. Sometimes it is laid into a corrugated wing surface and shows its aerodynamic effect through an increase of profile drag. In other cases, how-

[1] See p. 391, *supra*.

ever, the radiator is placed out in the free air and radiator resistance can be treated as a separate element of the parasite.

Its relative importance increases steadily with increasing speed of flight. If two airplanes are equipped with the same engine but fly at different speeds the amount of free-air radiator surface needed varies inversely as the speed, for it is known that the dissipation of heat from a surface over which a fluid moves is substantially proportional to the first power of the relative velocity of movement. The resistance per unit area, however, varies as the square of the speed, and the power consumed in driving that unit area through the air as the cube, so that notwithstanding the reduction in area the proportion of engine power used in propelling the radiator goes up as V^2. The production of low-resistance radiator types therefore receives special attention from the designers of pursuit and racing airplanes.

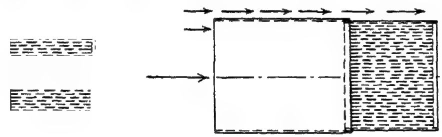
Although there are a great number of tests on the resistance of free-air radiators,[1] it is difficult to discuss the results in detail without going into the whole question of radiator design and selection, as a radiator of low unit resistance may easily be inferior to one having a higher resistance per unit of area but a larger cooling efficiency, permitting a smaller area. In a general way it can be said that the resistance coefficients of free-air radiators of the honeycomb type range from 0.5 to 0.7, the largest coefficients going with the radiators of greatest depth and of smallest tube diameter, or in other words with those which offer greatest obstruction to the flow of air. As a general rule the disk ratio for a honeycomb radiator not of abnormal proportions may be taken as one-half.

Without going into the relation between radiator form and position and cooling effect in any detail, certain round numbers may be given. Thus an airplane with a best climbing speed of 90 m. p. h. (the lowest speed at which it needs to develop the full engine power for any length of time) requires about 0.8 sq. ft.

[1] "Head Resistance Due to Radiators," by R. V. Kleinschmidt and S. R. Parsons, *Rept.* 61; "Experiments on the Resistance of Airplane Wheels and Radiators," Rijksstudiedienst voor Luchtvaart, *Tech. Memo.* 269; "S. T. Ae. Radiator Tests," an abstract of reports on tests made at the Eiffel Laboratory for the French Section Technique Aéronautique, *Bull.*, Expt. Dept., Airplane Eng. Div., U. S. Army Air Service, pp. 44–76, June, 1918.

of radiating surface in a free-air honeycomb radiator for every horsepower. If the best climbing speed be raised to 120 m. p. h. (which is as high as it is ever likely to go) the radiating surface can be reduced to 0.6 sq. ft. per hp. If prestone or ethylene glycol is used for a cooling fluid instead of water the amount of cooling surface needed is reduced by about 40 per cent. If the radiator is installed in a tunnel in the bottom of the fuselage, with the air escaping through louvres, the ratio of mean speed through the radiator to speed of flight is likely to be so reduced that the cooling surface should be increased by a third.

Furthermore, it can be set down as an approximation that in a typical honeycomb radiator there will be 8 sq. ft. of cooling surface for every square foot of frontal area per inch of depth of honeycomb core. Since the core depth generally lies between



(a) (b)

Fig. 311.—The fitting of an entrance cowl on a honeycomb radiator.

5 and 9 in. in current practice, the ratio of radiating area to frontal area lies as a rule between 40 and 70.

With a water-cooled engine, a free-air radiator 6 in. deep, and a 90-mile climbing speed, the requirement will then be for 1 sq. ft. of radiator frontal area for every 60 hp. Taking the disk ratio as one-half, the equivalent flat-plate area will be 1 sq. ft. per 120 hp.

The cooling effect of a radiator can be considerably increased, at the same time that the resistance is increased but in a larger proportion, by fitting an entrance cowl as in Fig. 311 to reduce the escape of air around the edges. It might at first sight appear reasonable to make the cowl divergent toward its open end to scoop in more air, and if that is done there is indeed a further increase of cooling effect, but it is accompanied with an even larger increase of resistance. Since the cooling effect rises comparatively slowly with increase of the speed of air through the tubes, the attempt to inject increasing amounts of air through a given core, at the expense of increased drag or direct expenditure of power through blowers or otherwise, is a process of rapidly diminishing returns.

The converse of a divergent cowl is a convergent one, and the disadvantage of the former has its counterpart in advantages gained through the use of the latter. It has been found[1] that such a cowl as is shown in Fig. 311(*a*), with its opening reduced to approximately 60 per cent of the frontal area of the radiator core, will still pass through the core as much air as would go through a similar core freely exposed with no cowling of any sort, and that it will therefore have a heat-dissipating power equal to that of such a core freely exposed. The drag of the cowled core, however, is some 20 per cent less than that of the free one.

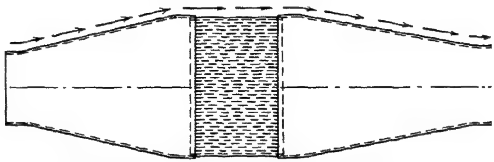


FIG. 312.—Honeycomb radiator with double convergent entrance and exit cowl.

A still further improvement can be made by supplying[2] a doubly convergent cowl as in Fig. 312, with the radiator enclosure beginning remotely to suggest a streamline body. In that case the flow through the radiator is reduced by a little over a third, requiring an increase of frontal area of the core of about 60 per cent to maintain the heat dissipation unimpaired. At the same time, however, the drag per unit of frontal area is reduced to about 20 per cent of the basic figure for the freely exposed core. Even after the necessary increase in area has been allowed for, the drag of the radiator system is but a third of that for a free core and but 40 per cent of that for the core with an entrance cowl alone.

Even where a radiator is enclosed within the fuselage, convergent cowling is desirable. An arrangement such as that shown in Fig. 313 offers much less total drag than any installation with the radiator more directly exposed to the view.

Although a radiator in free air always has less resistance than a normal flat plate of similar size and shape, one placed in the nose of a fuselage, or otherwise in close juxtaposition to some other element of the structure, may increase the resistance more than

[1] "Bigger and Better Radiators," by Weldon Worth, *Aviation*, p. 19, August, 1935.

[2] Worth, *cit. supra*.

it would be increased by a flat plate in the same position, as the flow into the radiator, around the engine, and thence out through louvres in the sides of the fuselage may disturb the flow of air around the body as a whole more than a perfectly blank flat surface would so. Direct evidence is afforded by a research conducted at the Bureau of Standards.[1] It was found that a fuselage of rather poor form, with a flat nose having an area about two-thirds that of the maximum cross section of the body, had a resistance coefficient of 0.43 when there was no flow through the nose radiator; and that when louvres were opened up so that the

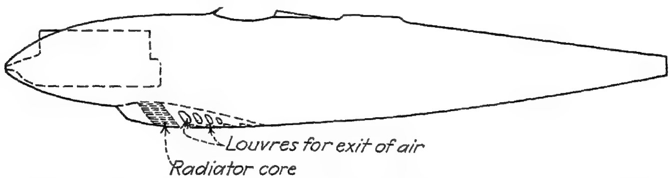


FIG. 313.—Fuselage tunnel radiator with convergent entrance cowl.

air flowed through that radiator freely but without any actual addition of obstruction on the sides of the body (the louvres merely being openings flush with the sides), the coefficient was increased to 0.60.

Fittings.—Fittings of an airplane, those parts which serve to assemble the major elements together, may make an important contribution to total drag, especially if the designer is a little careless in handling them. They play a progressively smaller part, as designers learn to take increasing care to keep the odds and ends under cover. A certain number of tests have been made on fittings,[2] but they are of little practical value, for the interference between the fitting and its environment is nearly always large enough to effect a major modification in the effective drag as compared with that measured in a test of the fitting alone. The interference of a fitting on the surface of a wing has already been discussed.[3] Where there are no direct tests, the magnitude of the interference can be guessed at from (1) the probable ratio of the local speed of air past the fitting to the speed of flight;

[1] *Tech. Memo.* 269, *cit. supra.* "Preliminary Report on Resistance Due to Nose Radiator," Pt. II, by R. V. Kleinschmidt, *Rept.* 61.

[2] "Scale Effect on Struts and Drag of Wiring Plates of a Bristol Fighter," by F. B. Bradfield, *R. and M.* 890.

[3] P. 334, *supra.*

(2) the degree of proximity, and the size, of objects of good aerodynamic form, the flow past which is likely to be disrupted and the drag appropriately increased by the disturbance occasioned by the fitting's introduction. Where it becomes desirable to estimate the resistance of an exposed fitting alone, preparatory to adding an interference allowance, the drag coefficient based on the total frontal area of the fitting can be taken as 0.8—

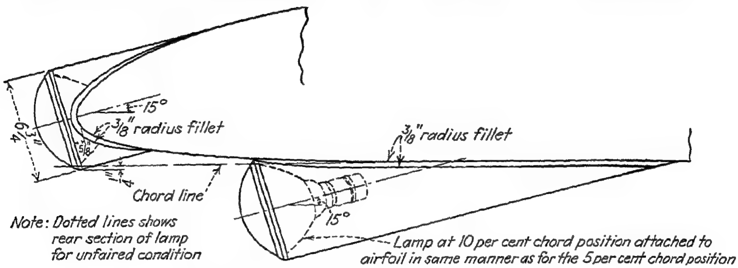


FIG. 314.—Landing-light installations in wing.

obviously a figure by no means rigorously constant, but a fair average.

Under the general head of fittings there may be included all manner of gadgets installed for all manner of specialized purposes, and more or less standardized as to form. An outstanding example is the landing lamp, commonly installed either in or just below the leading edge of a wing and inclined downward about 15 deg. with respect to the wing chord. Tests in the two positions shown in Fig. 314[1] show an increase of wing drag as follows:

TABLE IX.—EFFECTIVE DRAG OF EACH LAMP AT 100 M. P. H., LB.

	Lamp unfaired	Lamp faired
Lamp in leading edge..	2.7	1.4
Lamp on lower surface.	11.0	4.3

The drag of a flat plate, of frontal area equal to that of a lamp, in free air at 100 m. p. h., would be 7.5 lb. The effective disk ratio then ranges from 1/5.5 for the faired lamp in the leading edge to 1/0.7 for the unfaired installation on the lower surface.

[1] "Full-scale Drag Tests of Landing Lamps," by C. H. Dearborn, *Tech. Note* 497.

CHAPTER XIII

POWER-PLANT CHARACTERISTICS AND PERFORMANCE

In the fixing of airplane performance there are three determinant factors: the airplane structure and its aerodynamic form, the engine, and the propeller. The first fixes the power needed for flight under any condition; the second, the power basically available; the third, the proportion of the power delivered by the airplane that can be converted into a form where it can be directly applied to satisfying the airplane's demands. The aerodynamics of the airplane structure, or cellule as it is sometimes called, have been discussed at length in the last six chapters and will be further discussed in a later one.[1] The propeller, since the nature of its function is dependent on what the engine gives it to do and its characteristics are intimately bound up with those of the engine, will be deferred until the engine has been suitably treated. To the engine we proceed, without further delay.

Sea-level Power.—The basic curve, for any internal-combustion engine operating on an Otto cycle and with throttle control (which covers about 99.9 per cent of the engines now installed in aircraft), is that of full-throttle power against revolutions per minute. To be still more specific, since the adjective "full-throttle" has become a bit ambiguous through developments of the past few years, it is the plot of power developed with no artificial restriction of any sort in the way of the flow of air to the carburetor and the subsequent passage of the explosive mixture to the cylinders.

An Otto-cycle engine ought theoretically to operate at a mean effective pressure controlled only by the throttle. If the volumetric efficiency and the mechanical efficiency were constant, the torque developed by the engine ought also to be a constant. The curve of power output ought to be a straight line through the origin, the power directly proportional to the r. p. m. Actually

[1] Chap. XV, *infra*.

the volumetric efficiency tends to fall off with increasing rotational speed, the mechanical efficiency to rise. Both changes are exceedingly slow over a wide range of speeds, and the linear rule of power variation holds with substantial accuracy. As the speed increases, however, there comes a point beyond which the volumetric efficiency begins to fall off very rapidly indeed, the pressure difference between the outside air and the cylinder no longer sufficing to drive through a volume of mixture fully charging the cylinder within the time available. The thermal efficiency also begins to fall off, as the shortness of time allowed for the combustion of the charge and its subsequent expansion becomes inadequate to a normal functioning.

There need be no analysis of these factors in detail, for it is only the final joint result that matters from the point of view of performance of the aircraft. The final result is a gradual falling away of the power curve from its linear course, a falling away that ultimately reduces the slope of the curve to zero. Thereafter, the power output decreases with further increase of speed.

Practical operation (except in a steep dive, where the propeller is driving the engine rather than being driven by it) never extends past the peak of the curve. Figure 315 shows two typical full-throttle power curves, reduced for convenience of comparison to ratio scales. The definitions of rated power and rated r. p. m. are somewhat arbitrary and will be considered in detail in the next section, but for the moment they can be considered as being the highest power and rate of rotation at which the engine is ever likely to be operated in level flight.

The curves presented are quite different in form. The less steeply inclined is characteristic of engines (now seldom encountered except in the lower power ranges) that incorporate no mechanical aids to induction of any sort, depending exclusively on atmospheric pressure to feed the air to the carburetor and thence to the engine. The other curve, along which not only power but torque and brake mean effective pressure as well increase with increasing r. p. m., the power varying approximately as $(\text{r. p. m.})^{1.05}$ even up to speeds 10 per cent or more above the rated normal, is more typical of modern design. In particular it is more typical of radial engines, with induction stimulated by a rotary blower running at speed proportional to the engine speed

and with an available induction pressure which rises with engine speed as a consequence.

Manifestly, in an engine with no such artificial aid to induction the absolute pressure existing in the intake manifold during

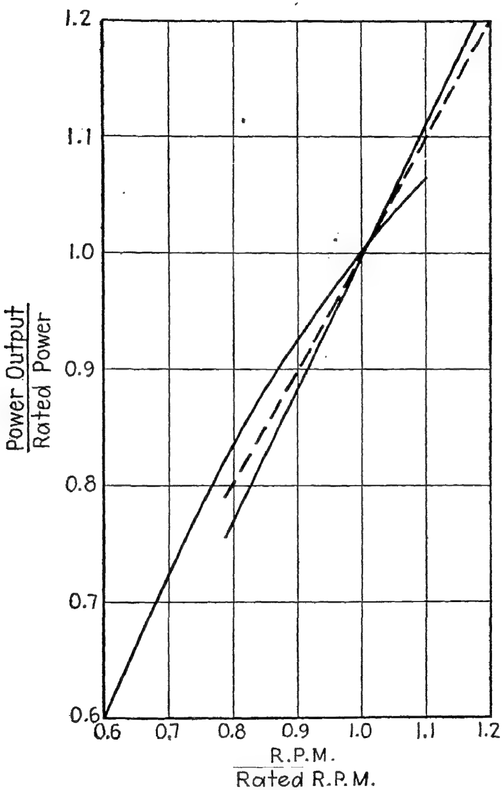


FIG. 315.—Typical curves of variation of power with r. p. m.

operation can never exceed atmospheric pressure; and must in fact be inferior to it by an amount sufficient to force the air through the carburetor and as far as the manifold. The pressure drop required for that purpose of course increases with the speed of the air through the passages and so with the engine speed, and at maximum operating speed with the throttle fully open it

may be as much as 3 in. of mercury. The normal atmospheric pressure level being 29.92 in., the manifold pressure in a typical engine without forced induction is likely to fall off from about 28.5 in. at low speeds (always at full throttle, the engine speed being governed by the load) to about 27 at the highest normal r. p. m.

With forced induction, now almost universal, there is quite a different story. The pressure maintained by a rotary blower while delivering a volume of air proportional to the blower's own r. p. m. is proportional to (r. p. m.)^2. If all losses could be

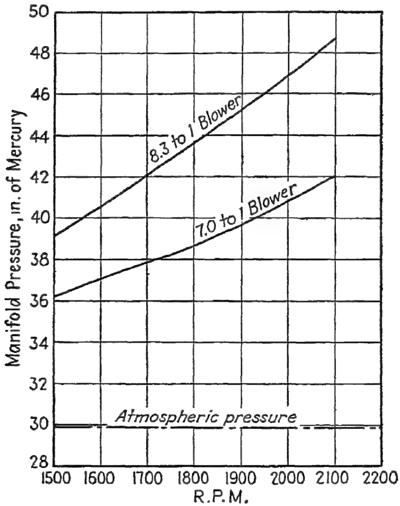


FIG. 316.—Variation of manifold pressure with r. p. m. and blower-gear ratio.

neglected, then, the full-throttle manifold pressure ought to exceed atmospheric pressure by an amount proportional to the square of the engine speed. What it actually does is shown in Fig. 316, where curves of full-throttle manifold pressure against rate of rotation are given for two engines identical except in their blower arrangements. In one case the blower is run at seven times the crankshaft speed; in the other case at 8.3 times. Still higher blower speeds are used on occasion, but full-throttle data then become unavailable. The engine cannot be operated at sea level for any length of time with the throttle wide open without disastrous results. Blower-gear ratios lower than 7 to 1

have, on the other hand, become uncommon. The increase of power by supercharging the cylinders with air forced in under more than atmospheric pressure more than compensates for the added weight of the high-speed blower, the gear train that drives it, and the reenforced engine parts necessary to stand the loads resulting from the higher working pressures. Supercharging made its first appearance, in 1917, as a device for employment at high altitudes only; its purpose was to maintain sea-level

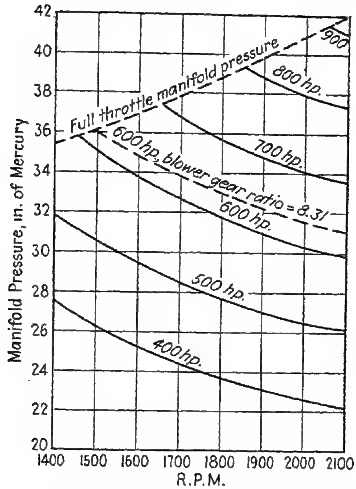


FIG. 317.—Dependence of sea-level power on manifold pressure and r. p. m.

manifold pressures or a close approximation thereto up to altitudes where the density is far below the sea-level standard and where manifold pressures under normal atmospheric induction would have fallen off accordingly, bringing a corresponding fall in the density, and total weight, and heat content, and potential power production of the explosive charge delivered to each cylinder in each cycle. Supercharging in 1935 pertains not only to high-altitude performance, though it is indispensable for all aircraft expected to perform efficiently above 8,000 or 10,000 ft., but to high power-plant efficiency at sea level as well.

The role of manifold pressure has come to be controlling. It is recognized as the primary agent influencing the appearance of

detonation. Where once the airplane pilot limited himself to watching his engine revolutions as an index of the load that the engine was being asked to carry, he now keeps an eye always upon the manifold gauge and operates his throttle with direct and constant reference to a specification from the engine builder that sets forth limits both of r. p. m. and of manifold pressure—and so of power, for at any given air density and temperature, and for a given engine, power is fully determined when manifold pressure and r. p. m. are known.

In Fig. 317, power has been plotted in a series of contour lines on a chart of which the coordinates are the two independent variables. The engine to which the curves relate is the less highly supercharged of the two models covered in Fig. 316, but a single curve (that of 600 hp.) has been plotted for the other model as well. The price paid for the higher power capacity of the engine with the high-speed blower is a loss of about 5 per cent of the power of the less highly supercharged model at the same speed and manifold pressure—the 5 per cent being a measure of the added power that has to be expended internally in driving the blower at its higher speed.

Engine Ratings.—The process and the method of rating power-plant capacity have varied from time to time. They still vary from country to country, and even in some cases from one engine manufacturer to another. As late as 1931 it was the common practice merely to state for each engine a rated power and speed, representative of the conditions under which the engine was supposed to be able to operate for prolonged periods without serious danger of structural breakage. At the same time it was not recommended that the rated conditions be the normal ones, for aside from somewhat enhancing the risk of engine failure they very definitely increased the rate of wear. The normal condition of cruising operation, to which the engine was to be throttled back as soon as the desired operating altitude had been attained and the flight path leveled off, reduced the r. p. m. to 15 per cent below the rated figure. Power being roughly proportional[1] to (r. p. m.)^3, cruising power was about 60 per cent of rated power.

Though that comparatively crude rating system is still encountered on occasion, better practice has overlaid it with refinements.

[1] See Chap. XIV.

It has come to be recognized that either excessive internal pressures or excessive speeds carry the risk of engine failure and that they must be separately limited. It has been accepted that there are three sets of limits: the maximum that can be regarded as safe over considerable periods, commonly referred to as the standard, or rated, output; the recommended level for normal operation, called the cruising output; and the limit of permissibility for very brief snatches (usually no more than 2 or 3 min., or 10 at the very outside), denominated the overspeed condition in Great Britain and the take-off rating in America.

Cruising power is likely to be determined as much by the ideas of the user of the engine as by those of the builder. The

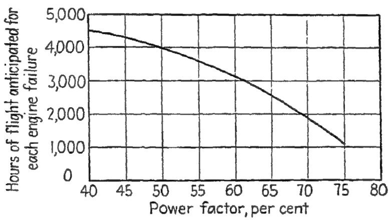


FIG. 318.—Cruising-power factor and engine reliability.

prevailing recommendation of engine manufacturers is that their products should be cruised at about 65 per cent of the maximum-continuous-output rating. It is commonly accepted, but the more conservative air lines use cruising ratings as low as 60 or even 55 per cent. On a flight against a strong head wind, on the other hand, the cruising ratio may be allowed to run up as high as 75 per cent. Though the data accumulated on the point hardly permit specific plotting, the curve in Fig. 318[1] gives what seems a probable relation between cruising-power percentage and frequency of failure for high-powered radial air-cooled engines. In 1936 leading American engine builders were beginning to rate their output primarily on cruising power, with the take-off limit and the climbing or maximum-speed limit given a secondary position.

[1] Based in part on "Economic Engine Operation for Cruising Reliability," by Edmund T. Allen and W. Bailey Oswald, *Aviation*, p. 89, March, 1935.

Take-off ratings of power for American engines are usually about 10 per cent above the maximum-continuous-output allowances. In Great Britain the rule is to allow a temporary overspeeding that increases power output to about 15 per cent above the rating.

So far, ratings have been discussed in terms of power. It remains only to break them down into manifold pressure and engine speed. For the concrete case of the engine already used in plotting several of the curves,[1] in the model with minimum supercharging the specification is that the r. p. m. must never at any time (except perhaps during a dive with closed throttle) exceed 2,100, that the manifold pressure ought never to exceed 34½ in. of mercury at any speed for more than a few minutes at a time but may be allowed to rise to 37½ in. for take-off purposes, and that the cruising limits with a 65 per cent cruising power factor are 1,800 r. p. m. and 27 in. of pressure. With a controllable-pitch propeller, as will be more fully explained in the next chapter, all these conditions, or practically any other set, can be attained on the same airplane. With a rigid propeller, however, the power absorbed by the propeller is necessarily a function only of r. p. m. and speed of flight and propeller design, and there is no assurance that the maximum-continuous-power condition and the take-off condition will be compatible with each other. They are likely to come fairly near to it, however, for it takes at least 10 per cent more power to turn a given propeller at a given r. p. m. at a rate of advance of 50 m. p. h. (taken as the average prevailing during the take-off run) than to drive it at the same rotational speed when flying 150 m. p. h. (assuming that to be the speed at which the engine is expected to put forth its maximum-continuous-output power, based on the particular combination of 2,100 r. p. m. and 34½ in. of pressure).

Performance at Altitude.—All this will come into play in calculating the performances of an airplane, and especially in analyzing the relations between the engine and the propeller. Of even greater importance to the airplane designer than the question of rating method, however—most important in fact of all engine characteristics—is the variation of engine power with altitude of operation.

[1] The Wright Cyclone, Model F.

If the thermal, mechanical, and volumetric efficiencies of the engine remained constant, the power output would be directly proportional to the density of the air, for with the volume of explosive mixture taken into the cylinder on each induction stroke constant and its weight proportional to the density it would follow that the weight of fuel, the number of heat units made available in the fuel, and the total amount of heat energy converted into mechanical work would all change in accordance with that same ratio. Actually, however, there are modifying factors which make for a somewhat more rapid loss of power.

As its importance deserves, the subject has undergone a vast deal of study both theoretical and experimental.[1] Out of it all, there develop three important factors supplementary to the primary one of reduction of weight of charge inhaled at each stroke.

The most important among them is the drop of mechanical efficiency with increasing altitude. While the pumping losses in an engine fall off with air density, the internal friction is independent of that quantity; and if a constant frictional power is subtracted from an indicated power falling off in direct proportion to the density, it naturally follows that the difference, or brake horsepower, will drop more rapidly than the density.

Second on the list is the difference, in their influence upon engine performance, between changes in density due to pressure

[1] "Effect of Compression Ratio, Pressure, Temperature, and Humidity on Power," by H. C. Dickinson and others, *Rept.* 45; "The Calculated Performance of Airplanes Equipped with Supercharging Engines," by E. C. Kemble, *Rept.* 101; "Performance of a Liberty-12 Airplane Engine," by S. W. Sparrow and H. S. White, *Rept.* 102; "Performance of a 300-horsepower Hispano-Suiza Airplane Engine," by S. W. Sparrow and H. S. White, *Rept.* 103; "Performance of BMW 185-horsepower Airplane Engine," by S. W. Sparrow, *Rept.* 135; "Effect of Altitude on Power of Aviation Engines," by Italo Raffaelli, *Rendiconti Tecnici*, July 15, 1924, translated in *Tech. Memo.* 287; "The Variation in Engine Power with Altitude Determined from Measurements in Flight with a Hub Dynamometer," by W. D. Gove, *Rept.* 295; "The Effect of Humidity on Engine Power at Altitude," by D. B. Brooks and E. A. Garlock, *Rept.* 426; "The Variation of Engine Power with Height," by D. H. Pinsent and H. A. Renwick, *R. and M.* 462; "Variation of Engine Power with Height," by H. L. Stevens, *R. and M.* 960; "The Variation of Engine Power with Height," by H. M. Garner and W. G. Jennings, *R. and M.* 961; "The Best Basis of Aircraft Performance Reduction," by J. L. Hutchinson and E. Finn, *R. and M.* 1532.

and those due to temperature. A reduction in atmospheric pressure may be expected, in an engine with little or no supercharging, to transmit itself through directly to the engine and to produce a drop in manifold pressure of magnitude substantially equal to its own. A rise in temperature, however, has no simply assured effect. The temperature of the mixture as it enters the cylinder is under the control of the temperature of the manifold and the cylinder walls, of the action of any local preheating or vaporizing device over which the flow may pass, and of the latent heat of evaporation of the fuel. Since, other things being equal, vaporization in the manifold will be most complete when the initial air temperature is highest, it follows that the drop of temperature due to the withdrawal of the latent heat of evaporation will tend to be largest when the initial temperature is a maximum. Extremes of initial temperature, in other words, are in some degree self-correcting, and the temperature in the manifold tends toward constancy. Changes in air density due to temperature are then likely to have less effect on power than those originating with atmospheric pressure. Manually controlled preheaters, too, are used to avoid wide variations in mixture temperature. On a warm day no heat is needed and none is applied. Let the temperature drop, and the heat will be switched on, both to promote complete vaporization and to prevent the formation of ice in the carburetor passages, which is a potent source of engine failure.

In going to high altitudes the pressure and air temperature both drop. The latter, in its effect upon density, partially counterbalances the former, and pressure therefore decreases more rapidly with increasing altitude (in round numbers, and up to 15,000 or 20,000 ft., a fifth more rapidly) than does density. Since pressure changes have a full effect on engine power and temperature changes only a partial one, it might be expected on that ground alone that the curve of power decline would follow a course midway between those of pressure and of density. Some students of the subject have argued that power is totally independent of air temperature and should be considered a function of pressure alone. Undoubtedly it is true that no single rule applies with perfect rigor to all engines, the exact influence of temperature depending somewhat upon the form and proportions of the intake system, but the general belief now is that on the

average the temperature has about half the relative effect on power that it does on air density and that the power ought therefore to be considered as dependent on $p/\sqrt{T}$ or $\sqrt{\rho T}$, where p, ρ, and T are the atmospheric pressure, the air density, and the absolute temperature, respectively.

However that may be, it is advisable to throw power variation into terms of density in deriving a standard formula. Whether or not engine power be a function of density, other elements that have to be balanced against engine power in the course of a performance calculation certainly depend upon density and density alone, and to introduce a variety of primary independent variables would make any orderly method of computation impossible. Even if it were admitted that pressure alone counted in the present instance, density could still be substituted as the independent variable by assuming a standardized relationship among altitude, pressure, temperature, and density. Fortunately the rate of temperature drop with altitude proves in practice to be nearly enough constant (especially at considerable altitudes) so that the assumption of a standard form does no very serious violence to the physical facts. It is now universal practice to use a "standard atmosphere" for purposes of performance prediction and reduction. The definition of such a standard is among the relatively few points in aeronautical science on which there is almost a full international agreement, and the curves plotted in Fig. 319[1] are official in the United States and in the British Empire, and in many other countries as well. From Fig. 319 it can be determined that, if engine power were directly proportional to atmospheric pressure, the mechanical-efficiency effect discussed on page 455 and all other modifying influences being negligible, the relation of power to standard density would be substantially as $\rho^{1.235}$.

The third of the factors premised—and by far the least significant of the three yet among the most troublesome in that it shows the largest differences within itself from one type of engine to another—is the variation of indicated thermal efficiency with altitude. Compression ratios selected for sea-level running do not permit the best possible performance in the upper atmosphere. Intake systems do not function with equal effectiveness

[1] Data from "Standard Atmosphere—Tables and Charts," by Walter S. Diehl, *Rept.* 218.

at all atmospheric pressures—a fact of which there is evidence in the changing form of the power curve at high altitudes on unsupercharged engines. A standard Liberty engine, for example, attained its maximum power at 30,000 ft. at an r. p. m. fully 6 per cent lower than the rate of rotation for maximum power

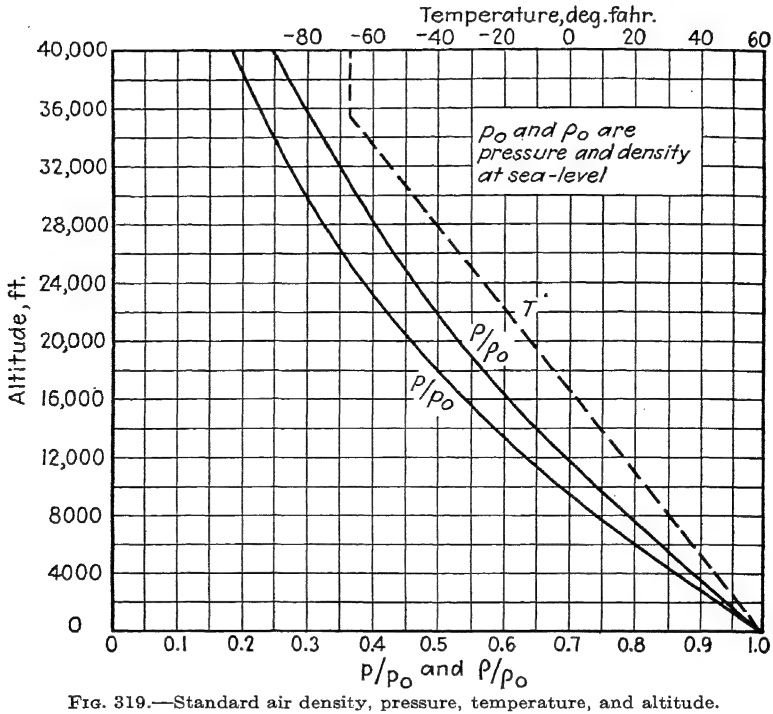


FIG. 319.—Standard air density, pressure, temperature, and altitude.

near sea level.[1] All this, however, is minor—small enough to neglect in dealing with most practical problems.

The British have done the most extensive experimental work on the relation between altitude and power. Gove[2] has provided the simplest and most complete theoretical formula, based on the assumptions (1) that indicated power varies directly

[1] *Rept.* 101, *cit. supra.*

[2] *Rept.* 295, *cit. supra.*

as the air pressure, inversely as the square root of the temperature; (2) that the losses representing the difference between brake and indicated horsepowers are divisible into two parts, the pumping losses which are proportional to indicated power and the frictional losses which are entirely independent of it, and that the two are roughly equal. The first two of the three factors outlined are thus taken care of, the third completely neglected, and the curves computed by Gove's formula prove to be in fair agreement with those experimentally obtained. In making use of the standard atmosphere to convert Gove's results back into terms of density, it develops that if friction is assumed to account for just half the mechanical losses the power at constant r. p. m. should vary approximately as $\rho^{1.19}$ where the mechanical efficiency is 92 per cent and as $\rho^{1.30}$ with an efficiency of 80 per cent.

The figures so obtained roughly bracket the field of practical experience, except with the now obsolete rotary engine, on which exceptionally high mechanical losses were accompanied by an exceptionally rapid drop of power with pressure. Though there are cases (the rotary engines excluded) of an apparent formula of variation as high as $\rho^{1.35}$ or as low as $\rho^{1.15}$, the typical exponent in modern engine-design practice clings very close to 1.2. If power is assumed to vary as $\rho^{1.2}$, no serious error is likely ever to result. The curve of power variation corresponding to that formula is given in Fig. 320.

Gove's method, or any other in which the mechanical efficiency of the engine is the controlling factor, actually produces a curve that is not of a true exponential form. There has been included in Fig. 320 a curve of the differences between the power ratios as given by $\rho^{1.2}$ and those obtained by Gove's formula on the assumption of a mechanical efficiency of 91 per cent (that being selected as the figure that fits best with an exponent of 1.2). The difference never exceeds 0.5 per cent of the sea-level power until the altitude exceeds 29,000 ft. If it be assumed (which there is no sound reason to believe) that the Gove method permits of a computation rigorously correct to a string of decimal places, the error resultant from using the exponential form would be no more than 0.3 per cent in maximum speed at 15,000 ft. altitude (and less than that at all other altitudes) or 250 ft. in the ceiling of a machine capable of climbing to 30,000 ft. Only in computing the performances of an airplane able to go to 35,000 ft. or better

would the difference of the two power curves begin to have a really appreciable effect, and machines with ceilings as high as that invariably use supercharged engines, for which the error due to the use of the exponential power curve becomes alarming only at some 30,000 ft. above the critical supercharging altitude.[1]

If the temperature of the air entering the carburetor differs from the standard temperature for that altitude as plotted in Fig. 319, the power ratio will be changed approximately, in the

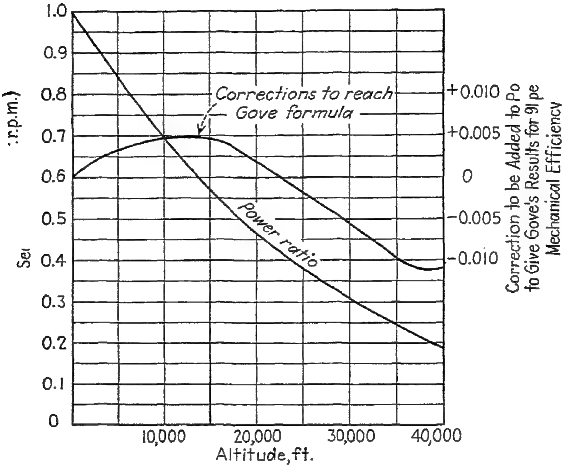


FIG. 320.—Variation of power with altitude, by $\rho^{1.2}$ rule and by Gove formula.

average case, in the ratio $\sqrt{T_o/T}$, where T is the actual temperature and T_o is the standard temperature for the altitude—both on the absolute scale. The exact relationship between air temperature and power in any particular case would depend upon the design of the intake system, upon where the air was taken in, and upon whether or not the incoming air was artificially heated, but no formula has been devised that will give results better than those obtained by a universal use of the square-root rule.

The next step, and the first on the road to the analysis of supercharging, is the reintroduction of manifold pressure as a

[1] Altitude up to which the power is maintained substantially at its sea-level value.

variable. With a constant throttle opening and either a complete absence of supercharging or a perfectly constant amount, the manifold pressure drops off approximately in proportion to, or a shade less rapidly than, the atmospheric pressure. For a radial engine with rotary induction for which particularly complete data are available, the pressure in the manifold varies in proportion to the 0.97 power of that at the air intake.

When the manifold pressure is held constant with changing altitude, either by increasing the amount of supercharging action at high altitudes or by closing the throttle progressively at low ones, the first impulse is to expect the power to remain constant.

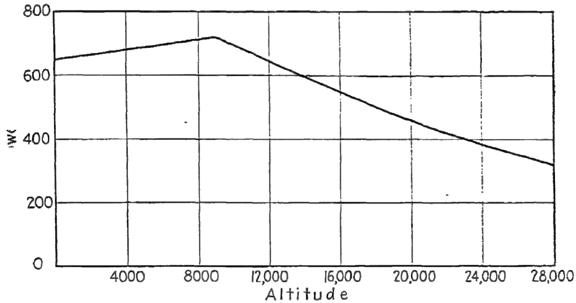


FIG. 321.—Power variation with altitude for a supercharged engine.

It would, were it not for the change of atmospheric back pressure against the exhaust and (where the control of manifold pressure is by throttling) for the reduced amount of energy expended in driving the intake blower in the more rarefied air of the upper altitudes. Taking those factors into account, the power at constant manifold pressure and constant r. p. m. shows a steady rise with altitude. On the average, the variation is as the inverse cube root of the density. To take a still simpler rule, the power of a typical rotary-induction engine increases by 1 per cent for every 1,000 ft. of altitude with the manifold pressure held to a fixed level.

It follows that if a fixed maximum limit is imposed on the manifold pressure the curve of altitude against power will have the form illustrated in Fig. 321. At a certain altitude (9,000 ft. in the illustration) the allowable limit of pressure can just barely be attained with the throttle fully open. That is the critical

altitude. At any lower level the throttle must be partially closed to prevent an excess of pressure and so of internal strain. At any higher one the full-throttle manifold pressure will be below the limit, farther and farther below it as the operating altitude rises farther and farther above the critical one.

Under this condition the common assumption that a supercharged engine develops full sea-level power up to a certain altitude and falls off in accordance with some function of air density or atmospheric pressure from there on is too simple. Only at one altitude is the maximum power attained, and from that maximum it falls off in both directions—slowly as the altitude diminishes, rapidly as it increases.

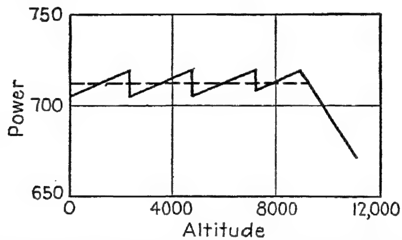


FIG. 322.—Power curve below critical altitude, with periodic increase of manifold pressure during descent.

There is, however, an alternative method; less common, but simpler for the performance analyst. Instead of limiting the manifold pressure, the limit may be placed on the power. In that case the pilot is instructed to control manifold pressure as a function of altitude, allowing it to rise in round numbers by ½ in. of mercury every 2,000 ft. below the critical altitude. The low-altitude segment of the power curve then takes the jagged form of Fig. 322, and with sufficient accuracy for practically any purpose the power can be assumed constant at the mean value shown by the dotted line.

To combine a series of such curves as that of Fig. 321, for as many different rotational speeds, and to cut across them with curves of constant manifold pressure to show the pressure corresponding to each altitude and r. p. m. with the throttle fully open, produces Fig. 323. Thus the representation of engine output and its variations becomes complete. Power, altitude, r. p. m., and manifold pressure are fully tied together by two

sheaves of curves, and the relationship among any particular pair of quantities can be read off and if necessary plotted. The illustration in this case is prepared for an engine with a high blower-gear ratio (10 to 1) and a high degree of supercharging. The heavy lines on the chart show the recommended cruising condition, the maximum-continuous-output condition, and the

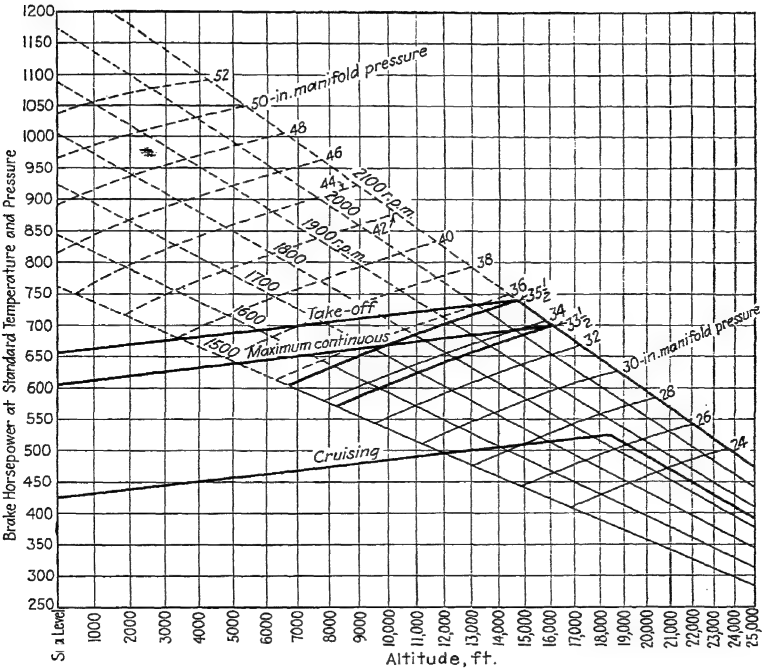


FIG. 323.—Full-throttle power chart for a typical radial engine.

allowable condition for take-off or momentary emergency. The nearly horizontal straight lines running over to the left-hand margin indicate the power that may be drawn off below the critical altitude at the appropriate limiting speed (2,100 r. p. m. for full power and take-off on this particular engine, 1,850 r. p. m. for cruising). The heavy arcs of constant manifold pressure show similarly the allowable maxima at varying r. p. m. without exceeding the set pressure maximum. Thus, for example, at

10,000 ft. the engine can with reasonable safety be made to supply 670 hp. at 2,100 r. p. m., but at 1,620 r. p. m. only 607 hp. may be taken. At any intermediate speed the allowable load can be determined by drawing inclined straight lines, parallel to the limiting power lines already drawn for maximum speed, through the intersection of the appropriate r. p. m. line and the curve of maximum allowable pressure. A construction of that order for the 10,000-ft. level would indicate an allowable power of 635 at 1,800 r. p. m.

The region in which the curves are dotted, above and to the left of the take-off line, is purely theoretical in the sense that the engine ought never under any circumstances to be allowed to operate there and that there is imminent danger of trouble when it does so. It must be emphasized once more before leaving the subject that Fig. 323, except the three nearly horizontal lines indicating the limiting allowable conditions at rated r. p. m. and below the critical altitude, is strictly a full-throttle chart. The same sort of diagram could be constructed, were the construction to appear of any practical interest, for any other throttle opening. The manifold pressure (and the power) for every engine speed and altitude would be reduced, all very much in the same proportion, as the throttle restricted the passage and so increased the difference of head between the air intake and the manifold. Conversely, a chart like Fig. 317, tying together manifold pressure, r. p. m., and power, could be constructed for any altitude.

The data for such curves as those of Fig. 323 are obtained in the laboratory, with the air flowing normally to the carburetor in response to the pressure gradient in the intake system. When the engine is installed on an airplane, however, air is often received through an intake of which the open end points forward. An additional pressure is built up by the motion of the airplane. The air pressure at the intake, instead of being equal to the static pressure prevailing at the altitude at which the machine is flying, is raised by the dynamic effect. The maximum possible pressure increase, referred to as *ram*, is equal to $\frac{1}{2}\rho V^2$, and the actual increase often comes very close to that figure. The exact amount of ram existing of course depends upon the form and location of the air intake.

At an altitude of 10,000 ft. and a speed of 150 m. p. h., $\frac{1}{2}\rho V^2$ is equal to 40 lb. per sq. ft., which in turn is equal to the pressure

change due to a decrease of altitude of from 10,000 to 9,300 ft. At 200 m. p. h. the maximum ram would be equivalent to an altitude decrease of more than 1,200 ft. An engine capable of maintaining its sea-level power only up to a height of 10,000 ft. under laboratory conditions may then, by taking advantage of ram, maintain power to an actual altitude of 11,000 ft. or more in flight at high speed. Though the effect obviously depends on the exact speed, one will not go far wrong for modern airplanes in general by assuming that ram increases the critical altitude

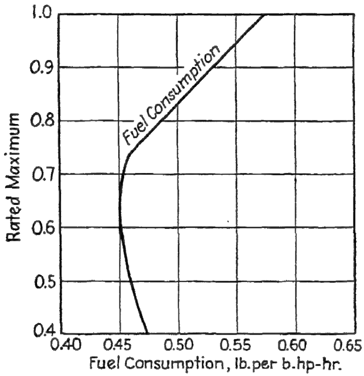


FIG. 324.—Variation of specific fuel consumption with power factor.

by 1,000 ft. at maximum speed or in cruising, by 500 ft. in climb or take-off.

There remains only the examination into brake thermal efficiency or its equivalent, which is fuel consumption. Consumption is primarily a function of power, variations with r. p. m. and altitude being secondary.

The design of intake system that is right for one speed and load cannot be ideal for all, and specific consumption, or fuel used per horsepower-hour, inevitably rises from its minimum for any change of conditions from the ideal. Naturally the engine designer's object is to keep the minimum as near as possible to the standard cruising condition, where most of the flying will be done, and Fig. 324 shows a typical sea-level consumption curve. The curve's minimum of 0.45 lb. per hp.-hr. is about average for good modern practice.

Five years ago 0.50 was an excellent figure for minimum cruising consumption. Improved engine design, and especially improvements in the quality of fuels, have now lowered that figure by about 10 per cent. Specific consumptions of 0.40 are already being approached in the laboratory, and may be hoped for in practical cruising operation within three or four years to come. The diesel engine holds promise of specific consumptions as low as 0.36 or even lower, but for the time being its use is largely experimental and the consumptions that must be counted on are those of the familiar four-stroke Otto-cycle construction.

In general the minimum runs lower for large engines (400 hp. and above) than for small ones, and it increases with the amount of supercharging, as an increasing proportion of the fuel has to be burned in overcoming mechanical losses in the high-geared intake-blower system. Consumption is lowest for a specific power at low r. p. m. and high manifold pressure, and if it were not for the bad effect of high explosion pressures on an engine's reliability and life it would be advantageous to keep cruising r. p. m. considerably below present limits and let the manifold pressure rise correspondingly above the allowance of current practice.

Consumption tends to decrease with increasing altitude, for the same reason that makes the power at a given manifold pressure tend to increase. Lowered back pressure improves the net efficiency of the engine, and at critical altitude the consumption is likely to be from 5 to 10 per cent lower than at sea level. In cruising, the consumption falls off with increase of altitude as long as the cruising power can be maintained without increase of r. p. m.; but when the altitude is reached where full-throttle operation will no longer give the desired cruising manifold pressure, cruising power must be maintained, if at all, by increasing the r. p. m. The specific consumption then begins to rise quite rapidly. In Fig. 323, for example, if 500 hp. were to be developed for cruising at a height of 22,000 ft., it would be necessary to run the r. p. m. up from the normal 1,850 to almost 2,000, since full-throttle manifold pressure at that altitude would be only 24.5 instead of the desired 26.8. The consumption in cruising at 500 hp. at 22,000 ft. and 2,000 r. p. m. would undoubtedly be at least 6 per cent higher than the consumption at the same power output at 18,500 ft. and 1,850 r. p. m.

CHAPTER XIV

PROPELLER CHARACTERISTICS

The air propeller is fundamentally a combination of an aerodynamic device and a dynamic one. Aerodynamically, its blades act as a couple of rotating wings, of which the drag acts (approximately) in the plane of the propeller and creates a resisting torque to be overcome by the torque applied through the shaft while the lift is directed (approximately) parallel to the shaft and becomes the propeller thrust. Dynamically, it is an instrument for receiving a certain mass of air approaching the propeller on converging streamlines from the front and accelerating it vigorously to the rear, the boundaries of the stream necessarily constricting as its velocity increases. The momentum law so generally applicable to the problems of fluid motion[1] applies here, and the propeller thrust is equal to the product of the mass of air handled per minute by the amount by which its sternward velocity is increased from the figure prevailing at a distance from the propeller. In symbolic form,

$$= \frac{\pi}{4}D^2\rho\left(\frac{V + V_s}{2}\right)(V_s - V)$$

where T is the thrust, V_s the velocity of the air in the slipstream behind the propeller (the velocity being measured relative to the propeller), D the propeller diameter, and V and ρ have their usual significance.

For further detail on the propeller's internal economy there are a variety of specialized texts available.[2] Concern here is only with the final performance, with the laws that govern it and control its mutations, and with the factors that bear upon the adjustment of the propeller to the characteristics of the engine, on the one hand, and the airplane, on the other.

[1] See p. 21, *supra*.

[2] The best of them being "Aircraft Propeller Design," by F. E. Weick, McGraw-Hill Book Company, Inc., New York, 1930.

For reasons which may be left to the propeller texts to explain, the expression of propeller behavior revolves around the non-dimensional ratio V/nD as airfoil performance revolves around the angle of attack. V is the speed of flight; n the rate of rotation of the propeller; D the diameter; and to make the ratio a pure number by keeping the units homogeneous the quantities must be expressed in ft. per sec., in r. p. s., and in feet, respectively. V/nD is commonly known as the slip function, for when it is equal to the ratio of the propeller's pitch[1] to its diameter, the slip as ordinarily defined is zero. For lower values of V/nD the slip is positive, and by an increasing percentage as the ratio is further reduced; for higher values it becomes negative. Following this definition and brief explanation the concept of slip can be consigned to limbo, for although marine engineers put it to extended use in solving their propulsion problems it plays but little part in aeronautics. Pitch, too, has virtually disappeared from the aeronautical vocabulary. A much more convenient geometrical measure is found in a specification of β, the blade angle, at some specified radius, usually either three-quarters of the extreme radius (three-eighths of the diameter) or 42 in. For a diameter of 9 ft. 4 in., which is a little below the average of current design practise, the two bases of specification coincide.

Propeller Coefficients.—Definite mathematical expression of propeller characteristics is limited to what happens at a constant value of V/nD, precisely as the fundamental equations of lift, drag, and moment for an airfoil can be applied with a given set of coefficients only at a particular value of α. With that condition met, the fundamental equations can be written

$$T = C_T \rho n^2 D^4 \tag{82}$$
$$Q = C_Q \rho n^2 D^5 \tag{83}$$
$$P = C_P \rho n^3 D^5 \tag{84}$$

[1] Commonly defined as the distance that the propeller would advance in one revolution if the blade sections were cutting their way through a substance so unyielding as to allow no slip. Mathematically, it equals $2\pi r \tan \beta$, β being the angle between the plane of rotation and the chord of the section of the blade at radius r. The pitch is not necessarily—and indeed it generally is not—perfectly uniform from root to tip. As a rule, however, it lies within 10 per cent of a constant value over the effective portion of the blade (excluding only the part within a few inches of the shaft).

Q being the torque and P the power consumed, and C_T, C_Q, and C_P being experimentally determined coefficients that are plotted against V/nD precisely as lift and drag coefficients are plotted against the angle of attack.

In dealing with propellers there are three semi-independent variables, V, n, and D. Sometimes it is most convenient to express the characteristics in terms of one group of variables; sometimes of another; and any one of the three can always be eliminated. Since V/nD is non-dimensional, and since C_T and its companions are determined only for particular values of that quantity, (82) to (84) can always be multiplied by V/nD to any desired power. The thrust, power, and torque formulas can then be thrown into the alternative forms

$$\begin{cases} T = C_T'\rho V^2 D^2 & (85) \\ P = C_P'\rho V^3 D^2 & (86) \\ Q = C_Q'\rho V^2 D^3 & (86a) \end{cases}$$

$$T = C_T''\rho\frac{V^4}{n^2} \tag{87}$$

$$P = C_P''\rho\frac{V^5}{n^2} \tag{88}$$

$$Q = C_Q''\ V^5 \tag{88a}$$

A primary object is of course to get as much work as possible out for as little as possible put in. The output is the product of thrust and speed; the input is the engine power; the propeller efficiency is the ratio of the two. Thus

$$\eta = \frac{TV}{P} = \frac{C_T'}{C_P'} = \frac{C_T}{C_P} \times \frac{V}{nD} \tag{89}$$

η being the propeller efficiency.

In Figs. 325, 326, and 327 are plotted the three sets of thrust and power coefficients and the efficiency for a typical propeller. The curves show certain characteristics that are universal. The propeller blade, as an airfoil, operates at an angle of attack steadily diminishing as the slip decreases. The consequence is a steady drop both of thrust and of power required (except at the very lowest speeds) with increasing V/nD. At a certain point the thrust becomes zero; at another point, farther to the right

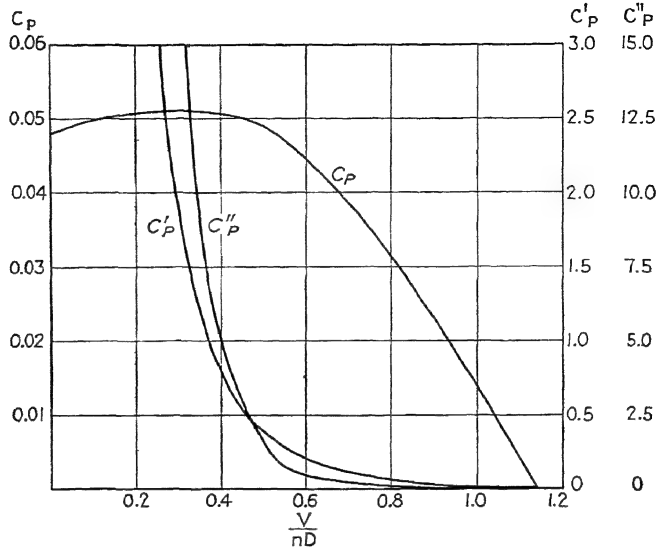


Fig. 325.—Power coefficients for a propeller.

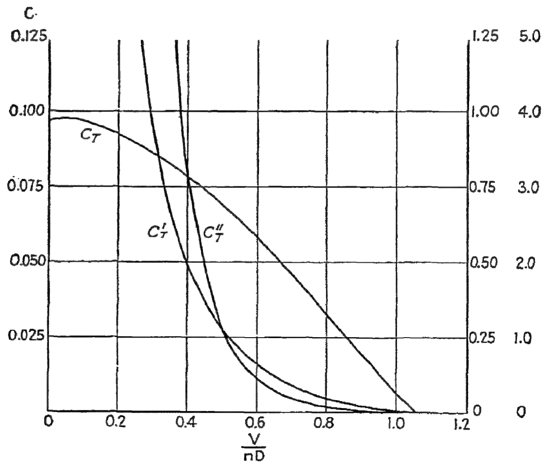


Fig. 326.—Thrust coefficients for a propeller.

along the scale, the power reaches zero; beyond the second point the propeller ceases to be one and becomes a windmill. In a steep dive the pressure of the air against the propeller blades may

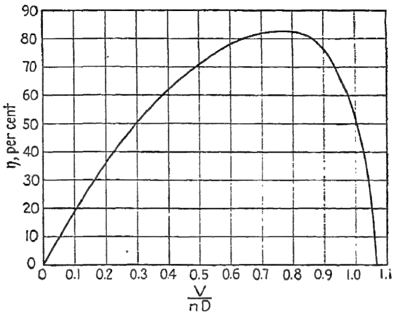


FIG. 327.—Efficiency curve for a propeller.

crank them around at a rate that threatens the dismemberment of the engine, even though the throttle is fully closed at the time.[1]

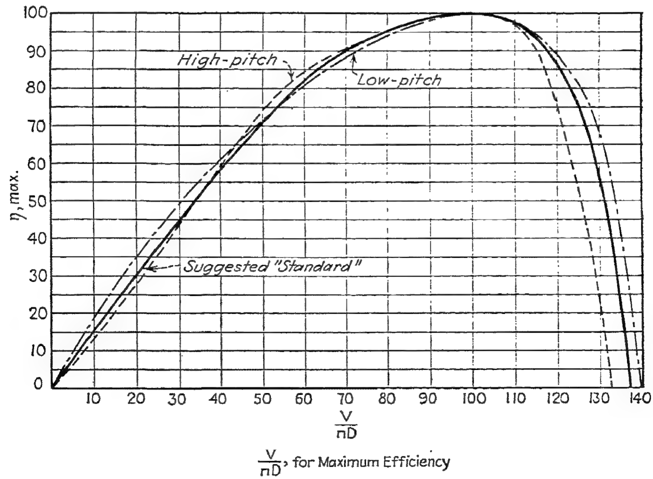


FIG. 328.—Two relative efficiency curves, and a suggested standard.

In the particular case for which the curves are drawn, zero thrust comes at $V/nD = 1.05$; zero power at 1.15.

[1] See p. 510, *infra*.

Propeller efficiency, being a fraction of which the numerator is the product of two variables, necessarily has two points of nullity: one for zero velocity, the other for zero thrust. Between the extremes is a maximum, usually at about 70 per cent of the V/nD for zero thrust. Though not all propellers have efficiency curves of identical general form by any means, there is enough similarity to invite the development of a standard curve that may be used in calculations where no specific propeller design is in question. Figure 328[1] shows two efficiency curves, for propellers of very high and very low pitch respectively, with both ordinates and abscissas reduced to ratios to facilitate comparison of form. It shows also a curve, lying between the other two, fairly representing average present practice and hence suitable for use as a standard.

Choosing a Propeller.—In selecting a propeller, the only quantities initially known are the engine power, the anticipated speed of flight, and the r. p. m. The combination of that particular group appears in (87), and C_P'' can be computed. The value of V/nD corresponding to the computed C_P'' can be read off from the experimental curve for any propeller that has been tested in a wind tunnel; D can then be calculated as the only unknown in the equation. Illustration best explains the method. Suppose it required, for instance, to find the diameter of a two-bladed propeller, of the form for which Figs. 325 to 327 were drawn, that will consume 250 hp. at 150 m. p. h. and 2,000 r. p. m. The work proceeds:

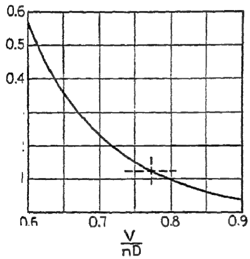


Fig. 329.—Power coefficient at high values of V/nD.

$$
\begin{aligned}
250 \text{ hp.} &= 250 \times 550 = 137{,}500 \text{ ft.-lb. per sec.} \\
150 \text{ m. p. h.} &= 220 \text{ ft. per sec.} \\
2{,}000 \text{ r. p. m.} &= 33.33 \text{ r. p. s.} \\
C_P'' &= \frac{137{,}500 \times (33.33)^2}{0.00237 \times (220)^5} = 0.125
\end{aligned}
$$

[1] The data for these curves were drawn from "Full-scale Wind-tunnel Tests of a Series of Metal Propellers on a VE-7 Airplane," by Fred E. Weick, *Rept.* 306.

$$\frac{V}{nD} \text{ (from curve)} = 0.771$$

$$D \quad \frac{220}{33.33 \times 0.771} = 8.56 \text{ ft.}$$

To make the reading of V/nD easier, the right-hand portion of the C_P'' curve has been plotted to a much enlarged vertical scale in Fig. 329. It is none too easy at best, and the method cannot be considered a highly sensitive one. It has the advantage of extreme directness, but after the result has been obtained it is

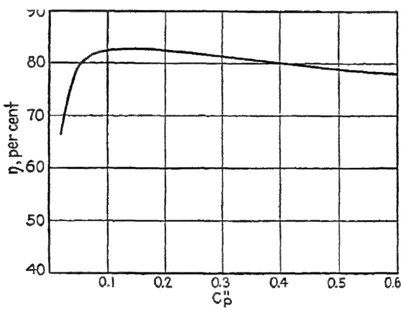


FIG. 330.—Efficiency and power coefficient, with diameter eliminated as a variable.

well to check it back through C_P. Thus, in this case, C_P at 0.771 being 0.0674,

$$P = 0.0337 \times 0.00237 \times (33.33)^3 \times (8.56)^5 = 136{,}300$$

A check within 1 per cent on power consumption, corresponding to a deviation of less than 10 r. p. m. from the rated figure, is as much as one has a right to ask for and is probably within the limits of accuracy of propeller testing in the wind tunnel. Seekers after the utmost exactitude, however, may add 0.013 ft. to the diameter and the check will be perfect.

V/nD for this particular case happens to have come out reasonably close to the value for maximum efficiency. As a general rule it will not, and to get the best possible results some one of the propeller's geometrical characteristics (usually the blade angle, or pitch) will have to be changed. To simplify the comparison of the various forms offered, Weick has devised the expedient of plotting efficiency directly against C_P'' or, better

yet, $1/\sqrt[5]{C_P''}$, since that avoids the very rapid change of slope with V/nD that characterizes the C_P'' curve and also because it gives a function which for any particular engine varies directly as V. The inverse fifth root of C_P'' is of enough importance to deserve a simpler symbol of its own. Weick has denoted it, and it will be denoted here, by C_S.[1] Figures 330 and 331 show the

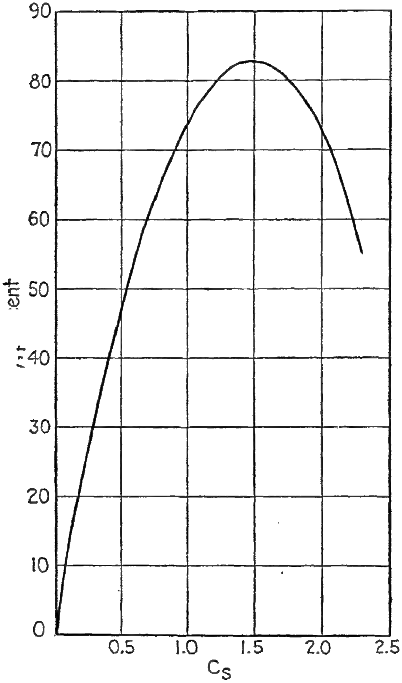


FIG. 331.—Efficiency and propeller-selection coefficient.

two forms of plot, applied to the propeller already studied. Figure 332[2] shows the latter of the two (C_S) for a conventional adjustable-metal-blade propeller tested with the blades set at several different angles. Most present design practice is within the range shown, though 40 deg. is not unknown. The dotted

[1] "Working Charts for the Selection of Aluminum Alloy Propellers," by Fred E. Weick, *Rept.* 350.

[2] Data from tests by the N. A. C. A.

line enshrouding the five full curves is the envelope of an imaginary infinite series of tests at blade angles infinitesimally differing,

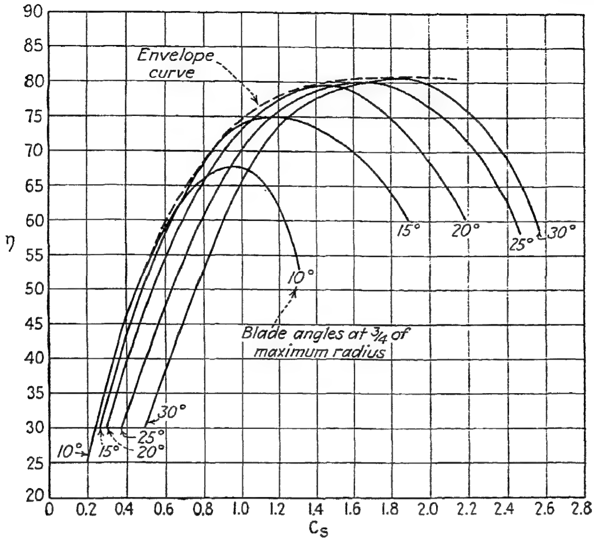


FIG. 332.—Variation of propeller-selection coefficient with blade angle.

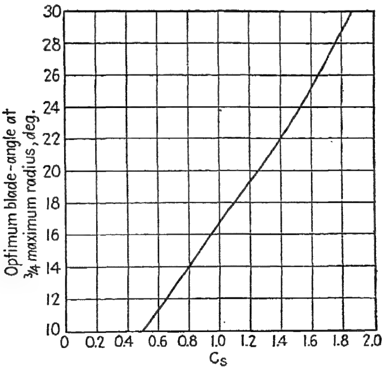


FIG. 333.—Variation of optimum blade angle with change of propeller-selection coefficient.

and Fig. 333 is a curve of optimum blade angle against C_S. Though drawn for a particular blade form, the values plotted in

Fig. 333 would be unlikely to change by more than 2 or 3 deg. for any other blade design within the range of normal metal-propeller practice.[1]

Figure 333 shows the optimum blade angle if nothing except performance at a single value of C_S is to be considered (as may be the case with an airship). It does not necessarily give the best angle for use on an airplane which has to operate as efficiently as possible not only at its design speed (which is usually the maximum flight speed or very near it) but also at much lower speeds (in taking off and climbing). The way in which the

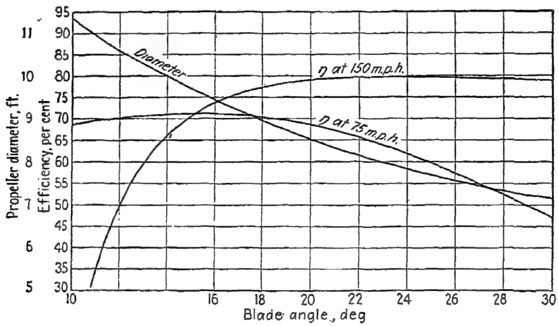


FIG. 334.—Variation of required diameter and propeller efficiency with change of blade angle used.

curves are staggered across each other makes each attain its brief moment as the optimum at a point considerably to the left of its own maximum efficiency. The 15-deg. setting, for example, shows its maximum efficiency in Fig. 333 at $C_S = 1.2$, but at that value of C_S a considerably higher performance can be secured by using a 19-deg. blade angle. If the larger angle is chosen, however, its output at the lower flight speeds will be considerably inferior to that of the 15-deg. blade.

This is somewhat confusing at best until one has become saturated in the subject by innumerable applications to innumerable special problems. To give some idea of the range of choice that a designer may have, there are presented in Fig. 334 curves of propeller diameter, of efficiency at 150 m. p. h., and of efficiency at 75 m. p. h., against blade angle—all for an engine developing

[1] They would undergo some change with changing conditions of interference, but that point will be taken up separately (pp. 519, 527, *infra*).

250 hp. at 2,000 r. p. m. and with a propeller required to absorb the full power at that rate of rotation and at 150 m. p. h. The best efficiency at top speed is to be had with a blade setting of 24 deg. and a diameter of 7.8 ft., but clearly it would be desirable to move well to the left of the peak of the high-speed efficiency

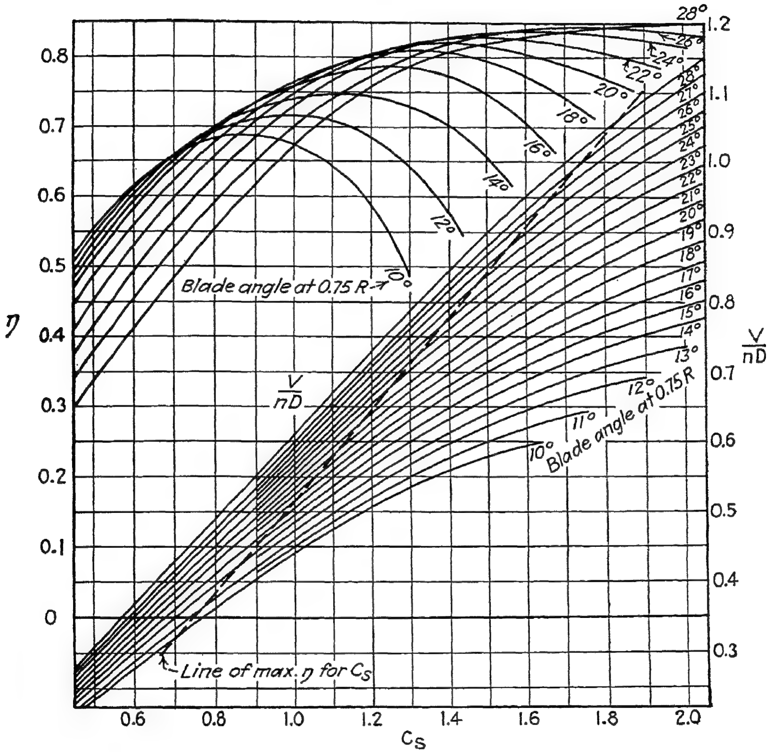


FIG. 335.—Propeller-selection chart.

curve to get a better low-speed performance. Specifically, the best arrangement for a typical plane with such a power plant would appear to be a 20-deg. blade angle and a diameter of 8.25 ft. The efficiency at top speed would be only 1 per cent lower than with the larger angle, and that at half speed would be 8 per cent higher. Where take-off is a serious factor, so that static thrust

must be given special consideration, a still lower blade angle and a still larger propeller diameter are likely to be indicated.[1] The angle could be anything between 17 and 22 deg. without really serious effects on the efficiency at either speed. The diameter, in turn, could be anything between 8.1 and 9.3 ft., and, having chosen a diameter within that range, the full-throttle r. p. m. at top speed could be brought to 2,000 by a trial-and-error adjustment of the blade angle. Such are the simplifications of the propeller problem that adjustable blades of metal have brought, for their predecessors of wood or molded plastic had to give the right answer the first time or be started over.

Figure 334 was built up, as many similar studies may be, from one of a set of propeller-selection charts developed by Weick from data taken out of the N. A. C. A. wind tunnel.[2] One of these is reproduced in Fig. 335. After C_S for a particular problem has been determined, the values of V/nD for as many blade angles as may be of possible interest can be read off from the lower sheaf of curves, and the diameter required with each angle computed therefrom—the corresponding efficiencies from the upper sheaf.

Propeller Diameter by Formula.—From these charts, also, we can derive a simple formula for approximating the propeller diameter that is to be allowed for in design, before the point of choosing a particular propeller has been reached. The line across Fig. 335 that shows the best propeller for each value of C_S corresponds roughly to the equation

$$\frac{V}{nD} = 0.5C_S^{1.2} \tag{90}$$

By combining terms and simplifying, transforming V and n into m. p. h. and r. p. m., respectively, and putting them into ratios conveniently grouped for mental manipulation, this becomes the formula

$$D = 10.0\frac{(P/100)^{0.24}}{(V/100)^{0.2}(n/1{,}000)^{0.52}} \tag{91}$$

[1] See p. 493, *infra*.

[2] "Working Charts for the Selection of Aluminum Alloy Propellers," by Fred E. Weick, *Rept.* 350.

Odd fractional exponents being somewhat inconvenient, minor adjustments are made to bring all the exponents to common fractions with a common denominator of 4. Taking into account at the same time the desirability of increasing diameter and reducing blade angle to improve the performance at flight speeds below the maximum, the final formula for optimum diameter becomes

$$D = 10.8\sqrt[4]{\frac{P/100}{(V/100)(n/1{,}000)^2}} \tag{92}$$

The variables, let it be noted again, are in hp., m. p. h., and r. p. m., respectively, and V is the maximum speed.

This formula is, of course, good for sea-level conditions only. If the propeller is basically designed to operate at some other altitude, the factor $\sqrt[4]{\rho_0/\rho}$ must be multiplied in to secure the diameter. This can be taken, with a probable error never exceeding 1 per cent, as increasing the required diameter by 0.8 per cent for every 1,000 ft. of altitude. Note, also, that the formula corresponds to the selection of a propeller for all-round performance over the whole operating range. If top-speed performance is of paramount interest, climb being comparatively negligible (as on airships), the coefficient can be reduced from 10.8 to 10.0.

These diameter formulas are limited in direct application, too, to two-bladed propellers. But if three blades are to be used it is only necessary to lower the coefficient from 10.8 to 10.0, while with a four-bladed propeller the reduction is to 9.5.

Efficiency and Pitch.—Figures 332 and 335 show a universal phenomenon, the increase of maximum efficiency with increasing pitch. Up to a pitch/diameter ratio of well over unity the gain continues, and in Fig. 336[1] the maximum that may reasonably be expected is plotted against the V/nD at which the maximum appears. The curve for wooden propellers represents the performance of propellers designed to absorb 300 hp. or more and to run at high tip speed, and with blade sections made inefficiently thick to insure structural safety. A wooden propeller for a low-powered engine may show a peak efficiency substantially on the uppermost of the two curves.

In practically every modern aircraft, either airplane or airship and for whatever purpose it may be destined, C_S lies between

[1] From Army Air Corps data.

0.8 and 2.4. In nine airplanes out of ten, if small machines built for private use are excluded, it falls between 1.4 and 2.1. At minimum speed, it may fall as low as one-third of its full-speed value. To simplify its calculation, it may be remembered that

$$C_S = 1.6 \frac{V/100}{\sqrt[5]{(P/100)(n/1{,}000)^2}} \tag{93}$$

where V on the right-hand side of the equation is in m. p. h., P in horsepower, and n in propeller r. p. m.

If a blade angle of 28 deg. is selected for maximum speed performance at a C_S of 2.1, and if that function then drops to 0.7 at leaving the ground, the propeller efficiency will (from Fig.

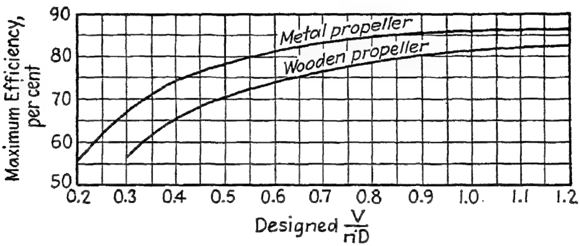


FIG. 336.—Maximum efficiency to be expected, as a function of V/nD.

335) drop to 48 per cent where with an angle of 18 deg. it would be 64 per cent. The desirability of a blade angle adjustable during flight is already apparent, but analysis of the effects of realizing the desire must wait upon a study of how changes of flight speed react through the propeller upon the engine.

Change of R. P. M. with Speed.—As the speed of flight changes, the aerodynamic load on the propeller blades changes with it, and the r. p. m. at full throttle show the result. With a drop of r. p. m., in turn, comes a loss of engine power.

This problem can best be attacked in general terms through C_P. Since

$$P = C_P \rho n^3 D^5$$

if C_P were constant there would be no change in n with changing V/nD. Actually there is little change of C_P with V/nD below 0.5, but above that level the change becomes more and more rapid. In the region neighboring maximum efficiency, C_P

can be taken as very roughly inversely proportional to V/nD. On that assumption, for a given propeller,

$$P = K_1 \rho \frac{n^3}{V} \tag{94}$$

If it is further assumed that the engine torque is constant and the power delivered therefore proportional to r. p. m., and that ρ is fixed, this further simplifies to

$$K_2 n = K_3 \frac{n^3}{V} \tag{95a}$$

$$\frac{n^2}{V} = \text{const.} \tag{95b}$$

The r. p. m. ought then to be approximately proportional to the cube root of the speed at high speeds, grading off into substantially complete independence of flight speed when the speed is low.

To confirm or to disprove that approximation, an actual calculation can be made for a particular case. Calculation can proceed either from the actual power curve of the engine concerned or from the assumption that the torque is independent of the r. p. m., the second alternative being close enough to serve all practical purposes in nine cases out of ten. If the constant-torque assumption is used, it is necessary to proceed directly from the equation involving C_Q' if the solution is to be direct, avoiding trial and error; that is the simplest basis, too, for the exact computation.

Since

$$Q = C_Q' \rho V^2 D^3$$

with the engine torque known and after the propeller has been chosen, it is possible to solve directly for C_Q'. C_Q' is plotted in Fig. 337 for the propeller for which Fig. 335 was prepared. The plot is logarithmic, so that the ordinate may be read off with an equally small relative error at any point.

If a constant torque were to be assumed, it would only be necessary to read off the value of V/nD corresponding to the value of C_Q' computed for each of several flight speeds and to convert V/nD into n into n for the known speed and propeller diameter.

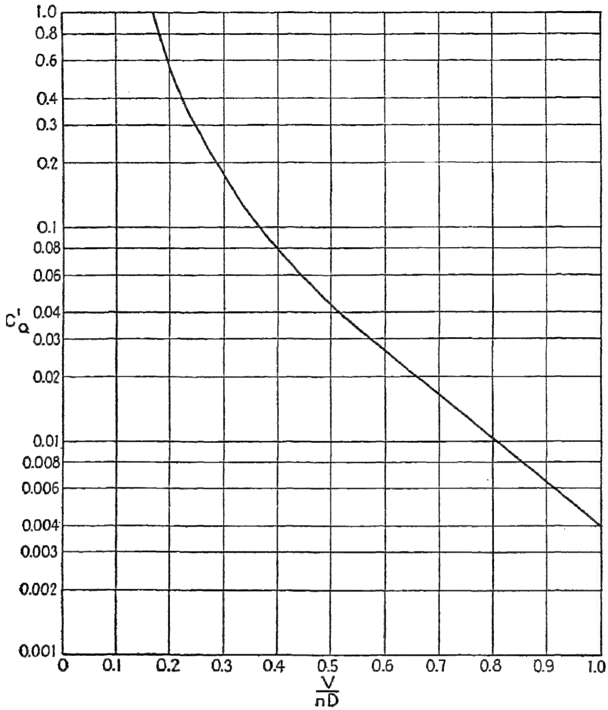


FIG. 337.—Variation of torque coefficient with V/nD.

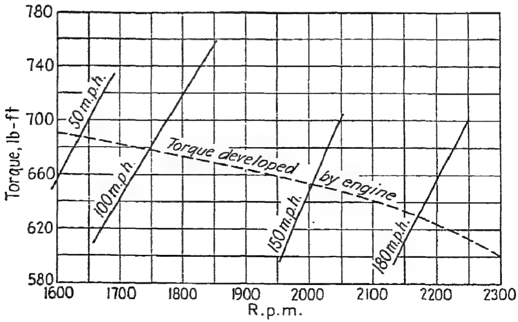


FIG. 338.—Determination of propeller r. p. m. at various flight speeds.

If the actual torque variation is to be allowed for, however, another graphical step is necessary. For each speed, there must be plotted a curve of torque against r. p. m., the former being directly proportional to C_Q' and the latter inversely proportional to V/nD. Each of the curves plotted in Fig. 338 is then a re-working of a short section of the curve plotted in Fig. 337. The actual torque curve of the engine can then be plotted on the same sheet, as has been done in Fig. 338, and its intersections with the several torque-required curves give the rates of rotation at the several speeds. At zero speed of advance these methods of course do not work, as C_Q' would then be infinite, but V/nD

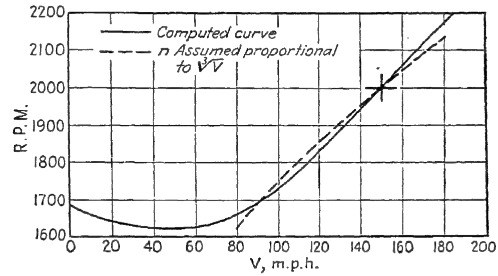


FIG. 339.—Variation of r. p. m. with flight speed, actual and approximated.

is then known to be zero, so C_Q can be used instead and the equation

$$Q = C_Q \rho n^2 D^5$$

can be solved directly for n.

The result of the calculation, for a particular case, is plotted in Fig. 339, with r. p. m. as the ordinate and speed the abscissa. $\sqrt[3]{V}$ has been superposed for comparison, and the crude analysis that led to its selection is supported by a fair agreement, over the range within which the preliminary assumptions applied even approximately, between the mathematical approximation and the actual curve computed from experiment. The r. p. m. vary roughly as $\sqrt[3]{V}$ down to 55 per cent of the designed operating speed, and below that point they are roughly constant.

Calculations for a particular propeller on this point are hardly necessary as a rule, for there is so little variation that a standard r. p. m. curve can be assumed. If any individual deviations

are to be allowed, propeller pitch is the factor that should be taken into account, for a given proportional change of air speed has less relative effect upon the engine speed with a low-pitch propeller than with a high-pitch one. Suggested standard curves for propellers giving their best efficiency at $C_S = 1.8$ and $C_S = 1.0$, respectively, are given in Fig. 340.

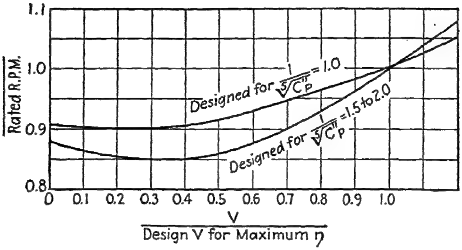


FIG. 340.—Dependence of r. p. m.-speed relationship on propeller pitch.

Propeller Revolutions and Altitude.—A propeller's power consumption is proportional to air density, other things being equal. If the power supplied at a given r. p. m. were to vary in the same ratio, the curves of efficiency and of r. p. m. against speed would be independent of altitude. As the previous chapter was largely devoted to showing, however, engine power follows no such simple law.

Reverting to the assumptions that created (95), treating the density now as a variable,

$$P = K_1\rho\frac{n^4}{V}$$

Following the rule given on page 459 for an unsupercharged engine, and again assuming that the torque is a function of the density alone,

$$K_2 n\rho^{1.2} = K_3\rho\frac{n^3}{V} \tag{96a}$$

$$\frac{n^3}{V\rho^{.2}} = \text{const.} \tag{96b}$$

n falls off approximately as the 0.07 power of the density ratio, and the full-throttle power actually developed by the engine at

the air speed for which the propeller is particularly designed varies as $\rho^{1.27}$.

To prove the case or to secure more accurate data for a specific engine-propeller combination, return to Fig. 337. The values of C_Q' for which V/nD is to be read off must obviously be changed from the sea-level figures in the same ratio in which Q/ρ is changed. For an unsupercharged engine, in other words, C_Q' will be changed in the ratio of $\rho^{0.2}$. In Fig. 341 are plotted

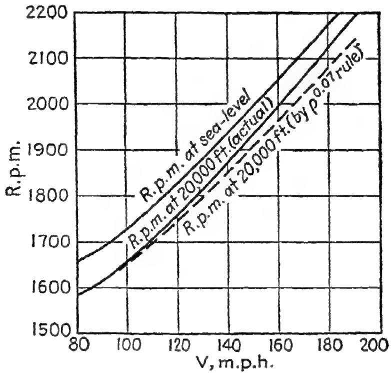


FIG. 341.—Variation of propeller r. p. m. with altitude on an unsupercharged engine.

the actual r. p. m.-speed curves computed for sea level and a 20,000-ft. standard altitude, and on the latter is superposed the curve constructed by reducing from sea-level r. p. m. as $\rho^{0.07}$. On the average, a variation as $\rho^{0.05}$ would give a better accord with the experimental curve in this particular case.

Propellers for Supercharged Engines.—If engine power is independent of altitude, (96) becomes

$$K_2 n = K_3 \rho \frac{n^4}{V} \tag{97a}$$

and

$$\frac{\rho n^3}{V} = \text{const.} \tag{97b}$$

n varies approximately as the inverse cube root of density, and with a 15,000-ft. critical altitude the engine would turn up 17 per cent faster than at the same air speed at sea level. Conversely,

if the throttle is closed enough to hold the r. p. m. to the rated figure, it will be found that the power developed must be decreased in almost exact proportion to the density. Eighty per cent of the effect of the supercharging will be nullified.

The only recourse, if a fixed-pitch propeller is to be used, is to an overloading of the engine at sea level so that the r. p. m. rising steadily with altitude will reach their rated value substantially at the critical height. Figure 342 shows the percentages of rated r. p. m. and power that should be taken as design conditions at 100 m. p. h. at sea level (for the propeller and engine to which Fig. 340 related) in order that the r. p. m. at the critical altitude and 150 m. p. h. may be 2,000. The assump-

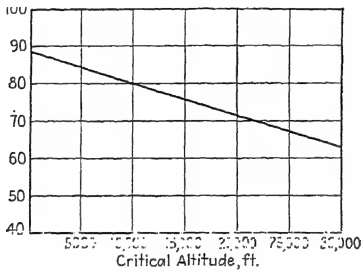


FIG. 342.—Degree of required overloading of a fixed-pitch propeller at sea level to give maximum critical-altitude performance.

tion here is a constant power curve from sea level to critical height. If (as is the more general case) the manifold pressure is limited so that the allowable output falls off with decreasing altitude as shown in Fig. 321, the drop of sea-level power and r. p. m. below the rated sea-level maxima would be about a third greater than those plotted in Fig. 342. Even the allowance of a 10 per cent rise in manifold pressure for take-off would increase the r. p. m. only about 4 per cent, the power developed about 14, leave the take-off power still below the rated power at critical altitude in all cases where the critical altitude exceeds 4,000 ft. All this is the price of a rigid power-transmitting mechanism. Again, and even more obviously than at any previous point, the need for a load-varying element is acute.

There is one other possible system of utilization of the fixed-pitch propeller on the supercharged engine. During the last

part of the climb to the critical altitude the throttle can be used to keep a check on the r. p. m., and after that altitude is passed the accumulated reserve of supercharging can be used to hold the engine steadily up to its rated speed. At constant air speed and r. p. m., the power consumed by the propeller varies as ρ, while that developed at the crankshaft is following the law (beyond the critical height) of $\rho^{1.2}$. If the engine gives more power at some altitude than the propeller can use, then as the altitude is increased the curves will converge and finally cross. The par-

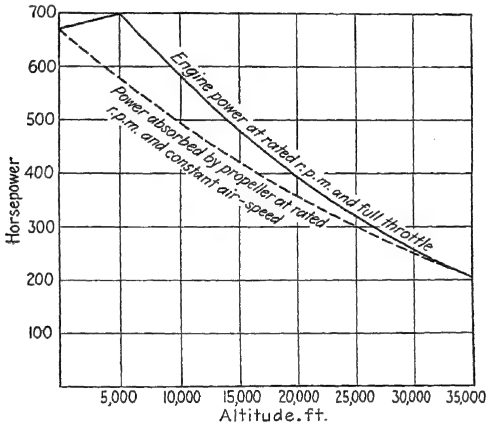


FIG. 343.—Power-absorption characteristics of a fixed-pitch propeller allowing a supercharged engine to turn full r. p. m. at sea level.

ticular case of an engine with constant manifold pressure maintained up to 5,000 ft. and full-throttle potentialities thereafter, in combination with a propeller that absorbs the rated maximum power at the rated r. p. m. at sea level, is illustrated in Fig. 343. The two curves drawn there collide at about 33,000 ft.—demonstration that a propeller and engine fitted to each other on this principle can be used to full capacity at sea level, and again at about six times the critical altitude, but nowhere in between. There are, of course, an infinite number of gradations and intermediate stages between the propeller that turns up the maximum allowable r. p. m. at full throttle at sea level and the propeller that does the same at the critical altitude. Those are the extreme possibilities. To choose a propeller by the first

extreme would be most unusual; to choose by the second extreme is common; but to depart from it in favor of an intermediate practice is also common, and often desirable.

Horsepower Available.—The last link in the chain that binds the propeller and engine together in performance calculation is the combination of engine power, r. p. m., and propeller efficiency to determine the power finally made available through the thrust at every speed. Figure 344, by processes obvious on inspection,

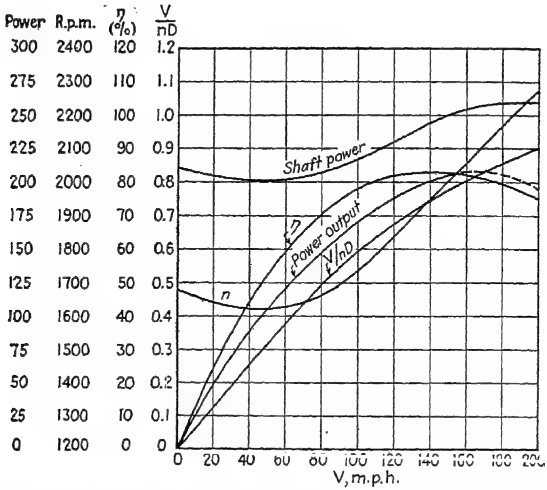


FIG. 344.—Variation with speed of actual useful power delivered.

forges that link. n comes from Fig. 339. From the combination of n and V, V/nD can be worked out. η is based on Fig. 327, the abscissas transformed to the speeds corresponding to various values of V/nD. Shaft power is taken from the engine power curve and the known variation of n, the kink at its outer end being due to the peaking of the power curve for this particular engine at about 2,200 r. p. m. Power output is the product of shaft power and efficiency, and beyond the design speed it is dotted as a reminder that the powers there indicated can only be attained by running the engine at full throttle beyond its rated speed. That process being disallowed, the power output at excess speed can best be obtained by assuming n constant

at its rated value, computing the engine power required to drive the propeller at that rate at each air speed (power requirement of course being proportional to C_P, and therefore a function of V/nD), and redrawing the efficiency curve for constant n. Figure 345 represents the right-hand part of Fig. 344 redone in that fashion. With r. p. m. limited, the drop of available power beyond the design speed becomes disastrous; it is then important, if the design speed is for any reason to be set below the maximum speed of flight, that the design r. p. m. be set enough below the rated r. p. m. of the engine to permit of continued full-throttle

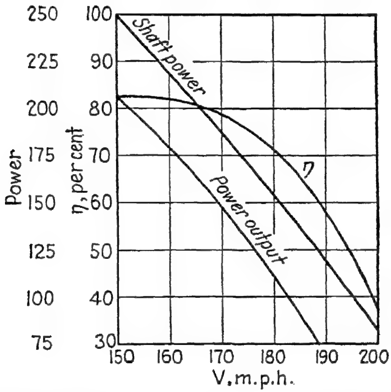


FIG. 345.—Effect of throttling required to prevent overspeeding of propeller at high speeds of flight.

operation at least up to maximum flight speed without producing a dangerous rotational speed. If a fixed-pitch propeller is being selected with primary reference to giving its maximum efficiency in climb, for example, the diameter should be such that at climbing speed the propeller will hold the engine down to about 10 per cent below the rated maximum r. p. m.

Horsepower-available curves for any other altitude can be done in the same manner. The method is essentially the same whether there is supercharging or none.

Tip-speed Effects.—The last fifteen pages of analysis rest on the implied assumption that propeller coefficients are functions of V/nD alone. As an approximation this serves, and in most cases it comes very close to the truth, but it has to be offered with reservations. The blade of the propeller works as an airfoil.

Like any other airfoil, it is subject to scale and compressibility effects. The first are almost always negligible; the second, far from being so.

Of the available wind-tunnel data on propellers, a great part come from the large wind tunnels of the National Advisory Committee, where propellers are run at full scale. Those drawn from model tests relate in practically every instance to models of one-quarter scale or more; the speeds of test are generally at least one-third of the maximum speed of the typical airplane; the ratio of extrapolation of Reynolds' number is therefore 12 to 1 or less. While that might be serious enough if a critical region of sudden change of type of air flow intervened, neither experience nor theory gives any indication of anything of the sort. The general rule is that performance gradually improves with increase of scale or speed (V/nD, of course, being kept constant) but that the improvement is small enough to need no detailed discussion. From the minimum Reynolds' number used in model testing up to full-scale conditions, there might be an average gain of 3 per cent in maximum efficiency, a corresponding increase in thrust coefficients, no change in power consumption.

Compressibility, as might be expected, amounts to more. The typical airplane propeller operates very much in the compressibility region at least at the tip, and sometimes over the entire outer third, of each blade. Tip speed obviously being πnD, the diameter formula on page 479 can be transformed into

$$\text{Tip speed (ft. per sec.)}^{*} = 560\sqrt[4]{\frac{(P/100)(n/1{,}000)^2}{V/100}} \qquad (98)$$

Increases in engine power in recent practice have tended to raise tip speed; increases in speed of flight and more extensive use of geared-down propellers to lower it; on present-day military and transport planes, from 800 to 1,000 ft. per sec. is the prevailing rule, with an occasional case running up to 1,100. The highest powered engine that ever drove a single air propeller, the 2,700-hp. Rolls-Royce that won the Schneider Trophy for Great Britain in 1931, thanks to gearing down of the propeller to a moderate r. p. m. and to a design speed of nearly 400 m. p. h., barely exceeded 1,000 ft. per sec.

*For two-bladed propellers. With three blades, the constant is reduced to 520; with four, to 480.

Since compressibility becomes an important factor in airfoil behavior, for most sections and at the intermediate lift coefficients at which propeller blades habitually work, at about $0.8v$ (v being the speed of sound)[1] and since the propeller as a whole might be expected to show the effects only when something more than the bare tip of the blade is in the superspeed region, compressibility trouble might be expected whenever tip speed exceeds $0.9v$ (1,020 ft.-per sec.), or when

$$\frac{P}{100}\left(\frac{n}{1,000}\right)^2 > 11\frac{V}{100} \tag{99}$$

That rule of thumb would hoist a danger signal whenever power absorbed in a two-bladed propeller exceeded 400 at 2,000 r. p. m., or 750 at 1,500 r. p. m. (assuming a speed of 150 m. p. h.). With three blades it is possible to increase limits of power about 40 per cent; with four blades, an increase of about 75 per cent is possible.

The experimental data,[2] though their average agrees fairly well with the previous paragraph's conclusion regarding the critical condition, at first sight appear somewhat at variance among themselves. American trials on adjustable-blade metal propellers, carried up to 1,300 ft. per sec., showed no measurable effect up to about 1,000 ft. per sec. but very marked effects thereafter. More specifically, their point of appearance was definitely a function of blade thickness, trouble becoming manifest at 1,050 ft. per sec. with a blade having a thickness of 6 per cent of its chord at 75 per cent of the maximum, at 960 ft. per sec. when the corresponding thickness ratio was 10 per cent. The appearance of compressibility effect was abrupt, and beyond that point the maximum efficiency of the propellers fell off about

[1] See p. 222, *supra*.

[2] There have been many tests in Great Britain, somewhat handicapped by the limited speed obtainable in the wind tunnel and the consequent impossibility of reaching high values of V/nD at high tip speeds, and some in the United States. Original sources are "The Effect of Tip Speed on Airscrew Performance," by G. P. Douglas and R. M. Wood, *R. and M.* 884; "Wind Tunnel Tests with High-tip-speed Airscrews," by G. P. Douglas and W. G. A. Perring, *R. and M.* 1086, 1091, 1123, 1124, 1134, 1174, and 1198; "Full-scale Tests on a Thin Metal Propeller at Various Speeds," by Fred E. Weick, *Rept.* 302; "Full-scale Tests of Metal Propellers at High Tip Speeds," by Donald H. Wood, *Rept.* 375.

9 per cent for each 100 ft. per sec. added to tip speed, reaching 40 per cent maximum efficiency at 1,300 ft. per sec. British tests on a wooden propeller, its blades, of course, much thicker than the metal ones, indicate a perceptible drop of efficiency at tip speeds as low as $0.55v$, a further drop thereafter in accordance with Fig. 346. A typical metal-propeller tip-speed-efficiency curve, quite different in form, is also included in the figure. Thrust and power coefficients, individually examined, show what might have been anticipated from the known compressibility effects on airfoils. The thrust coefficient for the propeller, like the slope of the lift coefficient for an airfoil, increases with increasing speed up to about $0.8v$ (at which point the fastest moving air flowing over the upper surface reaches the speed of sound) and decreases thereafter. Power coefficients increase more rapidly at first, reach their maximum at a higher speed, decrease less rapidly thereafter. Result: a steadily falling efficiency, a power coefficient at maximum η that is about a third higher at $0.75v$ than at $0.3v$, a third higher again at v. The metal-blade propeller's behavior is qualitatively the same but quantitatively quite different, the increase in power coefficient over the low-speed minimum being only 7 per cent at $0.9v$ and 33 per cent at $1.15v$.

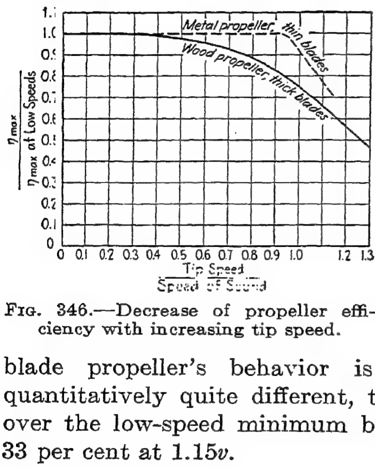


FIG. 346.—Decrease of propeller efficiency with increasing tip speed.

In accord with theory and with airfoil experience, too, is the difference between the wooden and metal propellers. Thin sections are notoriously resistant to compressibility effects,[1] and in fact the deliberate quest for a propeller capable of running at higher tip speeds than then seemed practicable was largely responsible for the pioneering work in the forged-metal-blade field. The summary of experience is that compressibility can be neglected on thin-metal-blade propellers up to $0.9v$; that it must be expected to produce a gradually accelerating drop of efficiency, a gradual increase of power coefficient, beyond that

[1] See p. 222, *supra*.

point; that on wooden propellers thick enough to run at such speeds and to absorb 200 hp. or more the loss of efficiency becomes perceptible at about $0.6v$, serious at $0.8v$; that the diameter of a wooden propeller running at $0.8v$ tip speed can be about 6 per cent less than would be computed for the same working conditions on the basis of data taken at low tip speeds; that for speeds above $0.7v$ (corresponding to 160 hp. at 1,800 r. p. m.) wooden propellers ought to be put aside in favor of metal ones wherever possible; that they may, however, be permissible where the propeller is geared down to r. p. m. very much lower than those of the engine, as such gearing both reduces the tip speed and, increasing the propeller diameter, increases the weight. The weight advantage of the fundamentally lighter wooden construction then becomes a more important factor in the choice. It is to the prevalence in Great Britain of propellers geared down in the ratio of 1 to 2 or even lower that the wooden propeller principally owed its continued hold upon the allegiance of British designers long after American opinion had branded it obsolete.

Static Thrust.—There is one tip-speed effect that does not depend on compressibility. Tip speed is the primary factor in determining the efficiency of the propeller as a blower for giving thrust on a stationary airplane—obviously an element of performance having some considerable influence on take-off quality.

The static thrust is, of course, equal to

$$T_0 = C_{T_0}\rho n^2 D^4$$

where C_{T_0} is the thrust coefficient at $V/nD = 0$. Similarly, the power consumed in producing it is

$$P_0 = C_{P_0}\rho n^3 D^5$$

The thrust per horsepower then becomes

$$\frac{T}{P} = \frac{C_{T_0}}{C_{P_0}} \cdot \frac{1}{nD} \tag{100}$$

or

$$\frac{T}{P} = \frac{C_{T_0}}{C_{P_0}} \cdot \frac{\pi}{v_t} = \frac{K_T}{v_t} \tag{101}$$

where v_t is the tip speed. It remains only to determine K_T.

Experiments by Durand and Lesley[1] and analysis of subsequent National Advisory Committee work by Diehl[2] have shown K_T as a primary function of the ratio of pitch to diameter, which in turn approximates to 1.1 times the V/nD of maximum efficiency. K_T ranges approximately from 5,000 to 2,500 over the span of common design practice, falling off as the pitch increases, and corresponds roughly upon the average to the formula

$$K_T = 6{,}250 - 3{,}900\left(\frac{V}{nD}\right)_\eta \tag{102}$$

$(V/nD)_\eta$ being the slip function for maximum efficiency.

No more is necessary for direct calculation, but as a first aid to generalization on take-off performance (102) may be combined with the general expression for diameter already given.[3] Then

$$\frac{T}{P_1} \quad \frac{6{,}250 - 3{,}900(V/n_0D)_\eta}{\pi n_1 D} = \frac{2{,}000}{n_1 D} - \frac{1{,}240V_0}{n_0 n_1 D^2} \tag{103}$$

where n_0 is the rated r. p. s., n_1 the r. p. s. at zero velocity, V_0 the design speed, and P_1 the power developed under static conditions. Taking power as proportional to r. p. m.,

$$\frac{n_1}{n_0} = \frac{P_1}{P_0}$$

Converting n_0 and V_0 into r. p. m. and m. p. h., respectively,

$$\frac{T}{P_0} \quad \frac{120}{\frac{n_0}{1{,}000}D} \quad \frac{655\frac{V_0}{100}}{(n_0/1{,}000)^2D^2}$$

$$\frac{11}{\sqrt[4]{\frac{(P/100)(n/1{,}000)^2}{V_0/100}}} \qquad \frac{5.6\frac{V_0}{100}}{\frac{(P/100)(n/1{,}000)^2}{(V_0/100)}} \tag{103a}$$

[1] "Experimental Research on Air Propellers," Pt. II, by W. F. Durand and E. P. Lesley, *Rept.* 30.

[2] "Static Thrust of Airplane Propellers," by W. S. Diehl, *Rept.* 447. For a general review of the whole subject and of the effect of particular modifications in propeller design, see this paper by Diehl and also "The Problem of the Helicopter," by Edward P. Warner, *Tech. Note* 4.

[3] See p. 479, *supra*.

From this rather formidable formula, the most interesting indication is that T_0/P, contrary to common supposition, reaches a maximum at some value of n. The casual presumption is, as it naturally would be from (100), that static thrust would increase indefinitely as the r. p. m. are indefinitely reduced by increasing the gear ratio between the engine and the propeller. But it is not so, with a fixed-pitch propeller. Reduction of n, if the design speed for flight performance is to be kept unchanged, demands increase of pitch. Increase of pitch reduces K_T, and ultimately a point is reached where the rate of decrease of K_T completely overbalances the effect of the corresponding reduction of nD.

The optimum value of n for maximum static thrust from a given engine, and with a propeller designed to put out its maximum impulse efficiency at a given speed, can be found by differentiating (103a). That process, too straightforward to need repetition here, shows a maximum of thrust from a two-bladed propeller[1] when

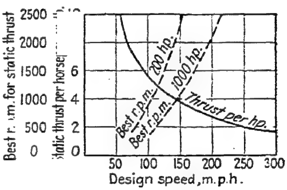


FIG. 347.—Best r. p. m. for maximum static thrust with two-bladed fixed-pitch propeller.

$$\frac{n_0}{1,000} = \frac{1.04(V_0/100)^2}{\sqrt{P/V_0}} \tag{104}$$

The thrust per rated horsepower under those conditions is of the amazingly simple form

$$\frac{T_0}{P} \quad \frac{5.4}{V_0/100} \tag{104a}$$

completely independent of power. It further develops from study of the general equation that T_0/P is highly insensitive to changes of n over wide limits. n can be increased to 50 per cent above its optimum value or reduced to 30 per cent below it, without decreasing static thrust by more than 4 per cent.

The maximum obtainable static thrust per horsepower, assuming the optimum gear ratio in each case, is plotted in Fig. 347, together with the optimum r. p. m. for two different powers.

[1] For three- and four-bladed propellers, respectively, the constant would be 1.2 and 1.3. The constant in (104a), would remain unchanged.

All this, it must be remembered, applies only with the fixed-pitch propeller. With controllable pitch quite a different result is obtained. In particular, the theoretically ideal r. p. m. are much lower than with fixed pitch—usually too low to be practicable.[1]

Propeller Reduction Gears.—The airplane designer has the option of using a propeller driven directly off the engine crank-shaft or, at a cost of from 2 to 5 per cent of the total power, used in overcoming friction in the transmission, one geared down to run from 20 to 60 per cent slower.

Gearing down the propeller increases the propeller diameter required to absorb the engine power. The diameter varies less rapidly than the r. p. m. (approximately as their square root only). nD accordingly drops. The tip speed is lowered, and compressibility effects on propeller efficiency are reduced or averted where they would otherwise appear. In very large engines, gearing is almost a necessity, both to avoid tip-speed efficiency losses and to reduce the risk of propeller flutter or structural failure.

Lowered nD also means a higher V/nD, and that in general means[2] higher efficiency, particularly if V/nD to start with was low. Still further, as has just been made apparent, in many cases a reduction of n increases the static thrust and the thrust at low speeds in general and so improves the performance in take-off. There are many other factors, of a "practical" order, that play a part in decision. Propeller weight and engine weight, landing-gear design, slip-stream effect on the controls, noise, liability to vibration, and engine cooling are all affected, favorably or otherwise, by gearing. All must be thrown into the balance on one side or the other, but the predominant influence is that of efficiency, both at design flight speed and at the lower speeds (down to zero) of climb and take-off. Once again, general analysis is in order.

Take static thrust first. Figure 348, drawn from (104), records the r. p. m. that will produce the greatest static thrust for all possible combinations of design speed and power. Generally speaking, optimum r. p. m. prove to be a primary function of speed, with the effect of power almost incidental. The

[1] See p. 500, *infra*.
[2] See Fig. 336, p. 480, *supra*.

highest desirable operating r. p. m. for modern engines, if the propeller could be neglected entirely, range from about 2,800 r. p. m. at 1,000 hp. up to about 4,200 r. p. m. at 40 hp. The first figure appears on the chart at 220 m. p. h.; the second at 140 m. p. h. Static thrust would then provide no reason for gearing on high-speed transports or on military craft, even with engines of the very highest power, if fixed-pitch propellers were used. On machines of the sporting class, with speeds of around 85 m. p. h. with 40 hp. up to 135 m. p. h. with 200 hp., the ideal r. p. m. would be from 1,500 to 2,000, and to get the utmost out of the

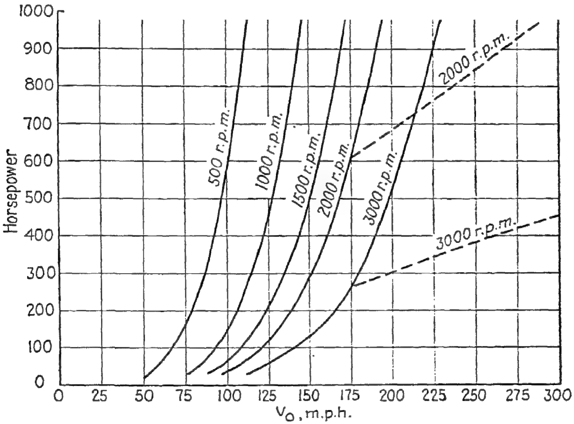


FIG. 348.—Best r. p. m. for static thrust with a fixed-pitch propeller, with additional limits on maximum tip speed.

take-off gearing should be used or (as is the commoner practice in types where economy and simplicity are more important than the last fraction of efficiency) the engine should be designed to be run at r. p. m. considerably lower than might offer the very highest possible output for a given weight and frontal area.

Before going too far with these conclusions, it should be repeated that the curve of static thrust against r. p. m. is exceedingly flat in the neighborhood of its maximum. A machine with 200 hp. and a design speed of 125 m. p. h. can turn a fixed-pitch propeller at anything between 1,100 and 2,200 r. p. m. without appreciable loss of take-off quality.

Broadly speaking, it is only on the somewhat exceptional airplane that take-off considerations would seem to demand

gearing, limiting the conclusions always to the fixed-pitch case. Limitations of desirable tip speed, however, may be much more severe.

To investigate that point, note that at the static-thrust optimum for n

$$\frac{n_0}{1{,}000} = \frac{1.04(V_0/100)^2}{\sqrt{P_0/V_0}}$$

$$D = 10.8\sqrt[4]{\frac{P_0/100}{(V_0/100)(n/1{,}000)^2}} = \frac{10.8\sqrt[4]{P_0/V_0}}{\sqrt{n/1{,}000}}$$

$$\frac{n}{1{,}000}D = 10.8\sqrt{n_0/1{,}000}\sqrt[4]{\frac{P_0}{V_0}} = 11\frac{V_0/100}{\sqrt{P_0/V_0}}\sqrt{\frac{P_0}{V_0}} \qquad (105)$$

$$\frac{V_0/100}{\frac{n}{1{,}000}D} = \frac{1}{11}$$

Transforming the units into homogeneous ones, so that V/nD may appear in the customary non-dimensional form,

$$\frac{V}{nD} = 0.8$$

So develops the surprisingly simple conclusion that in every case, however widely the power and design speed may range, the propeller speed giving the maximum static thrust from a fixed-pitch propeller is that which makes $V/nD = 0.8$ at design speed. This is substantially true even for propellers with more than two blades.

Only a very short step remains to tip speed.

$$\frac{V}{nD} = 0.8$$

$$v_t = \pi nD$$

Therefore

$$v_t = \frac{\pi}{0.8}V = 3.93V$$

where V is in ft. per sec., or

$$v_t = 580\frac{V_0}{100}$$

with V_0 in m. p. h. The tip speed at the static-thrust optimum of r. p. m. is a function of design speed alone. The design speed, in this case, is the speed at which maximum efficiency is developed. It generally lies between $0.85V_{max}$ and $0.97V_{max}$.

Since compressibility effects become troublesome at about 750 ft. per sec. with a wooden propeller and at 1,000 ft. per sec. with a metal one, it follows (1) that in following the curves in Fig. 348 across the sheet the change from wood to metal should be made not later than the passage across the vertical line representing 130 m. p. h. (so that, for instance, a wooden propeller should not be turned at over 1,750 r. p. m. by a 200-hp. engine or over 1,200 r. p. m. by one of 400 hp.); (2) that to the right of the 175-m. p. h. line the "optima" as originally plotted ought to be forsaken entirely and a further gear reduction made in the interest of tip-speed limitation as indicated by the dotted lines running obliquely upward and to the right. The method of constructing the lines, with their indication that the maximum power that can be absorbed at a given r. p. m. without exceeding the allowable tip speed is directly proportional to the design speed and inversely proportional to the square of the r. p. m., will be apparent from (105).

The dotted lines put a new complexion on the question of gearing. It now appears desirable to use it in practically all engines of over 650 hp. and in really high-speed engines of over 400 hp., as well as in lower powers on slow-flying airplanes.

Figure 348 is drawn, as the formulas in this section have been derived, for two-bladed propellers and standard sea-level air density. Where either the number of blades or the basic altitude is changed, the optimum value of n for static thrust is so changed that the design value of V_0/nD will still remain constant. With three blades, the values of n plotted on the chart should be increased by 16 per cent—the same rule applying both for the solid and for the dotted portions of the curves. Where the design condition for the propeller is taken at the critical altitude of the engine, rather than at sea level, the optimum r. p. m. is reduced by approximately 1.5 per cent for every 1,000 ft. of elevation of the critical altitude; V_0 is the design speed at the critical altitude; and T_0/P maintains the precise form of (104*a*). That the fixed-pitch propeller which absorbs the full power of

the engine at rated r. p. m. at critical altitude will overload the engine and hold it to a lower r. p. m. at sea level makes no difference in static thrust; for, with a constant torque, P and v_t in (101) vary at the same rate with r. p. m., and the total thrust remains equal to that which would be obtained from (101) on the basis of full rated power and of the v_t existing at the critical altitude.

Before leaving the subject, renewed attention should be given to Fig. 336 and its bearing on gear reduction. Although efficiency at full speed continues to increase up to $V/nD = 1.2$, the improvement with increase beyond 0.8 is slight. If top-speed efficiency were the only factor that had to be considered, it would be advisable to reduce the propeller r. p. m. to about 50 per cent below the figures plotted in Fig. 348. With a controllable-pitch propeller, reduction to that extent would benefit take-off also. Taking everything into account, including some of those factors of weight and encumbrance mentioned in the opening paragraphs of the present section, the best compromise will probably be found in the use of a gear box giving r. p. m. about 25 per cent below those indicated by the full curves when the pitch is fixed, or 50 per cent below when it is controllable, except (1) that the requirements of the dotted lines are fully controlling over those of the full ones; and (2) that where the indicated speed reduction would be less than 25 per cent it will not be worth while to make any reduction at all (again subject to the consideration that the zone above a dotted line must not be invaded except in the last extremity of necessity).

Controllable-pitch Propellers.—The advantages of being able to change the propeller pitch at will, to adjust it to the changing circumstances of flight, have already been several times hinted. Even with an unsupercharged engine they are apparent, but the present very widespread application of the controllable-pitch idea is the supercharger's offspring. Though the idea of easy adjustment of the blades is almost as old as flight itself, it was not until supercharging came into general use that it seemed worth while to buckle down to the hard and expensive process of working out the mechanical details to make a controllable-pitch propeller for high-powered engines a practicable service product. Now it has been done, and no designer of high-performance aircraft need deny himself the benefits of pitch variation, whether

the use of supercharging plays a large part in his particular design or not.

The principle is obvious. The blades are mounted on axial bearings in the hub, and they are free to rotate through a certain angle (usually a small one) either under the control of the pilot or at the dictation of some automatic governing device. The mechanisms used vary widely, as do the means of control, but the purpose is always the same.

To the questions of how far that purpose is accomplished, how much angular adjustment is likely to be needed, and upon what principle an automatic control ought to operate, the analysis of a typical problem offers the best approach. Take up once more the case already considered several times for various purposes, the case of the 250-hp. engine turning its propeller at 2,000 r. p. m. and fitted into an airplane with a design speed of 150 m. p. h.

The controllable propeller has to be judged from two points of view, those of full-throttle performance and of cruising, respectively. The first is the simpler, and the primary objective there is to secure the largest possible thrust at every speed throughout the operating range of the aircraft, taking into account the effects upon thrust of the variations both of propeller efficiency and of the r. p. m. at which the engine is able to turn and the power that it is able to develop.

For studying the effect of blade angle on thrust and r. p. m. over a wide range, the most convenient coefficient to use is C_Q'.

$$Q = C_Q' \rho V^2 D^3$$

It has been pointed out by Hartman[1] that, since the full-throttle torque of an engine is substantially constant at a given air density, if C_T/C_Q is plotted against $\sqrt{1/C_Q'}$, both the ordinate and the abscissa of the curve will be independent of r. p. m. For a given power plant and propeller, one coordinate will be directly proportional to the speed of flight, the other to the full-throttle thrust, and sheafs of curves can be plotted on that system for the various angular settings of the propeller. A set of curves so drawn, for the propeller to which Fig. 335 related, is reproduced in Fig. 349.

[1] "Working Charts for the Determination of Propeller Thrust at Various Air Speeds," by Edwin P. Hartman, *Rept.* 481.

Figure 350 gives curves of thrust against blade angle and of r. p. m. against blade angle, both at sea level, for several speeds of flight. Conditions calling for more than 2,000 r. p. m. are dotted on the curve. The engine and airplane chosen for the illustration were the same that have been used in all the previous examples, and the propeller diameter, which could of course

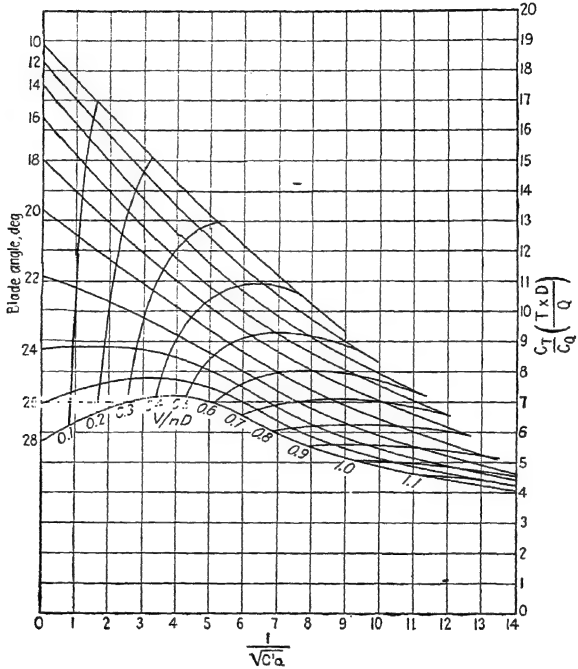


FIG. 349.—Hartman propeller-thrust chart.

have been varied over a wide range with variation of r. p. m. at a given speed and blade angle as the principal result, was taken as 8 ft.

It would have been equally feasible to plot propeller efficiency, rather than thrust, against blade angle. The efficiency is related to the curves actually drawn by the fact that it is proportional, at any given speed of flight, to the ratio of the thrust to the r. p. m. But for many purposes the thrust is the more convenient quan-

tity, since it takes into account the variations in engine power output with varying load, and at very low speeds of flight, where the efficiency would be approaching zero in every case, the thrust is the only quantity that is of any use at all.

Figure 350 supplies one generalization to be noted. The indication that both thrust and r. p. m. increase steadily with decreas-

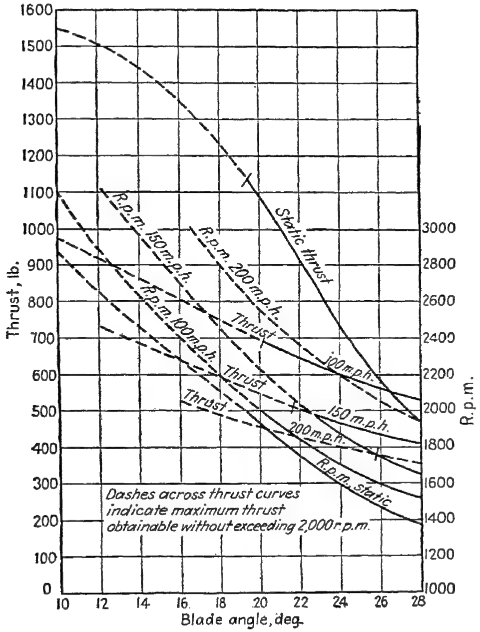


Fig. 350.—Variation with blade angle of r. p. m. and thrust at various flight speeds.

ing blade angle within the rather wide range covered by these curves is of the first order of importance. If there were no restrictions on engine speed, then, it would be advantageous to use a very small blade angle (10 deg. or less) at all flight speeds. Since there very emphatically are such restrictions, and since the r. p. m. with a 10-deg. blade angle would run up to nearly 50 per cent above the rated maximum even under static conditions, the alternative statement is that the blade angle should always be made as small as possible without allowing the engine to

overspeed. From the point of view of full-throttle performance, a controllable-pitch propeller should be so controlled as to hold the r. p. m. constant at its allowable maximum at all speeds and all altitudes. The dashes cutting across the thrust curves in Fig. 350 show the minimum angle allowable, and so the maximum thrust obtainable, without violating that stipulation.

It is apparent that the range of angles involved is small—seemingly, in the present problem, only a little over 2 deg. for speeds up to 150 m. p. h., or 6 deg. if a flight speed a third higher

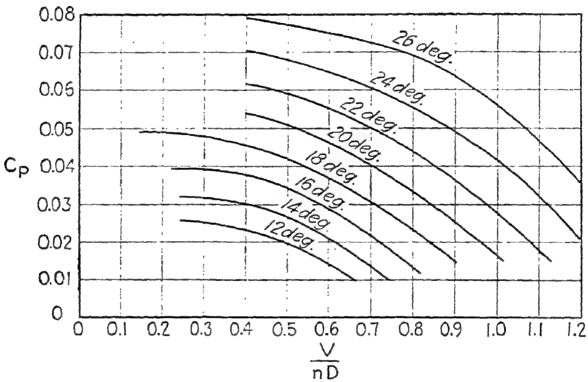


FIG. 351.—Variation of power coefficient with blade angle.

than the design speed is considered. To carry the analysis farther and with more refinement, turn back to a curve of C_P, plotted in Fig. 351 for half a dozen different blade angles.

If the engine is to be run at full throttle and the blade setting to be such that a constant r. p. m. will be maintained, the formula

$$P = C_P \rho n^3 D^5$$

has only one unknown. By substituting the appropriate values of P, n, D, and ρ, C_P can be determined. So, immediately thereafter, can the V/nD and the V at which the computed value appears for each blade angle. For the blade angles so found, in turn, C_T can be read off from a wind-tunnel curve and the thrust computed. In Figs. 352 and 353 the blade angles and the thrusts developed have been plotted against speed for the alternative

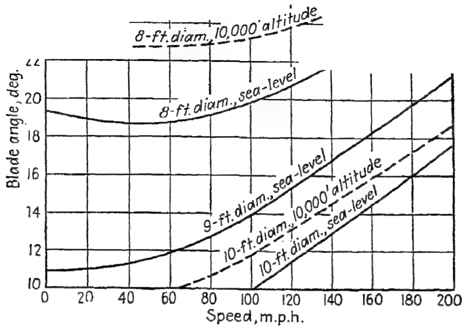


FIG. 352.—Variation with flight speed and diameter of blade angle of a controllable-pitch propeller.

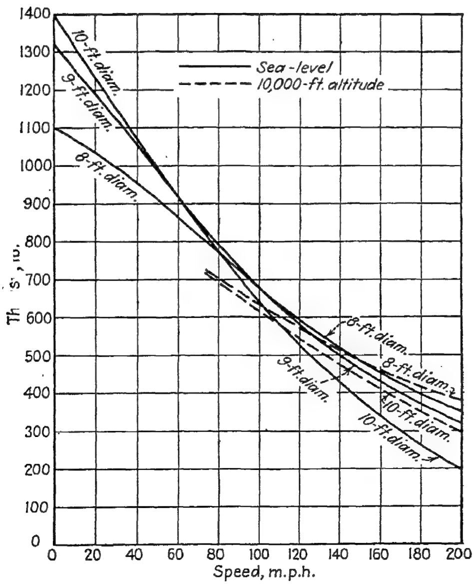


FIG. 353.—Variation with flight speed and propeller diameter of thrust of a controllable-pitch propeller.

assumptions of 8-, 9-, and 10-ft. propeller diameters. The 9-ft. propeller is clearly the best, with a thrust much in excess of that with an 8-ft. diameter at low speeds; it is much better than the 10-ft. diameter at high speeds.

The thrust curve has been determined, too, for a 9-ft. fixed-pitch propeller designed to absorb the engine power at rated r. p. m. at 150 m. p. h. It is plotted in Fig. 354, and compared directly with the controllable-pitch curve for the same diameter.

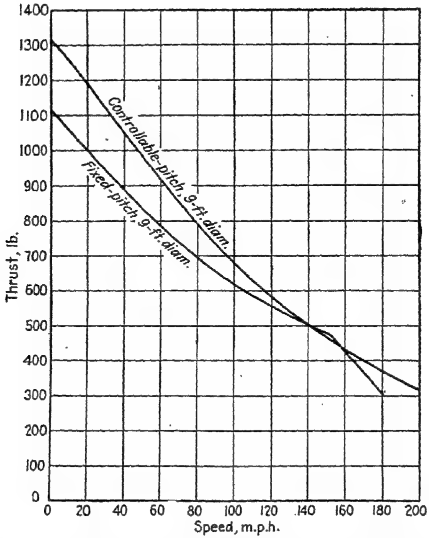


FIG. 354.—Comparison of thrust characteristics of a fixed-pitch and a controllable-pitch propeller.

Equalling the controllable-pitch performance at high speeds (since under those conditions the controllable propeller takes a form identical with the fixed one), fixed pitch proves inferior by more than 15 per cent under take-off conditions. In determining the possibility of clearing a restricted runway or of getting a heavily loaded seaplane off the water at all, the controllability of the propeller pitch may easily be decisive. If sea-level conditions predominate in the design, the best diameter for a controllable-pitch propeller is substantially equal to, or a very

little smaller than, the diameter that will give the best all-round results with a fixed pitch.[1]

On the same charts, also, there have been plotted curves for the performance at 10,000 ft. altitude on the assumption that there is full supercharging to that height and that the power there available exactly equals the sea-level power at the same r. p. m. Given a controllable propeller and supercharging, the thrust at 10,000 ft. is substantially equal to that at sea level. With a fixed pitch, a propeller capable of absorbing the full engine power either at sea level or at 10,000 ft. would give at least 10 per cent less thrust at the other level than at the one for which it was primarily calculated. It is there, as already intimated, that the controllable-pitch mechanisms most clearly pay their way.

At the 10,000-ft. critical altitude, the 9-ft. diameter (for which the curve is omitted) would be even more clearly superior to the 8-ft. than at sea level. As a general rule, where the power plant is supercharged and high-altitude performance has to be given consideration, the selected diameter should increase gradually with the critical altitude, ½ per cent increase of diameter for every thousand feet being a fair average.

There remains the question of cruising. For that phase of performance the thrust is known, being equal to the drag of the airplane at some reduced speed, and the problem is to find the minimum fuel consumption at a stated thrust and a variable blade angle and r. p. m. Minimum fuel consumption corresponds only approximately to minimum power consumption. Fuel consumption can be reduced by increasing the manifold pressure and torque, and so cutting down the r. p. m., as far as the engine manufacturer will allow, even though the resultant change of operating condition reduces the propeller efficiency somewhat and so increases the power to be delivered at a given speed. There is of course a limit to the length to which that process can be carried, but in general the most economical engine

[1] These conclusions regarding diameter may seem somewhat at variance with those arrived at for the same general conditions on p. 476. For reasons of relative availability of data presented in the various forms desired, however, the various illustrative examples have been worked out for propellers differing among themselves in basic form. The results must not, therefore, be considered directly comparable.

speed for cruising is appreciably less than the speed of maximum propeller efficiency. The difference shows up most conspicuously in cruising at relatively low speeds and very small proportions of the rated engine power.

The attack here is best made through C_T'. In studying the behavior in cruising at a fixed speed, both the speed and the thrust required to drive the airplane at that speed are known. In

$$T = C_T' \rho V^2 D^2$$

C_T' is therefore the only unknown. It can be computed, and the corresponding V/nD can be read off for each blade angle

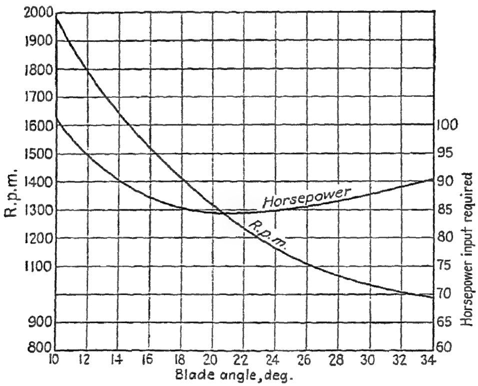


FIG. 355.—Dependence of cruising power and r. p. m. on blade angle in a controllable-pitch propeller.

in turn. Having V/nD, n is readily determined. The propeller efficiency is read off against V/nD on the appropriate curve, and the power input can be determined by dividing the output (which is the product of thrust and flight speed) by the efficiency.

Figure 355 shows the solution for the standard illustrative case, used throughout the present chapter, with the airplane cruising level at 90 m. p. h. and 10,000 ft. altitude. The thrust required at that speed and altitude is assumed to be 275 lb., a 9-ft. propeller is used, and C_T' is then 0.14. n and required power input vary as plotted in Fig. 355.

In this case the minimum power is required for level flight with a blade angle of 21 deg. The engine is then turning 1,270 r. p. m., or 63 per cent of rated speed. The engine power is 84 hp., or

34 per cent of the rating. The brake mean effective pressure (which is very roughly proportional to the manifold pressure), being proportional to power divided by r. p. m., stands at only 54 per cent of its rated maximum—too low a figure to approach the best economy. By increasing the blade angle to 26 deg. the power required would be increased by 2 per cent, but the r. p. m. would be reduced to 1,110 and the mean effective pressure in the engine would be raised to 62 per cent of rated maximum, and the specific consumption would undoubtedly be reduced by at least 4 per cent. The net effect on total consumption would be to effect about a 2 per cent saving, and somewhere around 26 deg. is where the optimum propeller setting would be found. Beyond that point, the propeller efficiency falls off too rapidly for the decrease of specific fuel consumption to keep pace with it.[1]

Stopped and Feathering Propellers.—With an engine stopped, either voluntarily or otherwise, the propeller ceases to propel and becomes an element of parasite resistance. Of some significance in its influence on the glide path of a single-engined plane descending with the engine idling or completely dead, the behavior of the undriven propeller assumes a really vital importance in multiengined aircraft for which safety depends on the possibility of continuing to fly with one engine (or more) out of operation. To know the action of the undriven propeller is essential to the calculation of performance under those conditions. To find means of reducing its drag may be to make a safe airplane out of one that had been dangerously dependent on the continued perfect functioning of all its power units.

[1] There has been no attempt to summarize the rapidly growing literature of controllable-pitch theory and computation. The most straightforward methods of building up the computation from fundamentals have been preferred to any acceptance of ready-made approximations or guiding conclusions. There are, however, a number of papers that should be cited as demanding study by anyone who wishes to go further with the subject. Leading examples are "Empirical Formulae for a Variable-pitch Airscrew," by A. D. Betts and H. A. Mettam, *R. and M.* 577; "Some Possible Advantages of a Variable Pitch Airscrew," by W. G. Jennings, *R. and M.* 1516 (particularly good on the relation between pitch control and fuel consumption); "Variable-pitch Airscrews and Variable Gears," by W. G. Jennings, *Jour.* Roy. Aero. Soc., 1934; "A Method of Calculating the Performance of Controllable Propellers, with Sample Computations," by Edwin P. Hartman, *Tech. Note* 484.

The matter has had a somewhat limited amount of experimental study, covering a couple of particular cases.[1] It proves as might be expected that the propeller drag varies widely with V/nD, which is more commonly replaced for this purpose by its reciprocal, in order that the characteristics may be plotted all the way up to $V/nD = \infty$ (case of the stopped propeller). Three particular conditions especially invite attention; the case of the propeller cranking over the engine against its frictional torque; the case of the free-wheeling propeller, declutched from the engine and allowed to spin ($C_Q = 0$); the case of the braked or locked propeller ($nD/V = 0$).

Douglas has shown[2] that for modern radial engines the frictional torque is substantially proportional to the r. p. m., and his data indicate that it corresponds roughly to the formula

$$Q_F = 0.065\frac{n}{1{,}000}d \tag{106}$$

where d is the piston displacement in cubic inches. Hartman gives curves from which the conditions that will permit the propeller, acting as a windmill, to overcome that much negative torque can be calculated. From his work there has been computed the curve of r. p. m. against blade angle, with the propeller cranking a dead engine, in Fig. 356. In the same figure is plotted the curve of r. p. m. of a free-wheeling propeller. In both cases the horsepower is 250; the normal r. p. m. 2,000; the propeller diameter 8 ft.; the piston displacement 650 cu. in.; the speed 125 m. p. h. That a proper blade setting will enable a propeller to turn a stopped engine over against friction at more than 80 per cent of its rated r. p. m. is, at first sight, little less than astounding.

[1] "Nouvelles recherches sur la résistance de l'air et l'aviation," by G. Eiffel, Paris, 1914; "Airscrews at Negative Torque," by C. N. H. Lock and H. Bateman, *R. and M.* 1397; "Negative Thrust and Torque Characteristics of an Adjustable-pitch Metal Propeller," by Edwin P. Hartman, *Rept.* 464; "Characteristics of Braked, Locked, and Free-wheeling Two- and Three-bladed Propellers," by F. J. Malina and W. W. Jenney, *Jour. Aero. Sci.*, May, 1936.

[2] "The Developments and Reliability of the Modern Multi-engine Air Liner, with Special Reference to Multi-engine Airplanes after Engine Failure," by Donald W. Douglas, *Jour.* Roy. Aero. Soc., November, 1935 (also in *Jour. Aero. Sci.*, July, 1935).

Figure 357 plots the effective drag of the propeller for the same two cases and supplements them with a curve for the drag of a propeller locked against turning (plotted on a different

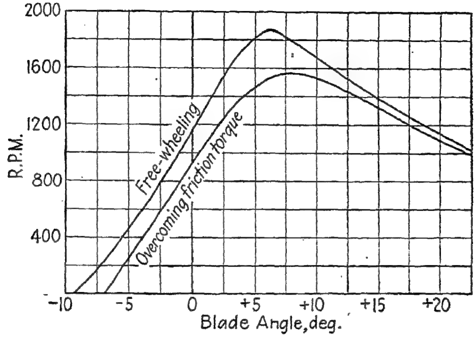


FIG. 356.—R. p. m. of a propeller on a dead engine, cranked by the air stream.

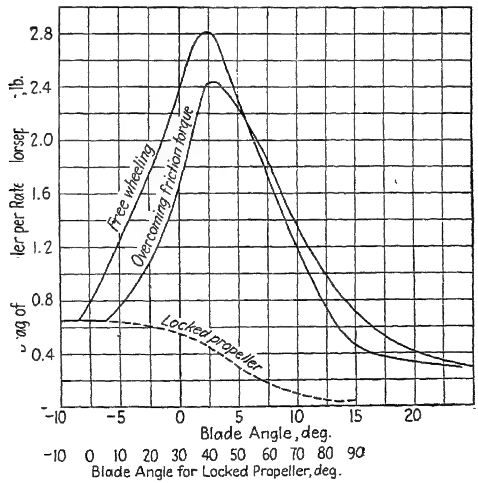


FIG. 357.—Drag of propellers on dead engines. (Note two scales of abscissas.)

horizontal scale in order that the range of blade angles may run up to 90 deg.). The sensitiveness to angle is tremendous. For a fixed-pitch propeller (blade angle about 20 deg.) the drag is a third less when turning over the engine than when locked, and still less for the free-wheeling case, but adjustment

of angle can multiply the drags that prevail at 20 deg. by 6 or divide them by 10. The best case is that of the blade feathered in its hub to an angle of between 80 and 90 deg.; the worst, from the point of view of airplane performance, that of the propeller continuing to turn with its pitch angle reduced to well below flight limits (about 2½ deg. for the maximum drag).

There is a popular presumption that a great saving can be made by fitting brakes on the propeller shafts to keep the propellers from windmilling after an engine failure, and in some cases they have actually been provided.[1] Figure 357 casts doubt on their benefits to performance. If the maximum angle to which the blades can be turned (taken at three-quarters of the maximum radius, as is customary) is 50 deg. or less, there is no advantage in a brake, for the drag of the stopped propeller is greater than that of the rotating one. If the controllable-pitch mechanism allows the angle to increase beyond 50 deg., very little additional effort should be involved in providing for an increase to 90 deg., and at some angle around 85 deg. the torque on the propeller drops to zero and the propeller stops turning without waiting for a brake's persuasion. The free-wheeling propeller and the locked one become identical. The ideal solution for the case of the stopped engine is then a complete universality of pitch adjustment. Given that, nothing else matters. Failing that, nothing else does much good in reducing drag, though a brake still has the practical value of preventing the engine from tearing itself to pieces after an internal breakage, and may improve the lift and the control characteristics by eliminating interference where the propeller turns close in front of the leading edge of the wing.

Propeller drag may be a positive as well as a negative factor in performance. One of the problems of design is the steepening of the glide path of an efficient airplane so that it can be brought down over the obstructions at the edge of a landing field and set on the ground without wasting too much space. Though wing flaps of some sort are generally used for that purpose,[2] propeller drag may be used either as an alternative or as a supplement. By setting the angle down to a proper level and letting the propeller spin, the effective drag becomes approximately

[1] "Propeller Brake," *Aviation*, p. 32, August, 1935.

[2] See p. 282, *supra*.

that of a normal flat disk of half the propeller's diameter. The total drag of the airplane is likely to be more than doubled. Needless to say, the effect on glide angle is tremendous.

Slip Stream and Drag.—Dynamically considered, all fluid-propulsion devices are alike. Screw propeller, paddle-wheel, jet or "rocket," and even oar, all serve as mechanisms for projecting a jet of fluid astern. The action that accelerates the fluid finds by Newtonian law an equal and opposite reaction, available for driving the craft to which the propulsive mechanism belongs. Considering the motion of the fluid relative to the aircraft or the boat that moves through it, an accelerated flow behind the propeller is an inescapable consequence of the fact that there is propulsion.

The propeller draws air in from the front at a certain speed and projects it astern at a higher one. The mass of fluid remaining constant, the stream must contract as its velocity increases. The diameter of the slip stream, at anything between a few inches and many feet behind the propeller, is less than the propeller's own.

Slip-stream diameter can be calculated when the velocity is known, and the velocity can be computed in terms of propeller performance. Thrust is equal to the product of the mass of air handled in unit time by the increase of velocity given it. From that general principle comes the formula

$$\frac{V_s}{V} = \sqrt{1 + \frac{0.05T\frac{\rho_0}{\rho}}{D^2(V/100)^2}} = \sqrt{1 + \frac{19\frac{P}{100}\eta\frac{\rho_0}{\rho}}{D^2(V/100)^3}} = \sqrt{1 + 2.55C_T} \qquad (107)$$

where V_s is the slip-stream velocity and the other quantities have their customary significance (V being in m. p. h.).

The estimation of thrust of course requires a preliminary guess at the propeller efficiency, but it can be quite rough, as an error of 10 per cent in the estimate of efficiency is unlikely to change the calculated slip-stream velocity by more than 3 or 4 per cent under any ordinary working conditions.

Figure 358 shows the variation of slip-stream velocity ratio in a typical case for a direct-drive propeller (running at 2,000 r. p. m.) and also for one geared down to half that speed.

Figure 358 denies any presumption that V_s/V can be taken as constant at maximum efficiency for all cases. It varies very widely with design conditions. If the formula for D is substituted in (106), and the expression for the ratio simplified as far as possible, and η assumed at 80 per cent, it becomes

$$\frac{V_s}{V} = \sqrt{1 + 0.13\sqrt{\frac{(P/100)(n/1,000)^2}{(V/100)^5}}}$$

Since the quantity inside the second radical may vary, within the range of reasonable design practice, from 0.2 to 32, V_s/V

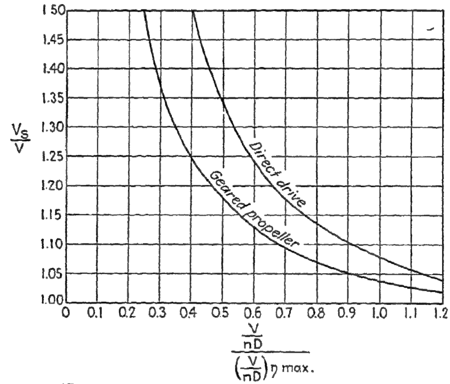


FIG. 358.—Ratio of slip-stream velocity to flight speed.

at maximum efficiency may be anything between 1.03 and 1.32. It can further be shown, by employing the same formulas but eliminating P instead of D, that

$$\frac{V_s}{V} = \sqrt{1 + \frac{0.11\eta}{(V/nD)^2}}$$

approximately. For the design conditions, at which maximum efficiency is to be realized, η can be taken, as before, as 0.80. Then

$$\frac{V_s}{V} = \sqrt{1 + \frac{0.09}{(V/nD)^2}}$$

The possibility that it offers of reducing the slip-stream velocity and so (as will later be explained in detail) of keeping down the

increase of parasite resistance of the parts of the structure that are in the slip stream is an argument supplementary to those already given in favor of using a gear reduction. It is, however, hardly such a paramount factor as to invalidate the general conclusions on optimum gear ratio that have previously been reached.

In itself the slip stream represents a certain loss of power through kinetic energy irrecoverably imparted to the air, but that appears as a part of the difference between the actual propeller efficiency and 100 per cent, and its analysis may be left to the propeller specialist. For the engineer charged with engine installation the slip stream's greatest importance is in its effect on engine cooling, often so decisive that an engine perfectly cooled with a direct-drive propeller becomes unsatisfactory when the slip-stream velocity is lowered by gearing. But from the point of view of airplane performance the most interesting aspect is in the interaction between the slip stream and the parts of the airplane that lie within its boundaries.

If an object (such as a brace strut on the stabilizer) is placed in the slip stream at a distance of a diameter or more behind the propeller, the effect is very simple. The slip stream is a stream of moving air with a certain velocity, and the object immersed in it shows a parasite resistance appropriate to that velocity. The drag of the imaginary brace strut, for example, would be expected to increase over that which it would have if mounted outside the slip stream in the ratio $(V_s/V)^2$, or $(1 + 2.55C_T')$. It is when the obstruction in the stream is brought close to the plane of the propeller that the relation grows complicated.

Under those conditions, the obstruction no longer plays a purely passive role. It begins to react upon the propeller. It retards the approaching air and reduces its mean axial velocity as it crosses the propeller's plane. The thrust per unit of power depends on the direction and magnitude of the air's resultant velocity relative to the blade sections, and the retardation of the air acts to increase the thrust. The term V in the expression

$$\eta = \frac{TV}{P}$$

remains unchanged, however, for the useful work done is proportional to the speed at which the aircraft is moved through

space, not to any local relative wind speed. If the retardation produced by the obstruction were uniform over the entire propeller disk, then, the efficiency curve would be modified by increasing both the ordinate and the abscissa of every point in the ratio $1/(1 - m)$, where m is the proportion by which the effective velocity of the air through the propeller is reduced by the obstruction's presence.

In practice, of course, the effect is not uniform. The obstruction's influence falls off rapidly with distance. It changes the air's direction as well as its velocity. An obstruction behind the outer part of the propeller disk destroys the symmetry of the conditions to which the blades are subjected, which is bad; but one behind the center of the disk, which is much the more common case, acts not only to retard the flow as a whole but also to guide the air outward from the ineffective to the more effective portions of the blades, and the effect is doubly good.

The gross effect of an obstruction is, in general, to increase both the power consumption and the thrust at a given V and n, but the thrust rises more than the power and an increase of efficiency is the result.

An enormous amount of work has been done on the interaction of propellers and bodies,[1] but most of it relates to the combination of a particular propeller with a particular airplane fuselage

[1] Among the papers most susceptible of general application are "The Effect of Slipstream Obstructions on Air Propellers," by E. P. Lesley and B. M. Woods, *Rept.* 177; "The Effect of Reduction Gearing on Propeller-body Interference as Shown by Full-scale Wind-tunnel Tests," by Fred E. Weick, *Rept.* 338; "An Analysis of the Mutual Interference of Aeroplane Bodies and Airscrews," by A. Fage and H. E. Collins, *R. and M.* 458; "On the Modification of the Performance of an Airscrew Due to the Propinquity of a Plane Surface (in a plane parallel to that of the propeller shaft), by A. Fage and R. G. Howard, *R. and M.* 682; "Experiments on a Family of Airscrews with Tractor and Pusher Bodies," by A. Fage, C. N. H. Lock, H. Bateman, and D. H. Williams, *R. and M.* 830; "The Effect of Gap between an Airscrew and a Tractor Body," by C. N. H. Lock and H. Bateman, *R. and M.* 921; "The Airflow round a Body as Affecting Airscrew Performance," by C. N. H. Lock, H. Bateman, and H. C. H. Townend, *R. and M.* 956; "Theory of Airscrew-body Interference," by C. N. H. Lock, *R. and M.* 1378; "Analysis of Experiments on the Interference between Bodies and Tractor and Pusher Airscrews," by C. N. H. Lock and H. Bateman, *R. and M.* 1445; "Interference between Bodies and Airscrews," by C. N. H. Lock and H. Bateman, *R. and M.* 1522.

of irregular form and is of limited use in an analysis of the fundamentals. There is, however, enough material of all kinds to permit the development of some general rules.

Direct experiment's first teaching is that the drag of a body in the slip stream may not strictly follow the formula computed from momentum considerations. In Fig. 359[1] the ratio of drag in the slip stream to drag of the same object in a free air stream of velocity V has been plotted against C_T'. By the simple

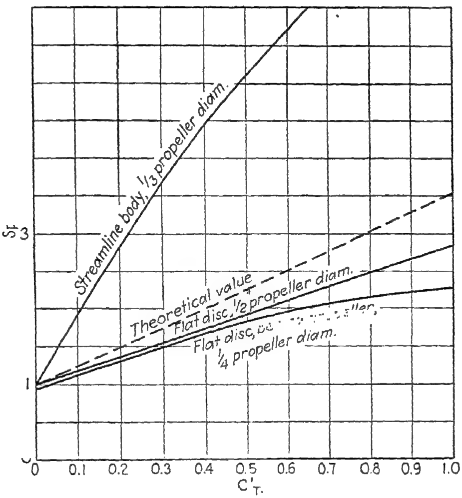


FIG. 359.—Drag of bodies in slip stream, by theory and by measurement.

theory, every body ought to describe the same curve on those coordinates, the line of

$$\frac{D}{D_0} = 1 + 2.55C_T'$$

What actually happened, for two flat plates and a streamline body of somewhat inferior form, all placed as closely as possible behind the propeller, is shown in the figure.

Slip-stream Drag on Streamline Bodies.—That the flat plates showed a slip-stream effect somewhat less than the theoretical

[1] Based on Durand, *Rept.* 177, *cit. supra.* Many other experiments, however, give the same general indications.

one is due in part to their extreme proximity to the propeller. The air flowing through the propeller cannot accelerate instantaneously. The boundaries of the slip stream are somewhat as indicated in Fig. 360, the full V_s only being attained at an appreciable distance downstream from the propeller's plane. Durand's experiments showed that a disk placed $D/6$ behind the propeller had appreciably more drag (about 6 per cent at low values of V/nD) than one at $D/50$. If that influence is taken into account, the difference between the theoretical curve and those representing the disk experiments in Fig. 359 may be considered as explained away, but the much more glaring dis-

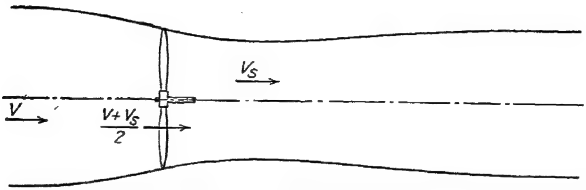


Fig. 360.—General form of, and velocity distribution along, slip stream.

crepancy between the dotted line of theory and the curve for the streamline body remains to explain.

Lock[1] has explained it. Immediately behind the propeller the pressure in the stream is well above atmospheric. At the minimum section it has fallen to atmospheric, and it maintains that value thereafter. If a body of considerable length is placed immediately behind the propeller, its nose is in the high-pressure region, its tail in that of normal pressure. Superposed on the drag due to the air flow around the body is a drag due to the pressure gradient in the stream, or what is sometimes called "horizontal buoyancy" from the analogy of the pressure distribution with that on a solid object immersed in a fluid and bearing pressures proportional to the depth of each point below the fluid surface. This second component of drag depends for its total magnitude upon the thrust per square foot of propeller disk area and the cross-sectional area of the body. It is almost unaffected by the degree of cleanness of form. Considered as a fraction of the body's total drag, therefore, it grows steadily in relative importance as the form is improved. The effect of

[1] *R. and M.* 1378, 1522, *cit. supra.*

pressure gradient on a circular cylinder behind the propeller may be hardly perceptible, while on the best streamline forms the increase of drag computed from slip-stream velocity may be multiplied by 8 or more. The body for which the results are plotted in Fig. 359 was a rather poor one, yet the total increase of drag in the slip stream was fully three times what could be accounted for as directly due to velocity increase alone.

Turn now to the propeller. The nature of the solution there has already been indicated. It cannot be followed through without delving deep into propeller-design principles, but its results can be given. In general terms, they are to slide the propeller curves to the right, so that everything happens at a larger V/nD than before; to increase the peak efficiency; to increase the power coefficient at maximum efficiency.

It is somewhat startling to discover that with a very large obstruction the apparent efficiency may rise to well over 100 per cent, but a little consideration shows the entire reasonableness of such figures. To prove it, imagine an obstruction well behind the propeller having a diameter equal to the propeller's own, and a tube extending forward from it to enshroud and pass the propeller, which would then be working inside a cylindrical box open only at its forward end. Its conditions would be akin to those of static thrust, and if the aircraft of which the propeller was supposed a part were to be moved at a variable velocity there would be no reason to anticipate any drop of thrust with increase of flight speed for a propeller so enclosed. Efficiency as ordinarily defined, proportional to the product of thrust and speed, could be increased to 100 per cent or 500 or more by imagining the speed indefinitely increased while the thrust is kept constant. "Efficiency" in that case would be an obvious misnomer. It is less so only in degree in the less manifestly absurd case of the propeller with a single obstruction close behind it. The obstruction becomes a part of the propelling system, and true efficiency cannot be determined without taking into account the force that the obstruction bears. Nevertheless, the apparent efficiency of the propeller alone is a useful device for studying the reaction that an obstruction in the slip stream sets up. It will continue in use for that purpose, always with reservations as to its real meaning and as to the legitimacy of the word efficiency in such a connection.

Apparent efficiencies for a propeller backed up by various obstructions are superposed on the true efficiency curve for the same propeller operating in free air in Fig. 361. They need no comment. In Fig. 362 curves have been plotted for the net propulsive efficiency of the same combinations.

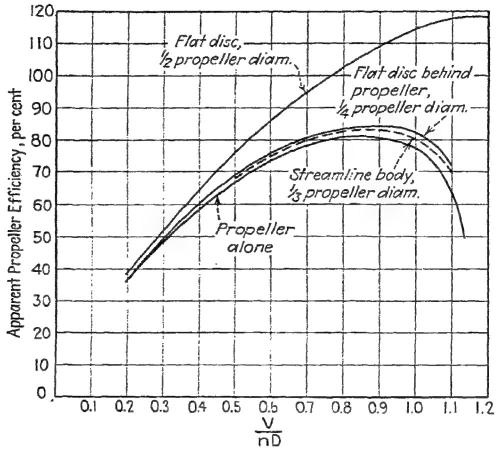


FIG. 361.—Effect upon propeller efficiency of obstructions behind propeller.

Net Propulsive Efficiency.—The concept of net propulsive efficiency has already been introduced in connection with the study of the comparative merits of various nacelle locations. Its formula is the same as that for propeller efficiency, except that T is replaced by $(T - \Delta D)$, where ΔD is the increase in drag of the body immediately behind the propeller due to the propeller's presence. The force term in the new expression can also be written $(T_0 + \Delta T - \Delta D)$, T_0 being the thrust of the propeller isolated in free air, ΔT the effect of the obstruction upon the propeller, and ΔD the effect of the propeller upon the obstruction. D_0, the drag of the body alone in a free stream of velocity V, does not appear in the expression.

Now we begin to get a grip on something that can be used for practical performance calculations. If it were to prove that the net propulsive efficiency of a given propeller was always the same, no matter what sort of an obstruction might lie behind it, it would be unnecessary for the airplane designer to give any

further thought to the individual mutations of ΔD and ΔT. They could be left to cancel each other out.

It is not quite so simple as that, but as Fig. 362 suggests it comes not far from being so. With a large flat disk behind the propeller the peak propulsive efficiency is raised by 4 per cent. With a crude streamline body, of slightly less diameter, it is reduced by 1 per cent. As a rough approximation both changes could be neglected. The net propulsive efficiency could be assumed to be constant, which is equivalent to saying that the standard efficiency curve for the propeller alone could be used, and the drag of the obstruction could be calculated as though it lay outside the slip stream.

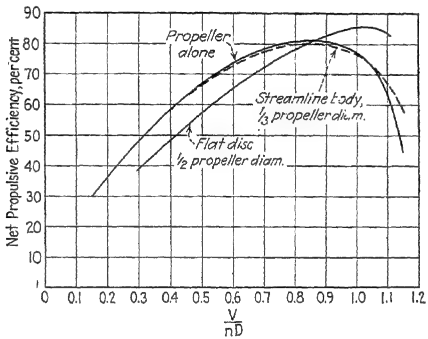


FIG. 362.—Net propulsive efficiency with obstructions behind propeller.

But it is not necessary to be quite so roughly approximate as that. A better rule can be devised. To show how, the net propulsive efficiencies have been computed for several different propellers from a series of British tests[1] and plotted against body diameter in Fig. 363. In this case, again, the body was of somewhat inferior form, blunt-nosed and parallel-sided and with a disk ratio of 1 to 14.

Propulsive efficiency falls off steadily with increase of body diameter, while the apparent efficiency of the propeller alone would be increasing as steadily, and even more rapidly. The idea of taking a mean between the two is an obvious one. Since the apparent efficiency would be used in calculation, if at all, in

[1] *R. and M.* 830, *cit. sufra.*

conjunction with an assumption that the body lay entirely inside the slip stream and that its basic drag was subject to a slip-stream allowance, while the net propulsive efficiency would correspond to an assumption that the body was outside the slip

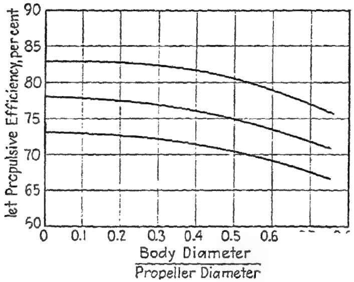


FIG. 363.—Variation of net propulsive efficiency with diameter of obstruction.

stream and that the slip-stream effect to be allowed for on its drag was nil, an intermediate assumption would correspond to leaving part of the body within, part without, the stream. It would correspond, in other words, to the assumption that the slip stream is annular in form, with a hole down the middle,

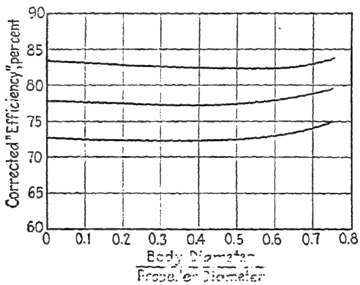


FIG. 364.—Variation of propulsive efficiency after application of correction factor.

and that one part of the body is subjected to the air speed V_s and another part to the speed V.

For this particular series of experiments the best results are obtained by assuming three-quarters of the body to lie outside the slip stream and the other quarter inside. The effective

propeller efficiencies calculated in that way, with thrust replaced in the efficiency formula by $(T_0 + \Delta T - \frac{3}{4}\Delta D)$, are plotted in Fig. 364. Within 1 per cent, the efficiencies so determined are constant, for each propeller, for all body diameters up to 70 per cent of the propeller diameter.

The three-quarters rule is a good one for the average fuselage, but it is not universal. With a flat disk as obstruction, as appeared from Fig. 362, not even an assumption that the whole obstruction is outside the slip stream does full justice to the improved propeller performance that the obstruction produces. With a fuselage or nacelle of exceedingly blunt form it would be safe to assume as much as 85 per cent outside the slip stream. With a good streamline form near the nose, on the other hand, such as may be attained in the fuselage of an airplane with a liquid-cooled engine, the ratio may run as low as a third. In the case now commonest, that of a radial engine enclosed within an N. A. C. A. cowl, about two-thirds may be taken outside the slip stream.

Some further rationalization is possible if the foregoing is combined with what has already been discovered[1] regarding slip-stream effect on drag and its relation to body form. It will be recalled that body drag was considerably increased by what was called the pressure-gradient component, and that the effect was greatest for a blunt-nosed form. Let the expression for total apparent body drag, including the effect of the body on the propeller, now be written

$$D = D_0[1 + 2.55C_T'(1 - K_1)K_2] \tag{108}$$

where D_0 is the drag that would exist in free air at velocity V, K_1 the proportion of the body that is assumed to lie outside the slip stream, and K_2 the factor of increase of slip-stream effect by pressure gradient. Within the range of practical fuselage design, K_1 ranges from 0.33 to 0.85, K_2 from 1 to 6, and both attain their largest values with bodies very blunt at the nose. $(1 - K_1)K_2$ ought always to lie between 0.65 and 2.0, and if a single value is demanded for universal use unity would be as good as any. The experienced designer seeking the greatest possible refinement in his calculations will estimate K_1 and K_2 separately and apply (107), but as an approximation that ought not to lead to any

[1] P. 518, *supra*.

very serious errors all of the various complicating factors to which the last half dozen pages have been devoted can be assumed to accomplish a mutual cancellation. The fuselage can be taken as within the slip stream, and its added drag can be computed by the simple formula dependent on slip-stream velocity alone. In that case the basic propeller efficiency used should of course be that of the propeller *solo*, tested in complete isolation from all disturbing influences. The commoner practice is to test propellers in conjunction with fuselages and to report net propulsive efficiencies, which are somewhat lower than the efficiency of the propeller alone would be. The net propulsive efficiency of course suffices for all calculations, and all this circuitous process of evaluating slip-stream corrections can be dismissed, when the wind-tunnel test has been made upon the same propeller-fuselage combination that is to be used in the airplane. That was done, for example, in the discussion of nacelles and of the propulsive efficiencies of propellers working in front of them.[1] There was no option, for the fundamental laboratory data in those cases were so taken and so presented as to permit of no distinction between the effect of the nacelle upon the propeller and that of the propeller upon the nacelle.

The discussion so far has borne entirely on bodies of smooth form. Excrescences and breaks of contour well behind the propeller have no favorable effect on propeller performance, but they are fully subject to slip-stream effect. If the method of (107) is applied, then, D_0 as used in the formula should be estimated for the body without excrescences. The difference between the figure so estimated and the true drag of the body at speed V should be treated separately and multiplied by the simple slip-stream factor $(1 + 2.55C_T')$. If the approximation of considering the whole body in the slip stream and using the simple speed-squared factor for the whole is used, the body can, of course, be treated as a unit and without special allowance for the excrescences.[2]

Body Interference on Pushers.—At first sight, a body in front of the propeller would appear immune from slip-stream effect.

[1] Pp. 396–411, *supra*.

[2] The effects of excrescences and the changes that those effects undergo with changing distance from the plane of the propeller have been particularly thoroughly investigated in *R. and M.* 1522, *cit. supra*.

If it terminated one-quarter propeller diameter or more in front of the plane of the propeller, it could actually be considered so, but a body coming close up against a pusher propeller has (1) a retardant effect on the air flow, and a consequent beneficial effect on propeller efficiency, essentially similar to that of a body behind a tractor; and (2) a lowered pressure on its after end, due to the drop of pressure with increase of velocity in the stream approaching the propeller, which increases the body's drag even though the speed at its nose is not increased to appreciably above V.

The British experiments already used for developing a rule for proportioning tractor body drag between the slip stream and the area outside[1] included pushers as well. The results were somewhat more variable than for the tractors, especially for the bodies of largest diameter. A pusher body with an almost square stern, immediately in front of a propeller, very effectively screens the inner portion of the blades from the normal approach of air, and when the body diameter exceeds half the propeller diameter the performance is likely to vary widely with the blade form and section and distribution of pitch along the radius. Under those conditions no general rule can apply, but for body diameters up to half the propeller diameter a rule of assuming 10 per cent of the body in the propeller stream and 90 per cent outside gave consistent good results for the form of body used in these particular tests. With a better streamlined body the proportion having to be assumed inside the propeller stream to give correct results in conjunction with the curve of propeller efficiency for the propeller alone would increase, but with such pusher bodies as are used in practice it would never be likely to rise above one-third.

The variation of drag of a body in the propeller stream ahead of the propeller [K_2 in (107)] is due, as previously indicated, almost entirely to the pressure drop in front of the propeller disk and scarcely at all to increase of air speed. Experiment shows an increase of drag above D_0, in most cases, even when C_T' is zero. It ranges in some cases as high as 50 per cent, and superposed on it is a value of between 2.2 and 4.0 for K_2 for bodies with a square stern, such as practically all pusher bodies must have, for it is there that the engine is placed. For a body faired in closely around a vertical engine, or for one carrying the

[1] *R. and M.* 830, *cit. supra.*

propeller on a long shaft extended aft of the engine, K_2 might drop to half the figures just given.

In sum, the effective drag of a pusher body, to be used with the standard propeller efficiency curve, is

$$D = D_0[1 + (K_3 - 1)(1 - K_1) + 2.55C_T'(1 - K_1)K_2] \quad (108a)$$

where K_1 varies from 0.67 to 0.90, K_2 from 1 to 4, and K_3 (the ratio of increase of drag, of the portion of the body considered to be in the propeller stream, at $C_T' = 0$) from 1.1 to 1.4. Again it is true that K_1 and K_2 both attain their maxima for bodies of bad aerodynamic form, though in this case K_2 depends almost entirely upon the degree of squareness of the stern, K_1 quite as largely upon the shaping of the nose. On the average (and that is about all that can be offered from the very limited amount of data that exist) $(K_3 - 1)(1 - K_1)$ can be taken as 0.05, $(1 - K_1)K_2$ as 0.5. The best general rule for dealing with the interference between propellers and pusher bodies is then first to increase the basic drag of the body in free air by 5 per cent and then to add to that an amount computed from the assumption that half the body is subjected to the air speed V_s (as though it were behind the propeller). In this case, since excrescences on the body help to increase K_1 (by contributing to the retardation of the air flow into the propeller) and tend to reduce K_2, they may actually add to the total effective drag something less than their own drag at speed V. Failing more specific data on that point, however, they may most safely be treated by making the complete calculation based on the drag of the body without excrescences and small irregularities and then adding in the simple value of D_0 that such irregularities are estimated to contribute. The idea is the same as in the tractor case, except that there the excrescences were considered to lie entirely within the slip stream and now they are to be taken as entirely outside the propeller influence.

Body Interference and Power Consumption.—The effects on propeller efficiency and drag are the only consequences of interference that enter directly into the calculation of performance. In choosing a propeller and in determining the blade angle to be used, however, changes of power coefficient and of the V/nD at which best net efficiency is obtained have to be taken into account.

As Figs. 361 and 362 indicated, there is a tendency to increase of optimum V/nD with increasing size and bluntness of the obstruction behind the propeller. The mean velocity of flow through the propeller being reduced by the influence of the body behind it, the true V/nD from the point of view of mean conditions actually existing in the air through which the propeller blades pass is less than the apparent V/nD. The true value of the slip function for best efficiency remaining constant, the apparent value increases.

It may be plotted against the size of the obstruction, for any particular form. For the blunt-nosed bodies used in the British tests already analyzed for other purposes,[1] V/nD for best propulsive efficiency proves to increase lineally with increasing body diameter, the percentage of increase in the basic V/nD being one-third of the ratio of body diameter to propeller diameter (measured as a percentage). With an extreme body diameter of 45 per cent of the propeller diameter, which represents the approximate mean of present practice, V/nD would increase by 15 per cent. Conversely, to maintain peak efficiency at the V/nD originally planned, the pitch of the propeller would have to be reduced in approximately the same ratio, calling for an adjustment of the familiar metal-blade propeller to decrease the blade angle by some 2 or 3 deg. (the exact amount depending on the value of the initial angle) as compared with the angle that would give best efficiency for the propeller alone, no body near it. With a cleaner body, and especially with a sharper nose, the effect for a given diameter would be reduced. With a typical cowled radial-engined fuselage the degree of change of V/nD may be taken as 30 per cent of the diameter ratio instead of a third. With a liquid-cooled engine (not having a nose radiator) it may fall as low as a fifth.

The effect of body interference on optimum V/nD with pushers is of the same sign, and of very much the same order, as with tractors. The correction needing to be applied is probably a trifle larger on the average in the pusher case, but the difference is too small to attempt a positive estimate from so limited a body of data. The tractor rule may be used.

If power coefficients are plotted against V/nD, as in Fig. 365, it develops that obstruction reduces the rate of change of C_P

[1] *R. and M.* 830.

with slip and increases its values in the high-efficiency range. From the first it follows that the r. p. m. of a propeller mounted in front of a large and blunt-nosed body will vary somewhat less rapidly with air speed (in extreme case, as much as a fourth less) than the C_P curve for the propeller alone would indicate. From the second comes evidence that a given propeller-engine combination at a given air speed will turn up a lower r. p. m. in front of a large body than a small one. To maintain standard r. p. m., then, the diameter would have to be reduced. But that is academic, for the problem ought not to be solved in that way.

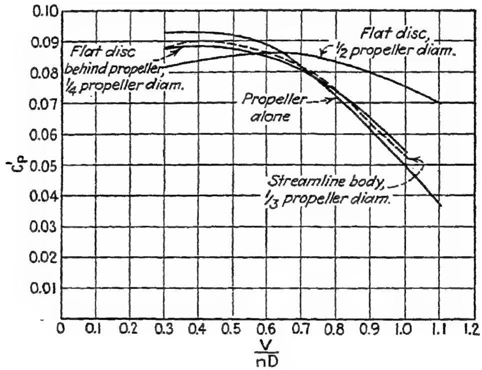


FIG. 365.—Effect of slip-stream obstructions on power coefficients.

After the change in diameter had been made, the propeller would not be running at its proper slip for maximum efficiency under the new conditions. It has already appeared that the same propeller should not be used in both cases. If the comparison is made on the basis of the V/nD of maximum efficiency for each case rather than at the same V/nD for all, C_P is found to be substantially independent of body diameter. All that needs to be done, then, is to change the blade setting. Diameter, blade width, and r. p. m. would remain as before.

Interference between Wings and Propellers.—Whether it is forward of the leading edge or behind the trailing one, a propeller in the neighborhood of an airfoil influences its performance. The wing like any other body sustains reactions proportional to V_s^2, but the effect does not stop there.

The apparent drag of an airfoil, it will be remembered, is exceedingly sensitive to changes in direction of the relative wind. A descending current tilts the vector of resultant force on the wing backward and introduces a component of apparent drag, referred to the hypothetical undisturbed wind direction, of $L\epsilon$ where ϵ is the angle through which the direction of the true relative wind has been turned—whence induced drag. A rising current has the opposite effect, the newly introduced component being a negative one. In the stream immediately behind a propeller, and for that matter for some distance in front of it as well, the lines of flow are convergent. Figure 366 shows it. Below the axis there is a positive vertical component; above it, a negative one. A propeller above a wing then tends, quite apart from any effect of increased velocity in the slip stream, to reduce the drag. One below the wing does the reverse. The effects are still quite perceptible, even with the wing definitely outside the boundaries of the slip stream. They may indeed reach their maxima under that condition.

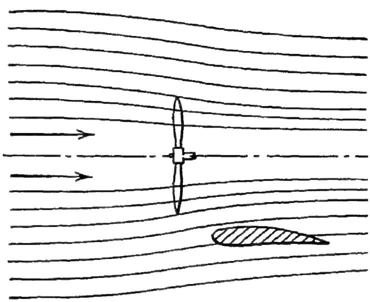


FIG. 366.—Flow of air around a propeller placed above and ahead of a wing.

Wieselsberger has shown[1] theoretically, with experimental confirmation by Prandtl,[2] that a wing of aspect ratio 6 may have its total drag at $C_L = 0.4$ increased or decreased as much as 15 per cent by the action of a propeller, of a diameter of 1.67 times the wing chord, placed just ahead of the leading edge and with its axis from $0.4D$ to $0.65D$ above or below the chord. Unfortunately there are other disadvantages, both aerodynamic and in the field of practical

[1] "Contribution to the Mutual Interference of Wing and Propeller," by C. Wieselsberger, *Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen*, translated as *Tech. Memo.* 754; see also "Uber die Wechselwirkung zwischen Luftschraube und Flugzeug," by H. B. Holmbold, *Proc.* Fifth Internat. Air Navigation Cong., The Hague, p. 575, 1930.

[2] "Gegenseitige Beeinflussung von Tragflügel und Schraube," by L. Prandtl, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," I, p. 112, 1921, translated as *Tech. Note* 74.

installation and balance problems, in the way of mounting a propeller more than half its diameter above a wing.[1] On rare occasions such a mounting might be both feasible and desirable, but as a general rule the saving in drag that could be made by adopting it dwells in the realm of the purely academic. With the propeller behind the trailing edge very much the same results were obtained, though in a somewhat smaller magnitude.

The effect of a propeller on lift is traceable primarily to speed, and the experimental increase of lift for almost any propeller position forward of the wing and so spaced vertically as to put the wing well into the slip stream has been just about what would have been calculated on the assumption that the force varies as $(V_s/V)^2$ and that the effect is limited to the portion of the wing immediately behind the propeller. As the wing passes beyond the boundaries of the slip stream, the effect very quickly drops to zero. A very striking additional point, however, particularly noticeable when the propeller is behind the wing, is the maintenance of maximum lift up to an exceedingly high angle of attack (in some cases as much as 6 deg. beyond the angle of maximum for the undisturbed wing). The explanation lies in the scouring action of the propeller stream, preventing stagnation and separation on the upper surface. The effect (even when the propeller is behind the wing) is that of boundary-layer control by pressure, the pull of the propeller on the air in front of its plane and its backward thrust on the air that has passed through it serving alternatively to accelerate the flow along the upper surface of the airfoil and to suppress any tendency of the air to linger there.

Inclination of the Propeller Axis.—Whenever an airplane changes its speed or the load that it carries, its angle of attack must change. And, failing a variable-angle mechanism, the angle between the propeller axis and the relative wind must simultaneously change by a like amount. Changes in propeller performance might be expected to follow.

Power coefficients and efficiencies might be expected to take new values, too, when the airplane is thrown into a side-slip or when for any other reason the relative wind is thrown to an angle of yaw to the propeller axis. Though yaw is generally a

[1] See discussion of nacelles and their interference effects; p. 398 *et seq.*, *supra.*

temporary phenomenon incident to maneuvers, and it might not matter very much what moderate changes propeller performance underwent in those brief periods, it is possible for it to become permanent. The best example is the case of the twin-engined airplane with its engines "toed out" so that both slip streams will strike squarely upon the tail surfaces, for the sake of better control, especially with one engine dead. Permanent yaw of that sort is never likely to exceed 5 deg.

Net propulsive efficiency (corresponding, it will be recalled,[1] to the assumption that the body lies entirely outside the slip stream) falls off slightly with either yaw or pitch. In no case is the effect likely to be more than 3 or 4 per cent, but even that amount can be reduced by adopting the device already recommended[2] of considering a certain fraction of the body as still inside the slip stream. If a proper value be chosen for K_1 in (107) and used in determining the effective propulsive efficiencies, taking account of the effect of inclination on the basic drag of the body, neither pitch nor yaw should affect the efficiency by more than 1 per cent as long as yaw or pitch does not exceed 15 deg. Given a proper method of calculation, in other words, the effect of inclination should simply be disregarded.

[1] Pp. 520, 522, *supra.*
[2] P. 523, *supra.*

CHAPTER XV

PERFORMANCE CURVES AND THEIR CONSTRUCTION

With the drag of the various elements of the airplane's structure and the characteristics of the power plant and the propeller duly and separately analyzed, it remains only to combine them to find out what the airplane that is the combination of the three in fact will be able to do. The medium of combination is the plotting

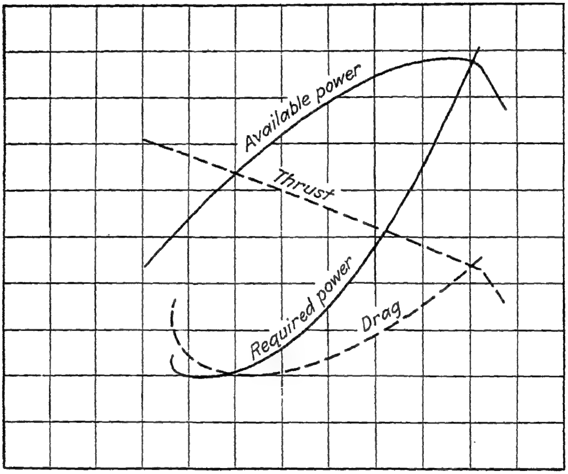


FIG. 367.—Typical set of performance curves.

of a group of performance curves; the most essential members of the group, sufficient in themselves for most purposes, are the representations of power required for flight and of power made available at various altitudes; they may usefully be supplemented by similar curves of propeller thrust and drag. A simple typical set, for sea level only, is illustrated in Fig. 367.

The power-available and thrust curves have been discussed at length in the preceding chapter.[1] The power required is the product of drag and speed, divided by the horsepower constant,

[1] See, most particularly, pp. 488–489 and 501–502.

which is 550 ft.-lb. per sec., or 33,000 ft.-lb. per min., or 375 mi.-lb. per hr. Unfamiliar as its units sound, it is the last constant that is most often used, for it is in m.p.h. that speed is most frequently expressed in the English-speaking countries. Where metric units are used, the horsepower constant (referred to the metric horsepower, a unit about 1.5 per cent smaller than its English equivalent) is 75 kg.-m. per sec., or 270 kg.-km. per hr.

All the elements are now laid out, and the problem becomes one of sheer straightforward computation.

Calculation of Drag.—Since drag rests ultimately upon wind-tunnel tests, the idea immediately occurs of securing it through a single test of a model of the finished airplane. Appealing as that method is in its simplicity, it will not work. It fails in that no model, unless made by a jeweler at enormous expense of money and time, can be a literally complete representation of *all* the external detail of an airplane; and again, and much more seriously, in that scale effects vary so widely on the various parts of the structure that no single scale correction can be anything more than an approximation.[1] The variable-density tunnel, allowing the attainment of full Reynolds' number at least for small airplanes, gives results that should be directly translatable into final performance. There is but one variable-density tunnel in the United States and one in England, however, and they are fully occupied on pure research. The estimation of the performance of new types can better be built upon some other foundation.

With the single-test possibility eliminated, performance can be simply and usually very closely approximated by formula[2] or more nearly determined by various chart methods, making greater allowance for individual peculiarities,[3] or derived from a painstaking step-by-step summation of all the drags contributed by all of the several elements that the plane exposes to the air stream. It is to the last method, most tedious but most direct and in certain instances surest, that the present chapter is especially devoted.

[1] "The Estimation of Airplane Performance from Wind-Tunnel Tests on Conventional Models," by Edward P. Warner and Shatswell Ober, *Tech. Note* 218.

[2] Chap. XVII, *infra*.

[3] Pp. 636–638, *infra*.

Though no effort will be made to carry through a complete illustrative calculation in the same form that might be used in a design office, it will be convenient to derive most of the partial illustrations from a particular airplane and to have some mental image of its appearance. Figure 368 shows the machine selected for the purpose. Its principal characteristics are

Gross weight	2,300 lb.
Rated power at sea level	200 hp.
Rated r. p. m.	2,200
Critical altitude of supercharging	6,000 ft.
Wing area	150 sq. ft.
Wing span	25.9 ft.
Aspect ratio ($=b^2/s$)	4.47
Propeller diameter	7.0 ft.
Airfoil section	Clark Y
Wing loading	15.33 lb. per sq. ft.
Span loading	88.8 lb. per ft.
Power loading	11.50 lb. per hp.

Total drag is the summation of wing drag and parasite. Further broken down for convenience of tabulation, it is the summation of induced drag, of profile, of parasite inside the slip stream, and of parasite outside the slip stream.

The first step is the determination of the relation between the speed of flight and the lift coefficient. For that it is of course necessary to know the lift of the wings, and while the lift is approximately equal to the weight of the machine, closely enough equal for a first approximation, the difference between the two figures cannot be neglected in a really careful calculation.

The difference results from two causes, the inclination of the flight path, which tends to decrease the wing lift, and the load on the tail, which tends in general to increase it. The first effect is very small except for fast machines having a steep angle of climb. Since the lift, total resistance, thrust, and weight of the airplane must at all times form a system of forces in equilibrium, a resolution of those forces along lines parallel and perpendicular to the flight path shows, as in Fig. 381,[1] that the total lift must be equal to the product of the weight by the cosine of the angle of inclination of the flight path. The variation of that inclination with varying speed can be guessed roughly in advance, either from experience with airplanes of similar types or by the use of such

[1] P. 554, *infra*.

formulas as are treated in Chap. XVII, and the appropriate correction can be applied to total lift.

The load on the wings is generally, as here, in excess of the total lift on the airplane, because the tail surfaces usually carry a

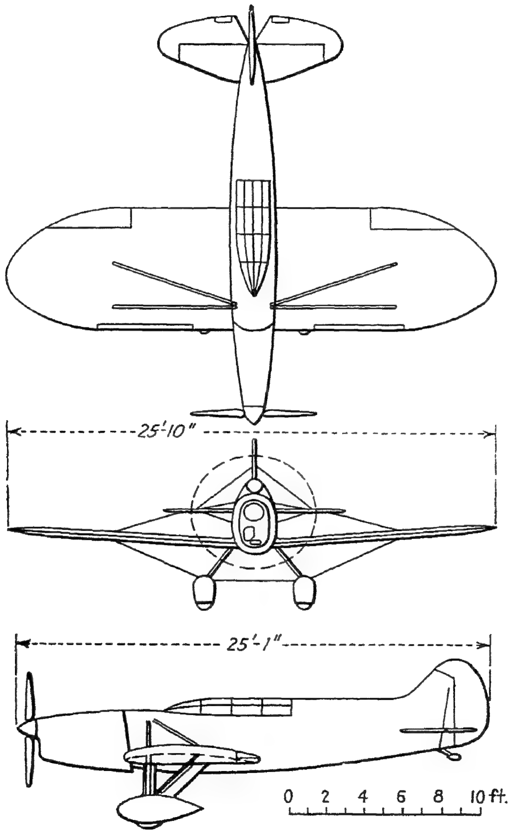


Fig. 368.—Airplane used for illustrative performance calculations.

down-load in steady flight. The total moment around the center of gravity must obviously be zero in order that the flight may be steady, and since the center of pressure of the wing is generally behind the center of gravity of the airplane the wing lift gives a

diving moment which must be balanced by a stalling moment from the tail, and that, in turn, requires a down-load. There are, of course, small pitching moments from other parts of the machine as well, but they are of too little significance to take into account in this connection. In this illustrative calculation it has been assumed that the mean wing chord is 5.8 ft., that the center of gravity lies 30 per cent of the way back on the mean wing chord, in accordance with good practice, and that the center of pressure of the tail (approximately one-third of the way back on its chord) is 14.5 ft. behind the center of gravity. The final result usually is, as it was in this instance, that the total lift of the wings differs appreciably from the weight of the airplane only at small angles of attack and consequently at near maximum speed.

Then

$$M = (C_{M_{0.25}} + 0.05C_L)c\frac{\rho}{2}SV^2$$

and the lift on the tail required for balance is

$$L_t = \frac{-M}{14.5} = -(C_{M_{0.25}} + 0.05C_L)\frac{\rho}{2}\frac{5.8}{14.5}150V^2 = -60(C_{M_{0.25}} + 0.05C_L)\frac{\rho}{2}V^2 \quad (109)$$

The lift on the wings having been found as the algebraic difference of the total lift and that on the tail, the calculation of the exact lift coefficient depends only on the fundamental equation

$$L = C_L\frac{\rho}{2}SV^2.$$

The calculations for the illustrative case here chosen as standard are tabulated in Table X. If still more accuracy were sought, the preliminary estimate of C_L could be based on a value of L including a correction for the estimated tail lift, but such rigor would be absurd. Indeed, if the effect of flight-path inclination were to be neglected entirely, and if $C_{M_{0.25}}$ were to be assumed uniform at -0.070 at all angles, the final value of C_L would be nowhere affected by more than 0.021, and at no speed above 100 m. p. h. would the effect exceed 0.005. Except perhaps for very accurate calculation of maximum climb, that degree of simplification is justified in all practical cases, and the number

of columns that were used in Table X can be reduced approximately a half thereby.

If the more elaborate method is used, it is necessary to recalculate completely for each altitude considered; but if inclination of path is disregarded, the correction to a new altitude can be made by the simple process of multiplying each value of V by $\sqrt{\rho_0/\rho}$. The last six columns in the table then remain unchanged. There is no equally simple converse relationship between V and C_L at new altitudes, and no straightforward means of finding the C_L that corresponds to a particular V without repeating the whole tabulative process. The V scale must be distorted as the altitude changes, if the work of transforming the curves is to be kept as simple as possible.

Induced Drag.—The total induced drag of the wings may be obtained either through the induced-drag coefficient or directly from the span loading. In the former case,

$$C_D = \frac{}{\pi R}$$

In the latter, readily derived from the foregoing,

$$D_i = \frac{W^2}{\pi b^2 \frac{\rho}{2} V^2} - \frac{1}{\pi q}\left(\frac{}{b}\right. \tag{109a}$$

Curves of C_{D_i} against C_L for various aspect ratios have already been plotted in Fig. 157.[1] Curves of total induced drag against speed for various span loadings are now given in Fig. 369.

The figure is plotted on the assumption of an elliptic distribution of load along the span. Actually that is not to be expected. Not only is the form of the wing generally too near the rectangular to allow a perfect distribution, but the span-load curve is further distorted by the presence of the fuselage. At best, the induced drag is likely to run from 2 to 5 per cent above the value given by (109a) if b be taken as the extreme span of the wing. At worst, the proportion of increase above the theoretical minimum may be 10 per cent or more.

Profile Drag.—There remains only, so far as the wings are concerned, the selection of a curve of coefficients to be used in

[1] P. 233, *supra*.

computing profile drag. In most cases some sort of scale correction has to be applied to the wind-tunnel data, but for the Clark Y full-scale data are available at Reynolds' numbers up to almost

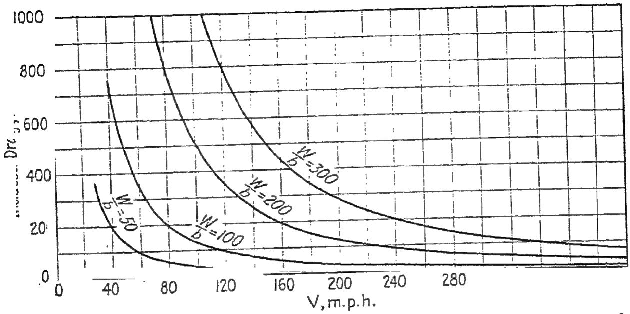


FIG. 369.—Variation of induced drag at sea level with span loading and speed.

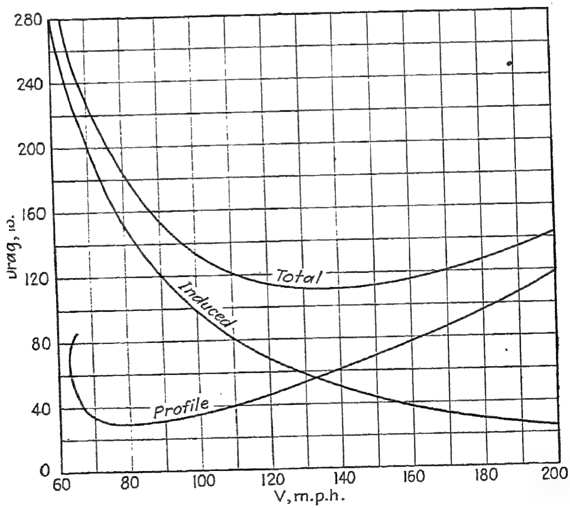


FIG. 370.—Airfoil drag for illustrative case.

10,000,000, where N for the wings of the illustrative airplane rises to only 7,000,000 at 200 m. p. h. The data from the source already cited in connection with the preparation of Table X can then be used directly, with no correction except for the rounding

of the wing tips (for which 0.0006 has been subtracted from C_{D_p} at each point). If the machine were a biplane, there would of course be an additional correction to induced drag on that account; and if the gap were a small fraction of the chord, the

TABLE X.[1]—CONDITIONS AT SEA LEVEL (ρ = 0.002378)

V, m. p. h.	θ, approx. deg.	L_{total} lb.	C_L (first approx.)	$C_{M_{0.25}}$	L_{tail}	L_{wings}	C_L (corrected, final)	α, deg.
65	8	2,277	1.41	−0.070	+ 1	2,276	1.409	14.9
75	12	2,250	1.05	−0.070	− 16	2,266	1.054	9.2
90	10	2,265	0.73	−0.071	− 43	2,308	0.746	4.8
105	7	2,282	0.54	−0.071	− 74	2,356	0.559	2.3
120	5	2,291	0.42	−0.072	−112	2,403	0.437	0.7
140	3	2,295	0.31	−0.072	−169	2,464	0.339	−0.6
160	2	2,298	0.24	−0.073	−239	2,537	0.259	−1.8
180	1	2,300	0.19	−0.073	−314	2,614	0.212	−2.4
200	0	2,300	0.15	−0.074	−407	2,707	0.177	−2.9

[1] Airfoil data taken from "Scale Effect on Clark Y Airfoil Characteristics from N. A. C. A. Full-scale Wind-tunnel Tests," by Abe Silverstein, *Rept.* 502.

TABLE XI

V, m. p. h.	Induced drag, lb.	C_{D_p}	C_{D_p} (corrected)	D_p lb.
65	232.8	0.0318	0.0312	50.0
75	174.5	0.0141	0.0135	29.1
90	121.3	0.0108	0.0102	31.5
105	89.1	0.0093	0.0087	36.8
120	68.0	0.0088	0.0082	45.1
140	50.0	0.0087	0.0081	60.8
160	38.1	0.0085	0.0079	77.3
180	30.2	0.0084	0.0078	96.6
200	24.5	0.0084	0.0078	119.3

profile drag might need a small positive correction as well. Both induced- and profile-drag figures are tabulated in Table XI, and in Fig. 370 they and their sum are plotted. The final calculation of total drag can of course be made either through the addition of the two components, separately determined, or through adding the components of C_D to find its total value and then multiplying

the total lift on the wings by C_D/C_L. Similarly, the power required to overcome wing drag is equal to

$$L_w \frac{C_D}{C_L} \sqrt{\frac{L_w}{C_L \frac{\rho}{2} S}} \frac{1}{550}$$

or

$$\frac{1}{19} L_w \frac{C_D}{C_L^{3/2}} \sqrt{\frac{W}{S}} \sqrt{\frac{\rho_0}{\rho}} \tag{110}$$

L_w being the lift on the wings.

Parasite Drag.—The calculation of parasite resistance is more tedious and carries greater liability of error, although it is no more difficult in principle. It involves simply the listing of all the parts of the airplane that offer such resistance and the treatment of each one separately, together with the interferences among them. The number of elements to be listed in the typical airplane has grown progressively smaller over the past fifteen years as designers have made progress in enveloping within the structure the innumerable minor parts that once would have been exposed as independent drag-catchers, but at the same time the relative importance of interference has grown and on the whole the problem is no simpler now than it was at the end of the World War.

The method is best explained by application. The fuselage is of excellent form, with a cowl analogous to the N. A. C. A. type around the vertical air-cooled engine and with no open cockpits or other breaks of a major order. Its fineness ratio, however, is too high for very low drag. It will be assumed to have a disk ratio of 1/9, and with a cross-sectional area of 9 sq. ft. the equivalent flat-plate area is just 1 sq. ft.

The main brace wires are from $5/32$ to $7/32$ in. in thickness across the wind stream, and they and all other exposed wires are lenticular in section. The Reynolds' numbers at which they work (based, as usual in such cases, on length of section) range from 50,000 to 100,000 at 200 m. p. h., progressively lower at lower speeds. The landing-gear struts, $1\frac{1}{8}$ by $3\frac{1}{2}$ in. in section, have a Reynolds' number of 500,000 at 200 m. p. h. Appropriate values of drag are inserted in Table XII.

The tail surfaces have been treated as airfoils, the area of the surfaces being given and a drag coefficient assigned. It is really hardly proper to count tail resistance under the parasite heading, as the tail surfaces are planned with a view to securing a favorable aerodynamic reaction, but their inclusion there has the sanction of common practice.

TABLE XII.—PARASITE DRAG

Part	Area	C_D			Total C_D (or C_DS)		
		80 m. p. h.	120 m. p. h.	200 m. p. h.	80 m. p. h.	120 m. p. h.	200 m. p. h.
Outside the slip stream:							
Fuselage	0.7 ft.^{2}	0.168*	0.150*	0.145*	0.118	0.105	0.101
Wires	11.3 ft.-in.‡	0.261*	0.203*	0.145*	0.245	0.191	0.136
Landing-gear struts	5.4 ft.-in.‡	0.105	0.090	0.075	0.047	0.040	0.034
Wheels†	6.50–10§				0.479†	0.445†	0.439†
Tail surfaces	10 ft.^{2}‖	0.019‖	0.013‖	0.012	0.190	0.130	0.120
Total					1.079	0.911	0.830
Inside the slip stream:							
Fuselage	8.3 ft.^{2}	0.168*	0.150*	0.145	1.395	1.245	1.204
Wires	2.2 ft.-in.‡	0.205*	0.162*	0.120	0.038	0.030	0.022
Struts	2.4 ft.-in.‡	0.100	0.087	0.075	0.020	0.017	0.015
Tail skid	1.5 ft. in.‡	0.125	0.110	0.105	0.016	0.014	0.013
Tail surfaces	20 ft.^{2}‖	0.019‖	0.013‖	0.012‖	0.380	0.260	0.240
Total					1.849	1.566	1.494

* Including allowance for interference.
† All landing-gear interference included under this item.
‡ Feet of length times inches of width of section.
§ Conventional size specification for medium-pressure tires.
‖ Area given in, and drag coefficients referred to, square feet of surface.

There are no fittings and other such minor details to be allowed for. All are stowed away within fairings. Interference is a comparatively secondary, but by no means a negligible, factor. The low-wing arrangement with but little fillet runs to large interferences, but the comparatively flat sides of the fuselage help to keep them down. Interference at that point is taken as increasing the fuselage drag by 10 per cent; that between the wires and the wings, which approach each other at a fine angle favoring serious mutual disturbance, as raising the wire drag 40 per cent; that within the landing gear as effecting a 50 per cent increase over the resistance of its parts as taken separately.

Division between the parts in the slip stream and those outside has been made on a basis of simple fact except in the case of the fuselage, where the reaction on the propeller must be allowed for. In that case, the very fine lines of the fuselage near the nose indicate a value of 0.50 for K_1 in (107),[1] of about 1.75 for K_2. Combining, the fuselage will be considered as seven-eighths in the slip stream, one-eighth outside, the effect on drag to be calculated in terms of V_s alone and to neglect pressure gradient. Since the break at the cockpit enclosure and the wing-fuselage interference ought both to be considered as entirely within the stream, the proportion to be so taken will be raised to 92 per cent of the fuselage drag including the interference. Slip-stream diameter is assumed 95 per cent of the propeller diameter.

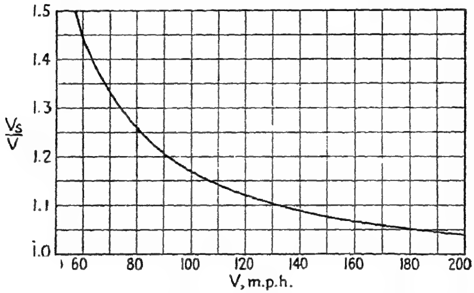


FIG. 371.—Slip-stream velocity in illustrative case.

Common practice in the past has been to sum up the parasite resistances for some particular speed and to assume that under any other condition they would vary in proportion to the square of the speed (always treating the slip-stream and non-slip-stream components separately), but enough is now known about scale effect to put so inexact a method into disrepute except for a machine having few struts and practically no lenticular wires. In Table XII the figures have been separately listed for three speeds, taking into account not only the scale effect but also the changes due to change of inclination of the fuselage and wing-bracing wires and to the change of angle of attack and of elevator angle required for trim on the horizontal tail surfaces.

In Fig. 371, V_s/V is plotted against V (assuming the propeller to give its maximum efficiency at 190 m. p. h.) and Fig. 372 gives

[1] P. 523, *supra*

the curves of the two components and the total of parasite drag. It remains only to add the drag so found to the total for the wings

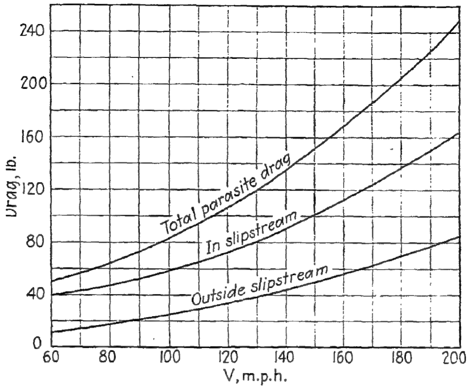


Fig. 372.—Parasite drag, in and out of slip-stream.

as given in Fig. 370, to plot the combined figure, and to multiply by the speed and divide by the horsepower constant to find

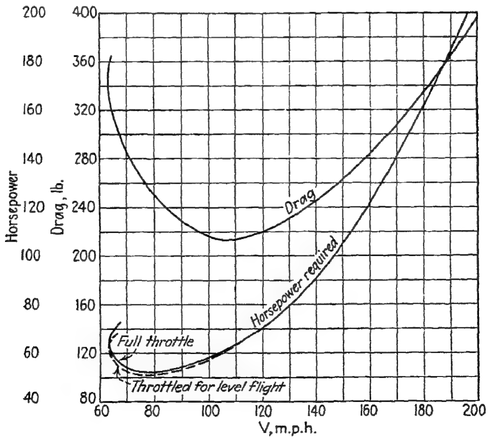


Fig. 373.—Total drag and power requirement.

the propeller horsepower required to drive the airplane at any or every speed. All that is in Fig. 373.

The appearance of minimum total drag at from 30 to 60 per cent above minimum speed, more nearly at the latter ratio when the machine is as clean and has as low an aspect ratio as in the present case, is characteristic. So is the minimum point on the power curve at from 15 to 30 per cent above V_{min}. The points of vertical tangency at the left of each curve correspond of course to the minimum speed of flight, controlled by the wing loading and

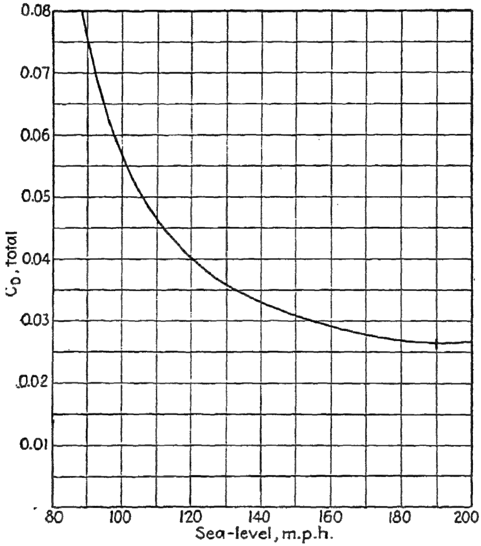


FIG. 374.—Coefficient of total drag for illustrative airplane.

the value of $C_{L_{max}}$, and the continuation of the curves beyond that point corresponds to the inefficient and undesirable, but perfectly possible (except for difficulties of maintaining control under that condition), regime of flight above the angle of maximum lift.

All the calculation to date has been for the open-throttle condition, with any surplus of power utilized in climbing. It was upon that hypothesis that the lift on the wings was figured; upon that basis that the slip stream was analyzed. It is the full-throttle curve that is of greatest interest as a rule, but the slip-stream velocity and the components of drag and the power needed can equally be determined for the case of level flight with a

throttled engine, the thrust always just sufficient to overcome the drag. Recalculation on that basis gives the power curve dotted in Fig. 373. The difference, it will be observed, is exceedingly small, though not small enough to neglect in careful studies of such matters as the maximum range of flight under the most economical conditions.

It is often useful to have a coefficient of total drag for the airplane, expressed either in total drag per unit of q or in drag per unit of q per unit of wing area. In symbolic form, the expression may be either

$$D = C_D \frac{\rho}{2} V^2$$

or

$$D = C_D \frac{\rho}{2} S V^2$$

For most purposes, the second form is the more convenient. It is the second form that has been used in plotting the curve in Fig. 374.

Required Power and Altitude.—With so radical a change in slip-stream conditions making so little difference in the total power consumed, it will be no surprise to find a very trivial effect from the changes of slip stream with altitude. It has already been noted[1] that the lift-coefficient curve can be fully corrected for change of density by multiplying all its V coordinates by $\sqrt{\rho_0/\rho}$. The same is substantially true for wing drag and for the parasite drag of members outside the slip stream. Their attitude is unchanged while $\frac{\rho}{2}V^2$ is kept constant, and $C_D S \frac{\rho}{2} V^2$ must be equally invariant. The only disturbing element, even in the most refined theory, is the change of Reynolds' number. But since N at a given $\frac{\rho}{2}V^2$ varies only as $\sqrt{\rho}$, the effect there, for the machine as a whole, may be confidently expected to be negligible.

There remains only the slip stream. With an unsupercharged engine, its velocity decreases gradually with increasing altitude. With supercharging it increases, less gradually. In Fig. 375 the slip-stream velocity at $V = 120$ m. p. h. is plotted against altitude on the assumption of constant power to 6,000 ft. and decline in

[1] P. 537, *supra.*

accordance with the usual rule thereafter. The maximum variation at a given true speed is less than 2 per cent, which would affect the total power consumption of the airplane by less than 1 per cent.

Figure 371 can then be considered substantially valid at all altitudes, and it follows that at a given *indicated* speed (which means a constant $\rho V^2/2$ and a true speed increasing with altitude) the slip-stream velocity falls off at the higher levels, very slowly if the engine is supercharged and rapidly if it is not. In going to 20,000 ft., for example, V_s/V at an indicated speed of 90 m. p. h. in the present case would decrease from 1.205 to 1.115. The slip-stream component of parasite drag would go off 15 per cent, and the total drag would drop by about 3 per cent. Under top-speed conditions the effect on total drag would again be just about 3 per cent. The seeker after extreme accuracy can make a separate analysis for each altitude, with a separately calculated slip-stream velocity curve for each, but very seldom is it worth while. A satisfactory intermediary between full recalculation and a complete ignoring of slip-stream changes is in the assumption that they lower the power curve by a uniform 1 per cent for every 8,000 ft. of altitude, as compared with the values that it would otherwise show, with an unsupercharged engine. With full supercharging the drop would be about 1 per cent for 20,000 ft. and may very well be neglected.

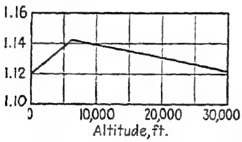


FIG. 375.—Variation of slip-stream velocity with altitude, at constant true velocity.

Subject to those small corrections, and to a good degree of approximation even without them, the power required at any altitude can be obtained from the sea-level curve by multiplying both the ordinate and the abscissa at every point by $\sqrt{\rho_0/\rho}$. The necessity of the multiplication of the abscissa has already been developed. The principle for the ordinate is exactly the same, for the drag at a given attitude is (again subject to reservations on slip-stream effect) unchanged, and the power therefore increases in the same ratio as the speed. Without the small correction proposed in the previous paragraph, then, the transformation of the power curve is attained by drawing radii vectors through the origin to a number of points along the original curve

and then extending the length of each in the familiar correction ratio $\sqrt{\rho_0/\rho}$ as in Fig. 376.

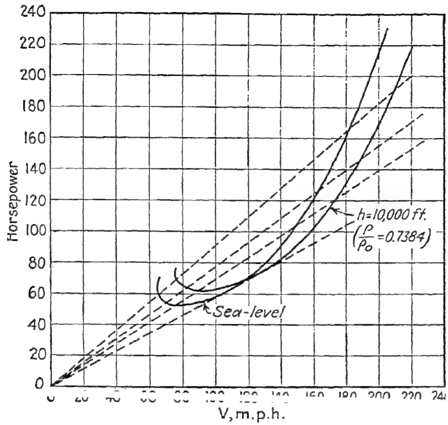


FIG. 376.—Relation of curves of power requirement at different altitudes.

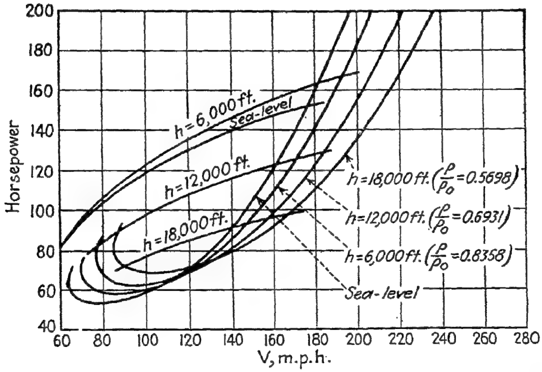


FIG. 377.—Power curves, supercharged engine and fixed-pitch propeller.

The intersection of the curves, with the sea-level curve lying at the top of the sheaf at high speeds and at the bottom at low, is universal. Horsepower required is

$$P_r = \frac{DV}{375} \approx \frac{WV}{375} \times \frac{D}{L}$$

and reaches a minimum, at any given speed, at the altitude at which the angle of attack for steady flight is that of minimum D/L (a condition shown by Fig. 373 to be attained in the present

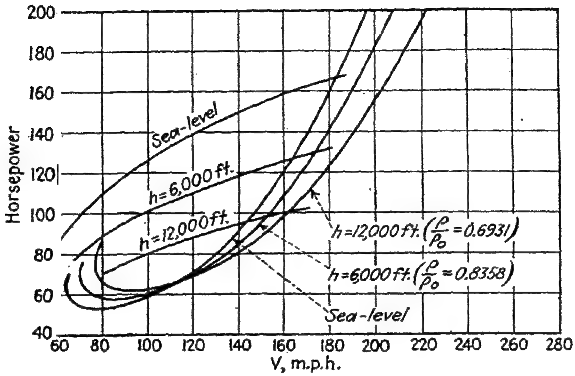


FIG. 378.—Power curves, unsupercharged engine and fixed-pitch propeller.

case at 107 m. p. h. at sea level). The statement, often loosely made, that "it takes less power to fly at high altitudes because the air there is thinner" is somewhat misleading, to say the least.

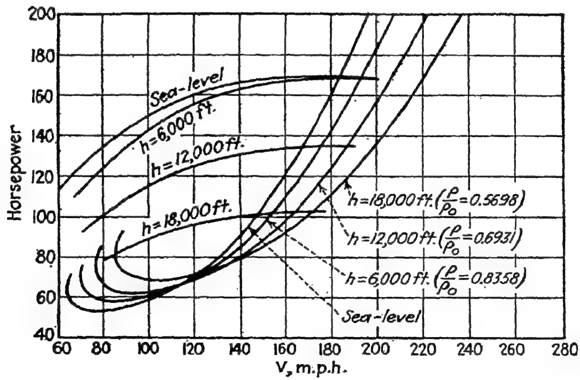


FIG. 379.—Power curves, supercharged engine and controllable-pitch propeller.

The power required for flight is less at high altitudes, *when* it is, because the altitude permits of a combination of a high speed and an efficient attitude. It is to be sure true that the power

required to overcome parasite drag, or in general to overcome any definite resistance of an object little affected by its angle to the air stream, falls off with falling density, but the overshadowing influence in the power-altitude relationship at a given speed is that of the wings, compelled at high speeds and at low altitudes to maintain a low C_L and so an inefficiently small angle of attack, while at low speeds the angle of attack is above the value of best efficiency and further increase of angle with increase of altitude is definitely damaging.

Performance-curve construction terminates with the horsepower available, and in Figs. 377 to 379 complete sets of curves are given for the illustrative case on the three alternative assumptions of (1) a 7-ft. fixed-pitch propeller absorbing the full power of the engine at maximum continuous-operation r.p.m. at 6,000 ft. altitude and 200 m. p. h.; (2) an unsupercharged engine, delivering 200 hp. to a propeller that absorbs it all at sea level; (3) the supercharged engine, as originally specified, with a controllable-pitch propeller applied. The horsepower-required curves are virtually the same in all three cases, the variations in slip-stream effect being almost too small to measure, but those of power available are widely different.

CHAPTER XVI

INTERPRETING THE PERFORMANCE CURVES

Maximum Speed.—All of the important elements of airplane performance can be calculated directly from the curves of power, thrust, and resistance; most of them, indeed, from the power curves alone. The simplest and most obvious of the determinations is that of maximum speed, read off directly at the intersection of the curves of available and required power for any particular altitude. Obviously the maximum speed of level flight is that at which all the power that the engine can produce and the propeller transmit is being used in driving the airplane along a

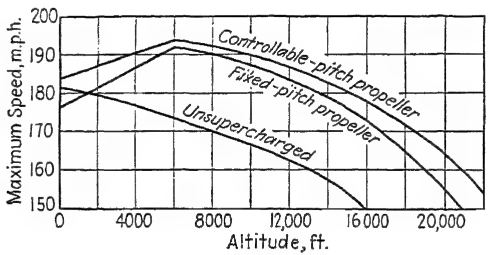


FIG. 380.—Variation oi maximum speed with altitude and with engine and propeller characteristics.

horizontal course. To travel faster than that it is necessary to secure additional power from gravity by diving.

The variation of maximum speed with altitude depends primarily upon the power plant, incidentally upon the wing loading and power loading and general cleanness of the structure. It will be obvious from inspection of the groups of performance curves given at the end of the last chapter that speed will generally increase with altitude with a supercharged engine and decrease with an unsupercharged one. The speed-altitude curves for the three power-plant cases here treated as representative are plotted in Fig. 380, where the steady increase up to the critical altitude, the accelerating rate of loss of speed beyond that point,

and the beneficial results of the controllable-pitch propeller are all apparent.

Approximations to a general rule covering the drop of speed can be secured by analysis. With reference to Fig. 374, it is apparent that the coefficient of total drag for the airplane is virtually constant (within 3 per cent) over the range from 2.8 to 4 times the minimum speed. If C_D is taken as constant, the power required will vary as V^3, and at a given speed as ρ. Power available follows (approximately) any one of the four alternative formulas:

Supercharged engine, controllable-pitch propeller.......	ρ^0
Supercharged engine, fixed-pitch propeller.............	$\rho^{-0.3}$
Unsupercharged engine, controllable-pitch propeller....	$\rho^{1.2}$
Unsupercharged engine, fixed-pitch propeller..........	$\rho^{1.25}$

Furthermore, combining the propeller-efficiency and the r.p.m.-speed curves, it can be taken as approximately proportional to $\sqrt{V}$ with a fixed-pitch propeller and to $\sqrt[8]{V}$ with one of controllable pitch. Combining all these expressions, for the unsupercharged fixed-pitch case:

$$C_1\rho V^3 = C_2\rho^{1.25}V^{\frac{1}{2}} \qquad (111)$$
$$\rho^{0.25} = C_3V^{2.5}$$

Raising both sides of the equation to the 0.4 power,

$$\rho^{0.10} = C_4V$$

and the maximum speed ought to vary as the 0.10 power, that is to say as the tenth root, of the air density. With a controllable-pitch propeller the variation should be as $\rho^{0.07}$, while with a supercharged engine the anticipated laws would be a proportionality to $\rho^{-0.5}$ with a fixed-pitch propeller and to $\rho^{-0.35}$ with pitch controllable.

The formulas so computed check very satisfactorily with the illustrative case when a supercharged engine is used. With no supercharging the formula indicates a loss of speed from sea level to 6,000 ft. only about half that actually observed, and the explanation of the discrepancy lies in the separation, rapidly increasing with increasing altitude, between the point of minimum-drag coefficient and that of maximum speed. Figure 374 shows that at $2.5V_{\min}$ the drag coefficient, instead of being con-

stant, is varying in inverse proportion to $V^{0.9}$. At $2V_{\text{min}}$ the rate of change of C_D is equal to that of $V^{-1.4}$. Under those conditions analysis indicates, by a process obvious from the work just done, that the maximum speed in the unsupercharged fixed-pitch case will vary as $\rho^{0.16}$ and $\rho^{0.23}$, respectively. While the illustrative airplane's maximum speed at sea level is 2.9 times the minimum, the maximum at 6,000 ft. is but 2.47 times the minimum at that level. The total loss of speed from sea level to 6,000 ft. might then be expected, taking the mean of the laws that govern at the two extremes of altitude, to vary about as $\rho^{0.16}$—which is not unreasonably remote from the rule actually found to prevail for this particular airplane.

It then appears that the rate of change of speed with altitude depends primarily on initial speed range, and that the rates derived from (111) and its companion formulas are optima, from which a small recession is likely in the case of supercharged airplanes and a very much larger recession for the unsupercharged ones. Further study will be given to this whole subject in Chap. XVII.

Minimum Speed.—The minimum speed, at the opposite end of the scale, is commonly determined by the maximum-lift coefficient. There are very few airplanes which have not enough power to pursue a level course at the angle of attack corresponding to maximum lift, although there are many of which the control and stability are not adequate to permit the maintenance of flight at true minimum speed for more than a few seconds except under very favorable conditions.

If the maximum lift of the airfoil used continued to be the determining factor at all altitudes, the minimum speed of flight would be simply proportional to the inverse square root of the air density, but at high altitudes, near the ceiling, another limiting factor comes into play. As reduction of air density causes a gradual raising of the curve of power required and the lowering of that of power available, the left-hand ends of the two curves approach each other until they finally intersect at a point below the power required for flight at maximum lift. Thereafter it is the location of the second intersection of the two power curves which determines the minimum speed of level flight, although a somewhat lower figure, the true minimum corresponding to the maximum-lift coefficient, can be attained in a stalled descent, in

which the work done on the airplane by gravity makes up for the inadequacy of the engine power. Occasionally airplanes are designed for special purposes with so little reserve of power that it is the second intersection of the power curves, not the maximum lift, which determines the lowest speed of level flight even just above the ground.

The minimum speed may actually be forced, by an exceptionally skillful pilot flying an exceptionally controllable airplane, to 2 or 3 m. p. h. below the figure calculated from the maximum-lift coefficient. The technique is to stall the airplane to an angle of attack appreciably beyond that of maximum lift, so materially increasing the power required for level flight and making it possible to hang a considerable proportion (sometimes as much as 15 or 20 per cent) of the plane's weight on the propeller, balancing it directly against the vertical component of thrust (further amplified by the slip-stream effect on the lift of the wings). Though the reduction of speed so attainable can be calculated if desired, the maneuver is purely a stunt and its result has no proper place among the records of an airplane's performance.

Slip-stream effect on the wings will lower the minimum in level flight slightly in any case, though seldom by more than a mile an hour. Even that is commonly ignored, for minimum speed is of primary interest in connection with the making of landings, and in landing the engine is idled. Take-offs are made at near, but usually appreciably above, the minimum speed, as an increase of 3 or 4 m. p. h. will reduce the total drag as much as 10 per cent, the power required by half that proportion.

Though the terms "minimum speed" and "landing speed" are often used as though they were interchangeable, they actually denote quite different quantities. Minimum speed is computed for, and should be measured at, an altitude great enough to be clear of all disturbing influences. Landing speed, on the other hand, is by definition determined immediately adjacent to the ground. Ground effect[1] may reduce it to as much as 3 or 4 m. p. h. below the true minimum for steady flight at a height equal to the wing span. Furthermore, the airplane may be accelerating downward during the last few feet of vertical travel before contact with the ground, so that the lift during that interval may be dropping to appreciably less than the airplane's

[1] See p. 335, *supra*.

weight. That, especially on a machine with a good rugged landing gear, may reduce the speed at contact by another 4 m. p. h.

Rate of Climb.—The calculation of the rate of climb is slightly more involved, and requires an analysis of the conditions of equilibrium. With reference to Fig. 381, a representation of the forces acting on an airplane flying along an inclined path, and by resolving W into components perpendicular and parallel to the flight path, since all the other forces in the diagram already act along

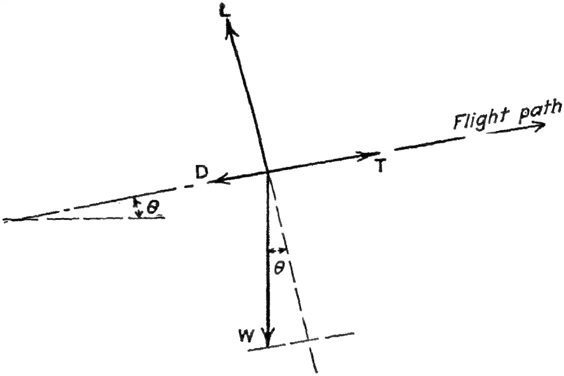


FIG. 381.—Elementary forces acting on an airplane in flight.

one of those axes, the necessary conditions of equilibrium appear as

$$L = W \cos \theta \tag{112}$$

$$T - D = W \sin \theta \tag{113}$$

It will be noted that the lift is actually less than the weight,[1] and that it decreases progressively as the climb becomes steeper, so that the statement, sometimes carelessly made, that the climb of an airplane results from the existence of an excess of lift is thrown out of court at once. Since $dh/dt = V \sin \theta$, dh/dt being the rate of climb and V being measured along the inclined path,

$$W\frac{dh}{dt} = (T - D) \times V \tag{114}$$

The product TV, however, is equal to the power available, while DV is the power required. Their difference is the excess of power,

[1] See also p. 534, *supra*.

or the vertical distance between the power curves on the performance charts, and the rate of climb is therefore equal to the excess power divided by the weight of the airplane. That might indeed have been taken as obvious from elementary mechanics, since excess of power represents excess of energy available for use in a given time and could be equated to the work done in raising the airplane against gravity. It is of course necessary that the units be consistent, so that if the weight of the airplane is in pounds and the excess of power in horsepower, the latter figure must be

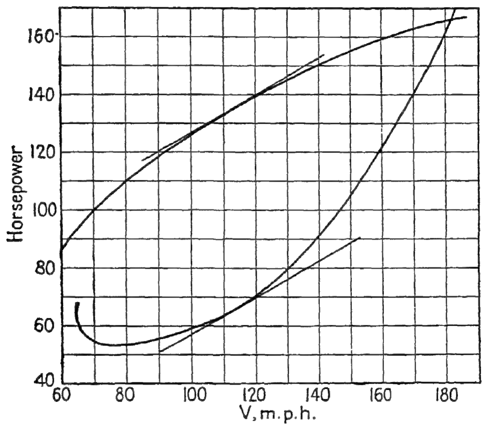


FIG. 382.—Graphical construction for determination of best climbing speed.

multiplied by 33,000 to get the climb in ft. per min., or by 550 if it is wanted in ft. per sec.

The speed of flight corresponding to the most rapid climb is obviously that for which the separation between the two power curves is greatest, and that point of maximum separation can be determined either by trial or by moving two scales along and always tangent to the curves at such relative rates that the two points of tangency lie always on the same vertical line. The point of maximum separation will obviously have been reached when the two tangents become parallel, as the curves are concave toward each other. The two tangents are shown in the parallel position, one point of tangency vertically below the other, in Fig. 382, and their location indicates the speed for best climb at sea level in the unsupercharged case to be 113 m. p. h. The

separation between the curves being 69.5 hp., and the weight of the airplane 2,300 lb., the maximum rate of climb works out at 995 ft. per min. For the two supercharged cases the best speeds for climbing from sea level are 110 m. p. h. with the fixed-pitch

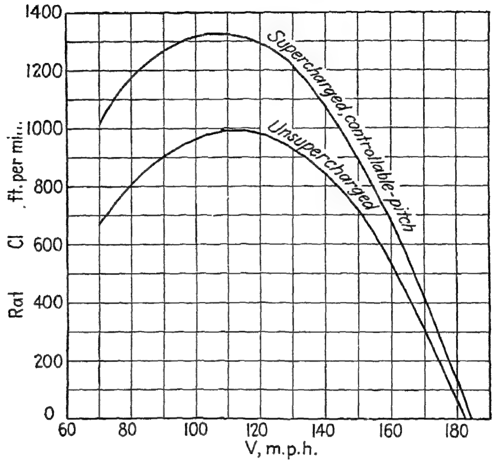


FIG. 383.—Effect of speed of flight on rate of climb.

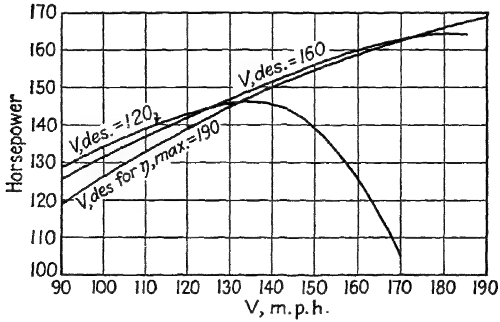


FIG. 384.—Effect of propeller pitch on power available.

and 106 m. p. h. with the controllable-pitch propeller; the best rates of climb, 875 and 1,330 ft. per min., respectively.

Small changes in speed do not of course have much effect on the climb. The rate of ascent is plotted against the speed

of flight for two cases in Fig. 383, from which it develops that the speed can be changed either way from its best value as much as 10 per cent without losing more than 3 per cent in rate of climb in this particular case.

It is now possible to analyze in more detail the effect of propeller pitch on performance. The unsupercharged case has been computed with a number of different propellers, all of fixed pitch, all absorbing 200 hp. at maximum speed at sea level, but having their maximum efficiency at various speeds from 120 to 190 m. p. h. The power-available curves are plotted in Fig. 384, the maximum speed and best rate of climb in Fig. 385. The indication is that for any service except racing it would have been better to choose a propeller that would show its maximum effi-

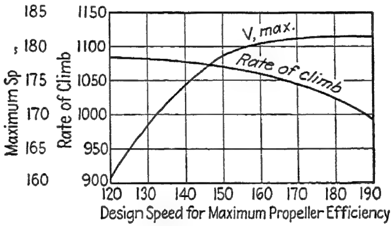


FIG. 385.—Effect of propeller pitch on speed and climb.

ciency at about 10 to 20 per cent below maximum horizontal speed. As compared with the performance with the best high-speed propeller the sacrifice in speed would be only ½ per cent, and an improvement of 3 per cent or more in the rate of climb at sea level (coupled with a substantial improvement in take-off) would more than offset it.

Change of Climb Characteristics with Altitude.—The rate of climb for other altitudes can of course be figured in exactly the same way as for sea level, and a curve of rate against altitude can be plotted as in Fig. 386. The curve usually has a slight upward concavity, as in the illustrations there given, but it is nearly enough straight so that a linear relationship between altitude and rate of climb is frequently taken as a sufficient approximation to the facts. Such a curve having been drawn, the absolute ceiling is read off at the intersection with the axis of ordinates or at the point where the rate of climb is zero, while the service ceiling is similarly defined by a rate of climb of 100 ft. per min.,

the two figures in the controllable-pitch case being 24,500 and 22,800 ft., respectively.

There is one abnormality in the form of the climb curves near sea level, especially apparent in the controllable-pitch case. The assumption in dealing with this illustrative problem has included an engine power maintained constant from sea level to the critical altitude, whereas the more customary case is a limitation of the manifold pressure which results in a gradual

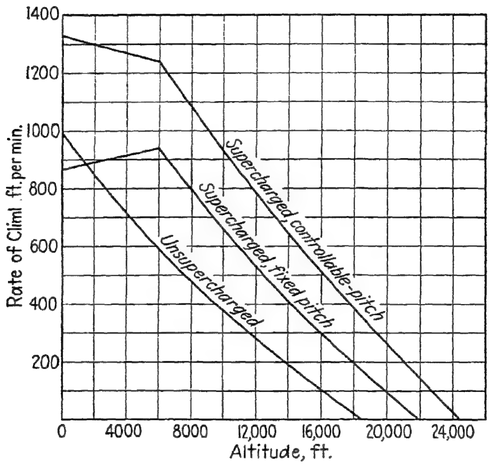


FIG. 386.—Variation of rate of climb with altitude.

increase of power with altitude through that zone. As a rule, with constant manifold pressure the power available increases with altitude, up to the critical altitude, more rapidly than does the power required. Even with a controllable-pitch propeller, then, the rate of climb should be a maximum at critical altitude and a little lower than that at sea level, instead of falling off slowly from a maximum at sea level as it does in the constant-engine-power case illustrated in Fig. 386.

If the propeller efficiency were the same at all speeds, the speed of best climb would always be that of minimum power consumption. Actually, however, the drop of efficiency toward the origin of the curve makes it profitable to fly at a speed somewhat higher than that of minimum power. If the slope of the power-available

line were constant at any value whatever, and the same for all altitudes, the best speed for climbing would vary with density in accordance with the familiar inverse square-root rule. The indicated speed ($V\sqrt{\rho/\rho_0}$, which is the figure read by any of the commonly used forms of air-speed indicator) would be the same at all altitudes. The best climb would correspond to the same angle of attack at all heights.

All that comes exceedingly close to being true when the engine is supercharged and a controllable-pitch propeller is used. Under those conditions the best speed for climb is likely to vary approximately as $\rho^{-0.45}$, which in turn implies a variation of indicated speed (what a pitot or venturi tube shows) as $\rho^{0.05}$, and a change of optimum angle of attack of less than a degree in going from sea level to 20,000 ft.

With a fixed-pitch propeller, for which the power-available curve generally has a larger curvature, the rate of change of true air speed is likely to fall off to about $\rho^{-0.40}$. The replacement of a supercharged by an unsupercharged power plant makes considerably more difference, and the rate of change of optimum speed with density in that case averages about $\rho^{-0.30}$, while the angle of attack for best climb may increase as much as 5 deg. in the course of an ascent to a 20,000-ft. altitude. In the present case the air speed for best climb at 12,000 ft. would be approximately 124 m. p. h. either with an unsupercharged engine or with one fully supercharged to a 12,000-ft. critical altitude or above, although at sea level the best speed was 7 miles higher for the unsupercharged than for the supercharged case.

The angle of climb is often more important than speed, as when it becomes necessary to get out of a field over high obstacles at the edge, and the angle can be determined from the maximum distance between the curves of thrust and total drag in just the same way that the climbing speed is determined from the power curves. With reference to Eq. (113), it is seen that

$$\sin \theta = \frac{T - D}{W} \tag{115}$$

Although the minimum power requirement occurs at a lower speed than the minimum drag, because of the entry of the factor V, the maximum reserve of power is found at a higher velocity than the maximum excess of thrust. In trying to get as high as

possible in a given distance of advance over the ground, it is therefore profitable, as might be expected, to pull the airplane up to a larger angle of attack and a lower speed than in seeking to gain the greatest possible altitude in a minimum of time. Curves of angle of climb against speed are plotted for two cases in Fig. 387, from which it appears that the best speed for angle is approximately 30 m. p. h. lower than the best for vertical velocity.

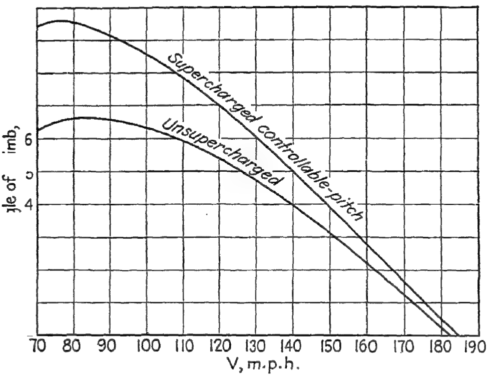


FIG. 387.—Effect of speed of flight on angle of climb.

Time of Climb.—The general climb performance of airplanes is commonly measured by the time which they require to reach some designated altitude. That factor is of considerable direct importance in military machines, which may have to climb to a great height in order to repel an attack or before venturing across heavily guarded lines and which should waste as little time as possible in the preliminary ascent, and also in long-distance transports, which commonly operate most economically at altitudes of 10,000 ft. or more and for which time unnecessarily spent in reaching operating height means fuel unnecessarily burned. The time needed to reach any particular height is easily found from the rate-of-climb curve, or rather from its inverse.

For this calculation the curve of rate of climb against altitude should be supplemented by one of dt/dh, most conveniently expressed in minutes per thousand feet of climb. Such a curve,

corresponding to the rate of climb plotted in Fig. 386, is given in Fig. 388 for two power plant cases. In analytic form

$$\text{Time to climb to height } h = \int_0^h \frac{dt}{dh} dh \tag{116}$$

and the integration is graphically performed by taking the area under the curve from sea level up to the height under consideration. The cross-hatched area in Fig. 388, for example, represents the time required to climb 10 000 ft. with an

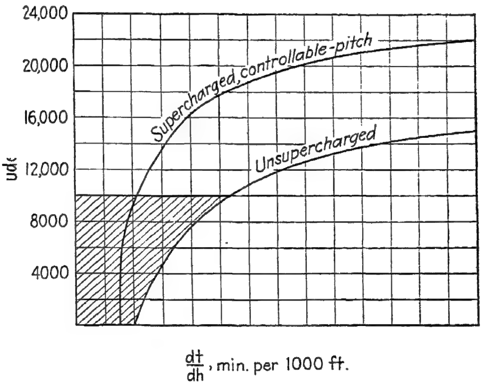


FIG. 388.—Determination of time of climb to a given altitude (cross-hatched area shows time to 10,000 ft.).

unsupercharged engine. The area is 16.15 units (each square on the diagram representing a unit). One unit vertically represents 2,000 ft., one unit horizontally 0.5 min. per thousand feet, and one unit of area is therefore 1 min., which makes the total time of climb 16.15 min.

A much simpler method of calculation of time of climb is available if it can be assumed that the rate-of-climb curve is either a straight line or a broken one of two or three segments, and it has already been pointed out that above the critical altitude even the simpler of those assumptions ordinarily furnishes a fair approximation to the facts. It is then possible to write

$$\frac{dh}{dt} = C_1 - C_2 h \tag{117}$$

C_1 and C_2 being constants depending on the performance of the machine. C_1 is manifestly equal to the rate of climb at sea level, while it is equally clear that C_2 can be determined in terms of C_1 and ceiling height, since the rate of climb at the ceiling is zero. Evaluating analytically the integral already treated graphically, denoting ceiling height by H and rate of climb at sea level by C_0,

$$t = \int_0^h \frac{dt}{dh} dh = \int_0^h \frac{dh}{C_1 - C_2 h} = \left[-\frac{1}{C_2} \log (C_1 - C_2 h) \right]_0^h = \frac{1}{C_2} \log_e \frac{C_1}{C_1 - C_2 h} \tag{118}$$

In the unsupercharged case

$$C_1 = C_0, \quad C_2 = \frac{C_1}{H}$$

$$t = \frac{H}{C_0} \log_e \frac{1}{1 - \frac{h}{H}} \tag{119}$$

Applied to the case just calculated by the graphical method, this would indicate a time of 14.5 min. to climb to 10,000 ft., only a very rough check with the figure previously obtained.

Various things can be done to this method to make it more exact. Diehl[1] fits the mathematical method to the experimental facts by taking the relation of rate of climb to altitude as quadratic instead of linear and determines the time by integration as

$$t = \frac{1,000}{\sqrt{a^2 - 4bC_0}} \log_e \frac{2C_0 + \frac{h}{1,000}\sqrt{a^2 - 4bC_0} - a\frac{h}{1,000}}{2C_0 - \frac{h}{1,000}\sqrt{a^2 - 4bC_0} - a\frac{h}{1,000}} \tag{120}$$

where

$$\frac{dh}{dt} = C_0 - a\frac{h}{1,000} + b \ \frac{h}{1,000}\Big)^2 \tag{121}$$

and the rate of climb must be known at some three well-separated altitudes to determine the constants. In the present case

[1] "Engineering Aerodynamics," by Walter S. Diehl, Chap. VIII, Ronald Press Company, New York, 1928.

(unsupercharged) C_0 is 995, $a = 72$, and $b = 1.0$, and the computed time to 10,000 ft. is 16.20 min. The error is less than ½ per cent.

Undoubtedly the most accurate method, this one suffers somewhat from complexity of determination of the coefficients. A fair approximation for most purposes is given by a modification of (119), which in its unadulterated form underestimates the time in all cases and often rather badly. If C_0 and H both are arbitrarily reduced by 5 per cent, the time will come out correctly within 5 per cent at all altitudes up to service ceiling, and within 3 per cent at most.

Another alternative, perhaps the most practical of all, and one that has to be used in any case when a supercharger and a critical altitude are involved, is the treatment of the rate-of-climb curve as a broken line. The integration is then separately performed for each section of the climb, and the formula becomes the sum of a series of segments of the form

$$\Delta t = \frac{\Delta h}{C_1 - C_2} \log_e \frac{C_1}{C_2} \tag{122}$$

where C_1 is the rate of climb at the lower limit of the particular range of altitude covered by that particular segment and C_2 the rate at the upper limit. Thus, for the supercharged controllable-pitch case,

Range of altitude	C_1	C_2	Δt	t (cumulative)
0 to 6,000	1,300	1,240	4.73	4.73
6,000 to 12,000	1,240	785	6.03	10.76
12,000 to 18,000	785	385	10.68	21.44
18,000 to 22,000	385	145	16.29	37.73

Both for that case and for the unsupercharged one, time-of-climb curves have been computed and are reproduced in Fig. 389. Such a curve forms a part of every performance-prediction study.

In the broken-line method just described the formula becomes indeterminate when $C_1 = C_2$, and accurate calculation grows progressively more difficult as they more nearly approach each other. Where they lie within 20 per cent of each other, the work

can be simplified and the probable accuracy improved by replacing the logarithmic form with the algebraic one

$$\Delta t = \frac{\Delta h}{0.45C_1 + 0.55C_2} \tag{123}$$

C_1 in this case always being taken as the larger of the two rates of climb. With a supercharged engine below the critical altitude and a fixed-pitch propeller, the significance of C_1 and C_2 would then be reversed from that given in connection with (122).

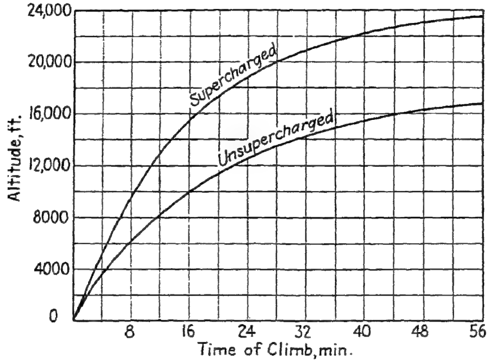


FIG. 389.—Time to climb to altitude, for the illustrative airplane.

Cruising Speed.—As set forth in an earlier chapter,[1] airplanes do not cruise at maximum speed or power. The cruising power factor may be anything between 50 and 75 per cent, and in present practice it generally runs near the middle of that range for transport operation. In military and naval flying no single percentage can be given, as the cruising condition will depend entirely upon the nature of the exercise. Where speed of execution is the essence of safety it may rise to 90; where range and fuel economy are paramount and the flight is to be made over undefended country it may fall to below 50.

Whatever the factor's value, the method of analysis is the same. A new power-available curve is drawn at x per cent of the rated power output. If extreme accuracy is sought a new power-required curve may also be constructed to allow for reduced slip-stream effects, but, as the dotted line in Fig. 373

[1] Chap. XIII, *supra.* See also pp. 507–509, *supra.*

served to demonstrate, the allowance on that account is exceedingly small. The graph of power available with a fixed-pitch propeller may take either of two forms, depending on whether the cruising limitation is specified on power alone or on r. p. m. and manifold pressure separately. Under the second alternative the torque will be substantially constant, precisely as in full-throttle operation, and the r. p. m. will reach the cruising limit only at one speed of flight. Under the first, the r. p. m. and manifold pressure can be so varied with respect to each other that the power (substantially proportional to their product) will reach the fixed limit at any flight speed whatever.

This difference is of some importance, for notwithstanding the fact that cruising limitations are applied only in level flight and therefore only at a single speed at each altitude the power absorbed by the propeller at a given r. p. m. varies materially with the altitude. In supposing a 75 per cent cruising factor applied to the standard illustrative airplane, and analyzing the fixed-pitch unsupercharged case, the r. p. m. at sea level would be 1,990, or 90.5 per cent of normal, while at 6,000 ft. altitude they would rise to 96 per cent of normal in order that the propeller might continue to absorb the same power. Conversely, if there were a definite limitation to 91 per cent of rated r. p. m. ($=\sqrt[3]{0.75}$) among the cruising conditions the power that could be put out in cruising at 6,000 ft. would not be 75 per cent of the rated figure but only 66 per cent. The natural expectation is that, other things being equal, the power absorbed by the propeller at a given r. p. m. will be proportional to the air density. The percentage of power available at cruising r. p. m. at the higher altitudes is, however, rather more than the product of power factor and density ratio (66 instead of 62.7, in the present instance) because the cruising speed under these conditions falls off with altitude, and the resultant drop of V/nD increases the power coefficient of the propeller.

There is, therefore, a substantial gain in cruising speed at high altitudes if the engine builder and the maintenance department can be persuaded to approve of putting a cruising limitation on power alone and allowing the r. p. m. to run up toward the full rated value as the propeller torque runs down and the manifold pressure is dropped with decreasing density. If that degree of freedom of action is refused, a new evidence of the virtues of a

controllable-pitch propeller appears, for pitch control allows, within very wide limits, the absorption by the propeller of *any* desired power at *any* desired r. p. m. and at *any* altitude.

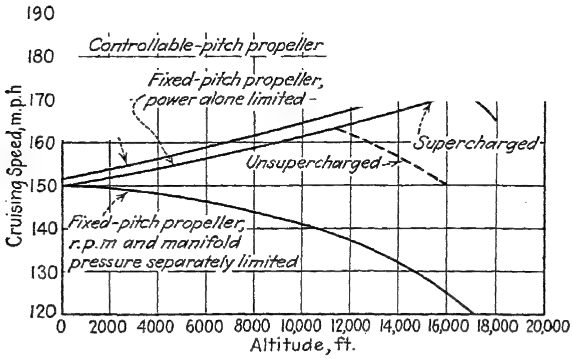


FIG. 390.—Cruising speed at a 65 per cent power factor.

Implicit in the preceding paragraphs is a fact so pregnant with practical significance, and seemingly so obvious, that it is difficult now to understand how it could have been generally ignored through a decade of growing air-transport operation. Only in 1933, as the result of the epoch-making work of Allen and Oswald,[1] did it fully and generally sink home that in cruising even an unsupercharged engine has the equivalent of a supercharging effect and that in every case where the cruising limitation is stated in terms of power alone, or where a controllable-pitch propeller is used, the cruising speed should increase with altitude. In Fig. 390 the cruising speed with a 65 per cent cruising factor is plotted against altitude for all three engine-propeller cases and for the alternative assumptions that the 65 per cent limitation stands alone and that it is combined with a

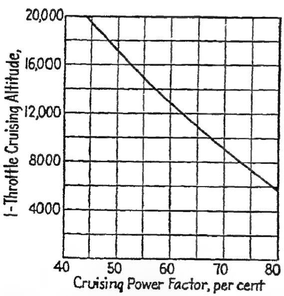


FIG. 391.—Cruising power factor and critical altitude.

[1] "Cruising Control in Transport Operation" (and other articles of kindred title), by Edmund T. Allen and W. B. Oswald, *Aviation*, ten articles printed between May, 1934, and June, 1935.

limitation of the r. p. m. to 87 per cent ($= \sqrt[3]{0.65}$) of the full-power rating of 2,200. Up to the altitude where the engine is unable to deliver the allowable cruising torque at full throttle it makes no difference whether there be supercharging or not, except that there may be some difference in propeller diameter and efficiency in the two cases. In Fig. 391 is a curve (universally applicable, not limited to any particular airplane) of the altitude at and above which full-throttle cruising is possible with an unsupercharged engine. That is the altitude up to which the cruising speed ought to show continuing increase. If a supercharger is introduced into the problem, it is only necessary to add the critical altitude to the altitudes plotted against cruising power factors.

Gliding.—Occasionally it is of interest to compute the angle of descent of an airplane with its engine dead, and in the case of a permanently engineless soaring plane the gliding angle and the rate of gliding descent are the essential measures of efficiency. The calculation is simple, for if T is removed from Fig. 369 and the remaining factors brought into balance

$$\tan \theta = D \tag{124}$$

The drag as plotted in Fig. 373, then, is the index of gliding angle, and at the speed of minimum drag the flattest possible glide will be found. The glide path steepens very rapidly as the speed is diminished to more than about 5 per cent below the optimum, as many a pilot trying to "stretch a glide" by pulling the nose of the plane up, and so increasing the angle of attack, has found to his cost when he settled swiftly into the very obstruction that he was trying to clear.

It need hardly be mentioned that there is no slip-stream effect in a glide. All parts of the airplane are subjected to the same relative wind, but there has to be added the drag of a stopped or windmilling propeller, and that, unless there is provision for feathering the blades,[1] is likely to add more drag than the subtraction of the slip-stream effect takes away. In the present case the minimum drag as plotted was 213 lb., at 108 m. p. h. The disappearance of the slip stream reduces the total by 15 lb.,

[1] P. 512, *supra*.

but the drag of the dead propeller at 108 m. p. h. would be 69 lb.[1] The final outcome is a total drag of 267 lb., a reduction of the best gliding speed to 98 m. p. h., a minimum gliding angle of 1 in 8.65.

As power is to drag, so speed of gliding descent, or sinking speed as it is generally known, is to gliding angle. It is the product of the speed by the sine of the angle of descent, and in the present case it reaches its minimum at sea level (after allowance is made for the drag of the dead propeller) at about 72 m. p. h. The minimum sinking speed is approximately 13 ft. per sec., but a soaring plane, with its induced drag cut down by an aspect ratio of 15 or more and with no engine installation to disturb the smoothness of the fuselage's outline, may show a sinking speed as low as 4 ft. per sec. Where it does, its ability to soar indefinitely without power in any air current having a vertical velocity component of 4 ft. per sec. or more is guaranteed, provided the pilot can find such a current, remain within its boundaries, and keep steadily to the angle of attack that makes the "speed of descent" a minimum.

Diving.—The extension of the gliding approach to a landing is the dive, used to lose altitude in a hurry or as a military maneuver to take an enemy by surprise or to gain speed for subsequent combat. In the normal gliding descent, with an angle to the horizontal that seldom exceeds 10 deg., the difference between the lift of the wings and the weight of the airplane can be disregarded. But in a dive, of which the angle may be anything from 10 deg. up to 90, the decrease of lift becomes a major factor. Equation (124) still applies in determining θ, but in calculating V, which is usually the item of primary interest where steep diving is in prospect, the equilibrium equations

$$L = W \cos \theta$$

and

$$D = W \sin \theta$$

should be combined to give the form

$$W = \sqrt{L^2 + D^2} = \frac{\rho}{2} S V^2 \sqrt{C_L^2 + C_D^2} \qquad (125)$$

$$V = \sqrt{\frac{W}{\frac{\rho}{2} S \sqrt{C_L^2 + C_D^2}}} \qquad (125a)$$

[1] P. 511, *supra*.

C_L and C_D have of course to be taken for the complete airplane not for the wing alone. The angle of attack for which they should be read off is determined from (124).

In Fig. 392 the coefficients of total drag for the illustrative airplane (already given in Fig. 374, but now with the slip-stream effect on drag subtracted out and the drag of the dead propeller added in) have been replotted in a polar diagram against the lift coefficients computed for the same points. In Fig. 393 the diving speed has been plotted directly against inclination of path. It will be noted that it takes a path angle of just under 15 deg. for gravity alone to drive the machine as fast as its propeller will pull it on the level—which is about the average of experience in that respect. Noteworthy, too, is the very small difference in velocity between a truly vertical dive and one as much as 30 deg. off the vertical.

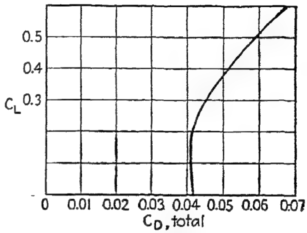


FIG. 392.—Coefficient of total drag for illustrative airplane, with dead engine.

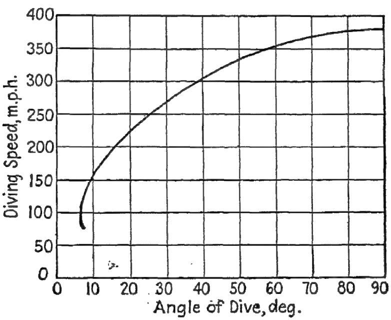


FIG. 393.—Diving speed and path angle.

Though wing flaps and "air brakes" in other forms, such as rotatable struts, can exert a considerable influence on path angle and speed, the great controlling factor is an adjustable propeller. With the propeller blades set down to a 3-deg. angle at 75 per cent of maximum radius and allowed to spin freely, the terminal velocity in a vertical dive would be reduced from 381 m. p. h. to 251 m. p. h. With the blades feathered to about 85 deg., on the

other hand, the velocity would be increased to 470 m. p. h.—faster than most pilots would care to travel without some exceptionally good reason.

Performance with a Dead Engine.—As the multiengined airplane approaches complete dominance in transport service and takes a gradually increasing place in the military establishment, the question of what it will do when one or more of its engines abdicate their function becomes progressively more important. Although there are some cases where their use becomes necessary to provide better vision from the extreme nose of the fuselage (as in bombers) or because no single engine can be made to produce the total power required, the multiplying of engines is in most instances a safety device. If the airplane is unable to continue normal flight with one engine missing, duplication reduces safety instead of promoting it. Whether or not normal flight can in fact continue is purely a performance problem.

The case of the twin-engined airplane with one engine out of action is the most critical, as it involves the largest relative loss of power that any multiengined combination is ever likely to experience. It is of course possible that two out of three engines might fail, but no one expects to be able to continue level flight in that event. As a general rule, whatever the number of engines may be, calculation bears only upon the loss of a single one among them, and the loss of one out of two is more serious than the loss of one out of any larger number. Furthermore, approximately 95 per cent of the multiengined airplanes built since 1932 are of the twin-engined category. For illustrative purposes in the course of the discussion, however, the almost obsolete three-engined arrangement will be used, for it is more general in that the failure may be either in a central or an off-center power plant.

When an engine stops, there are four effects in every case and a possibility of a fifth. The certainties are (1) a loss of $1/n$ of the total power, where n is the number of engines (assuming them all to be of equal size); (2) the introduction of the drag of the stopped propeller; (3) a reduction of the slip-stream effect; (4) a change (generally a drop) of the efficiency of the propellers still operating, due to the reduction in flight speed and the consequent lowering of V/nD. The uncertain factor is an increase of drag due to the necessity of setting the rudder and aileron controls off the neutral

position to compensate for the offsetting of the remaining resultant thrust with respect to the plane of symmetry, and it obviously appears only when the engine that failed was itself off-center.

Figure 394 gives the basic sea-level power-available and power-required curves for a transport plane weighing 16,000 lb. and mounting three 500-hp. engines. When the center engine fails, the power available is immediately reduced by a third. The power required is reduced by the elimination of the slip-stream component on the fuselage and the central portion of the tail

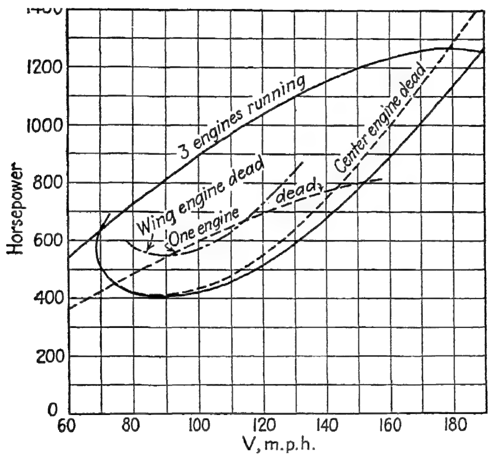


FIG. 394.—Power curves for a three-engined airplane, with various engines stopped.

surfaces, which is likely to amount to about 3 per cent of the total power required at maximum speed and 5 per cent at 60 per cent of maximum; it is increased by the drag of the stopped propeller, which averages about 10 to about 30 per cent of the total drag with a fixed-pitch propeller and 2 per cent with controllable pitch and the blades feathered (the exact percentage of increase itself increasing with increasing speed). The final effect with fixed pitch is as indicated by the dotted line on the power chart. With feathering blades the aggregate power requirement would be diminished below the original.

The power-available curve has only to be scaled down by a uniform fraction at every point. If all the power plants worked

under identical conditions, the fraction would of course be one-third; and it is substantially correct to take it so, as long as the interaction of the propeller and the body is being computed as described in the last chapter. If the short cut of merging slip-stream effect with propeller efficiency into a net propulsive efficiency is to be used, however, the high slip-stream effect on the fuselage, referred to on the previous page, produces a net propulsive efficiency for the central power plant much (sometimes as much as 10 per cent) lower than for the wing engines.

The shape of the power-available curve, plotted against speed, remains as before. Where there is a loss in propeller efficiency under the reduced-power condition it is due only to the lowered speed at which the machine is then flying; for the relation between the propeller and the air flowing through it depends only on the speed of flight. The functioning may be very slightly affected, where it is an off-center engine that fails, by the angle of yaw at which the airplane then flies, but it was demonstrated in an earlier chapter[1] that that factor is almost too small to measure. The power available with one engine dead, whether a center engine or one of those on a wing, is then as shown by the dotted curve in Fig. 394. The speed for best climb is lowered from 120 m. p. h. with all engines running to 105 m. p. h. with the center one out, and the rate of climb from 1,060 ft. per min. to 350. Operation is still perfectly safe, though restricted, so long as the engine failure occurs at a speed of 80 m. p. h. or better. It is far from safe, however, as many operators of airplanes supposed to be able to fly with one engine dead have learned to their sorrow, in the event of failure immediately after take-off at near minimum speed. If the machine is pulled off the ground at 73 m. p. h., which would be perfectly possible, and the center engine then fails, the rate of climb would be −60 ft. per min. It would be necessary to descend as rapidly as that to hold the 73-mile speed, still more rapidly to gain speed to attain the self-sustaining regime which begins at 75 m. p. h. The zone below a speed about 10 per cent above that of minimum power requirement is always a suspect one for multiengined airplanes, to be avoided as far as possible at low altitudes.

Where it is a wing engine that fails, conditions are far more serious. Not only is the residual thrust off center on one side,

[1] P. 531, *supra.*

but the drag of the dead propeller is off center on the other—offering a new argument for feathering blades. Not only do the rudder and ailerons have to be set over, with resultant increase of their own drag, but the whole attitude becomes unsymmetrical. Fuselage, struts, etc., all have to edge through the air at an angle of yaw, and the resistance is increased in each case. There may even be a slight increase of induced drag, due to distortion of the curve of distribution of lift along the span.

The relative increase of drag is least, of course, for an airplane with an efficient control system and a circular fuselage and, most particularly, with a minimum length of exposed strut and streamline wire, as struts and wires are most sensitive to yaw. Douglas has shown[1] that the relative increase of drag under critical conditions, with little or no reserve power in the stopped-engine case, is approximately equal to

$$K\left(\frac{d_T}{h}\right)\left[1 + \frac{1}{2}\left(1 + \frac{d_p}{d_T}\right)f_p\right] \tag{126}$$

where d_T is the distance from the plane of symmetry to the center of action of the remaining thrust, h the distance from the center of gravity of the airplane to the center of pressure of the vertical tail surfaces, d_p the distance from the plane of symmetry to the stopped propeller, f_p the proportion of the total parasite drag (including profile) for which the stopped propeller is responsible, and K a constant ranging from 0.4 for a very clean airplane with a retractable landing gear up to 1.3 for one with a large flat-sided fuselage and much exposed strut. d_p/d_T is 1 for a twin-engined airplane, 2 for a three-engined airplane, and the final indication of the formula is that the increase of airplane drag when a wing engine fails (not including the drag directly due to the stopped propeller) ranges from about 15 to 45 per cent for a twin-engined airplane and 10 to 35 per cent for a three-engined machine, and that the proportion of increase can be reduced by about a tenth by turning the blades of the stopped propeller up

[1] "The Developments and Reliability of the Modern Multi-engine Air Liner, with Special Reference to Multi-engine Airplanes after Engine Failure," by Donald W. Douglas, *Jour.* Roy. Aero. Soc., November, 1935; a tremendously exhaustive and careful examination into every detail of the problem, including many not touched here.

to just under 90 deg. The direct drag of the stopped propeller adds another 6 to 15 per cent to the total drag if the blades are not feathered.

Assuming a total increase of 25 per cent in this case, the power-required curve with all adjustments made appears as the dot-and-dash line in Fig. 394. The maximum rate of climb is now reduced to 50 ft. per min., at 100 m. p. h.; the speed range to from 92 to 114 m. p. h. The ceiling would be about 700 ft., and practically speaking the machine would be unflyable. The use of a controllable-pitch propeller with the control extending up to a 90-deg. blade angle would so increase the propeller efficiency, on the one hand, and reduce the propeller drag, on the other, as to increase the rate of climb from 50 ft. per min. to 320, making an airplane positively dangerous after engine failure into one that might be passably safe at low altitudes.

Range and Endurance.—The problems of how long and how far an airplane can fly without pause for refueling may be attacked by three methods of successively greater accuracy: on the assumption of a constant amount of available power; with the variation of propeller efficiency taken into account but assuming a constant brake thermal efficiency, or constant fuel consumption per unit of useful work for the power plant; or with full allowance for all of the factors involved. On the first basis it is quite obvious that the maximum duration would be secured by flying at the speed of minimum power, as the fuel consumption per hour would then be a minimum. It is equally true, although a little less evident, that the greatest radius of action would correspond to flight at the speed of minimum total drag. Assuming a constant ratio between units of energy supplied in the fuel and units of useful work done by the propeller, which was the condition of the first approximation, the radius of action will be greatest when the amount of work done per mile flown is least. Work being the product of force and distance, it has its minimum value for a given distance covered when the force that must be exerted is a minimum.

In practice it proves advisable to fly at speeds a little higher than those in order to take advantage of the improvement in propeller efficiency. Coming then to the second approximation, it is evident that the real condition of maximum duration is not that of minimum power output at the propeller but of minimum power input at the crankshaft. A curve of power input can

be constructed by dividing the power required at each speed by the corresponding propeller efficiency. The process of determination of that efficiency requires first a calculation from the curve of C_T' for the propeller[1] of the V/nD at which the propeller will have to operate to develop the necessary thrust for level flight at each speed. Knowing V/nD, the r. p. m. can be calculated, and the efficiency can be read off from the propeller curve. The power required for flight is then divided by the efficiency so obtained, to give the power input needed at each

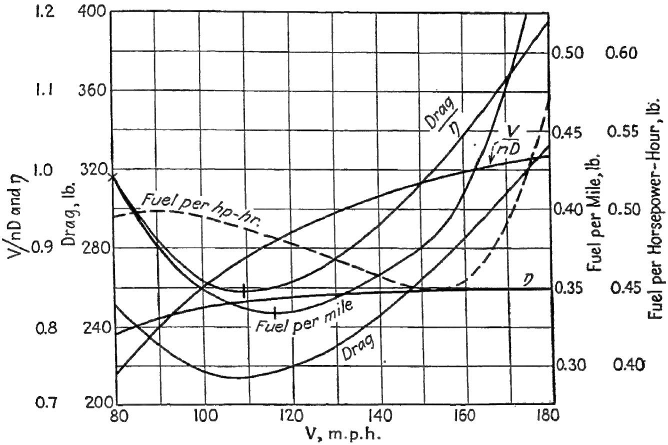


FIG. 395.—Curves for calculation of extreme range.

speed. The speed for maximum duration is, of course, that of minimum ordinate of this power-input curve.

Maximum duration is generally unimportant. No longer is it even granted official recognition as the subject of a "world's record." Maximum range, on the other hand, is of immense practical significance in very nearly every employment into which an aircraft can be put. The technique of computing range is closely analogous to that of computing duration, but with the total-drag curve instead of that of power required now serving as a basis and divided by the efficiency of the propeller at each point to build up a new curve. An illustrative example appears in Fig. 395, but its examination may best be postponed until after reading the next two paragraphs.

[1] See p. 469, *supra*.

Turning to the third approximation, and this in fact need hardly be referred to as an approximation, the fuel consumption is taken into account in the same way. In addition to dividing the power required by the propeller efficiency, it is multiplied by the specific fuel consumption in pounds per horsepower-hour, the ordinate of the resulting curve being fuel consumption in pounds per hour. The variation of consumption with r. p. m. and throttle setting of the engine does not in general have appreciable effect on the best speed for maximum duration. Its exact effect on the optimum speed for radius depends on the characteristics of the engine's induction system and upon the carburetor adjustment. In general, the specific consumption is a minimum at somewhere between 40 and 65 per cent of full power.[1] In general, except for a machine heavily overloaded at the start of a long flight, the power factor corresponding to minimum power input in level flight is 40 per cent or less. With rare exceptions, then, long-range flight is accomplished with the engine operating well below the minimum-consumption power. Following the usual rule of compromise where a minimum of the product of two oppositely varying quantities is required, it proves advantageous to increase the speed of flight somewhat and improve the fuel economy per unit of power, even at the expense of having to increase the propeller thrust slightly.

The actual process of calculation for maximum radius again parallels that for maximum duration, but with thrust substituted for power. If the specific fuel consumption is divided by 375 and then multiplied by the ratio of propeller thrust required to propeller efficiency, the result comes out in pounds of fuel per mile. The curves for the second and third approximations for maximum radius are drawn in Fig. 395 for the case of the fixed-pitch propeller mounted on the unsupercharged engine and operating at sea level.

The speed for maximum range of flight calculated on the second approximation would be 109 m. p. h.; on the third and more accurate method 116 m. p. h., that speed giving the minimum fuel consumption per mile.

It goes without saying that whether the object is maximum radius or maximum duration the conditions of flight will change

[1] For full discussion on this point, including the effect of varying the relationship between manifold pressure and r. p. m., see p. 465, *supra*.

with the passage of time as fuel is exhausted and the total weight decreases. It is therefore necessary for a rigorously correct solution to work out the rate of fuel consumption for several different gross weights, corresponding to different periods of the flight, and to plot the reciprocal (hours or miles of flight per pound of fuel) against weight. The total duration or distance is then the area under the curve between the abscissas corresponding to the initial gross weight and to the weight after the fuel has been completely exhausted. The form of the curve and the area measured are illustrated in Fig. 396. It will be observed

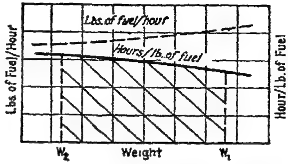


FIG. 396.—Graphical integration for determination of total endurance.

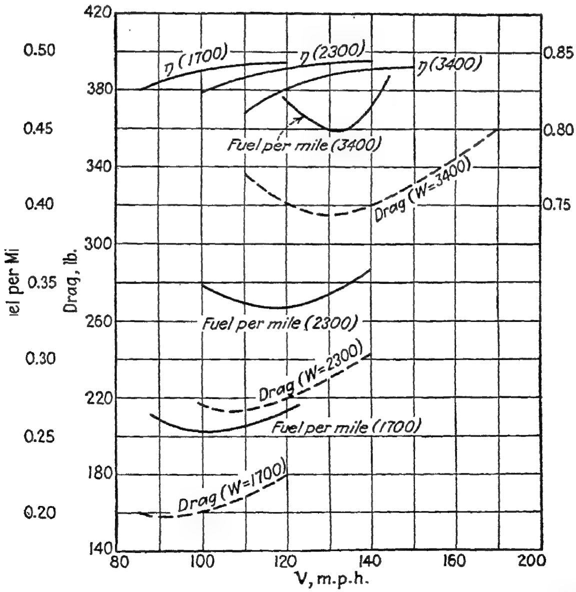


FIG. 397.—Variation of minimum fuel consumption with change of gross weight.

that the process is analogous to that used in the accurate solution for time of climb to any altitude.

The computation of new power-required curves for successively diminished gross weights is somewhat analogous to the computation for a new altitude. Since D/L at a given angle of attack remains unchanged, a function of the external form of the airplane alone, the total drag at a given attitude varies directly as the weight. The speed is of course proportional to the square root of the weight, which makes the power required proportional to $W^{3/2}$. The total effect of a weight increase, then, is to slide the curve of power required a certain proportion to the right, and upward (roughly speaking, and as long as the percentage of weight change is moderate) three times as much.

In Fig. 397 the curves have been drawn for the now familiar experimental airplane, with controllable-pitch propeller and on the assumption that the machine can start a long-distance flight with a 50 per cent overload, bringing the gross weight up to 3,400 lb., and that the weight empty of fuel is 1,700 lb. At the beginning of the flight, then, the fuel weight makes up just 50 per cent of the total. In Fig. 398 the curve for integrating for range appears and shows a range of 4,860 miles.

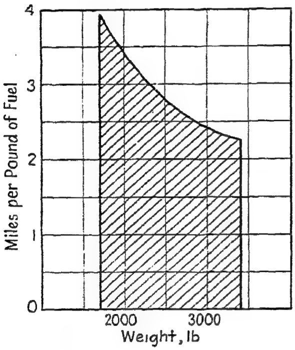


FIG. 398.—Integration for extreme range for illustrative airplane, with 50 per cent of total weight in fuel and oil.

The propeller-efficiency curve and the drag curve are in the same relation to each other throughout the flight. With the drag increasing and the thrust therefore having to increase in direct proportion to the weight and so to the square of the speed, the law of variation of thrust with r. p. m. indicates a constant V/nD at any given angle of attack, however the weight may change. There is, therefore, little to be gained from the use of a controllable-pitch propeller, where attainment of absolute maximum range is the primary factor governing the initial choice of a propeller for the airplane, except that the specific consumption may be reduced, especially under lightly loaded conditions near the end of the flight, by increasing the pitch to hold the r. p. m. down. The use of controllable pitch gives a pronounced

advantage, too, where the initial take-off run is in any way limited, in that it accelerates the take-off and makes it possible to lift a larger load safely into the air.

All these calculations have been for sea-level flight. They could be repeated for any other altitude, but in general the final result would be very much the same. The minimum drag is independent of altitude, but it appears at a progressively higher speed as the altitude is increased. V/nD is unchanged, as n like V must vary inversely as the square root of density to keep the thrust constant. The only variable factor is consumption, and that, as pointed out in Chap. XIII,[1] is likely to improve slightly with altitude. In a particular series of studies through actual flight tests[2] the range in still air was found to be a maximum as a rule at from 5,000 to 16,000 ft. and to exceed the sea-level range by from 2 to 5 per cent. The air at high altitudes is never still, however, and in practice the controlling factor in choosing an operating altitude for a long flight is never the still-air range, but rather the prevailing wind condition.

The best speed of flight for maximum radius when there is a favoring or opposing wind is different from that in a calm. Manifestly if there is a strong head wind it is advantageous to fly faster and so suffer from the wind during a shorter time, while a favoring wind makes it desirable that the speed be reduced nearer to that of minimum power and that the machine be allowed to float along with the wind for a longer time. The best speed for any condition can be found by drawing a tangent to the curve of power required, suitably corrected for variations of propeller efficiency and fuel consumption as just explained, from the point on the axis of abscissas corresponding to the wind velocity. The velocity is taken as positive if it is a head wind, and negative if it favors the flight. In a calm, then, the tangent would be drawn through the origin, and it is manifest that the slope of a line through the origin to any point in the power curve would represent the thrust required at that point, since the ordinate of a power curve is the product of speed and thrust requirement, while the abscissa is speed. It follows that a line from the origin would become tangent to the power curve at the point of mini-

[1] P. 466, *supra*.

[2] "Range of Aircraft with Air-cooled Radial Engine, Using Altitude Control," by A. E. Woodward Nutt, A. F. Scroggs, and E. Finn, *R. and M.* 1399.

mum thrust requirement, or of minimum total drag if the simple power-required curve were used without corrections. In Fig. 399 the curve of power with correction for fuel consumption is presented for the illustrative case, and tangents are drawn showing the best sea-level speeds with a head wind of 30 m. p. h., a calm, and a tail wind of 30 m. p. h. to be 123, 113, and 110 m. p. h., respectively.

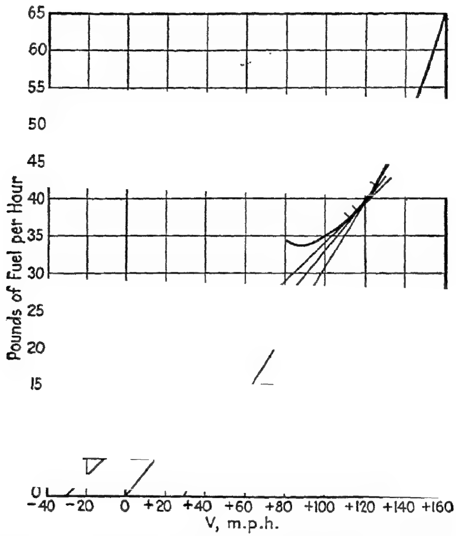


FIG. 399.—Correction of optimum speed for extreme range for the effect of wind.

Take-off Distance.—The analysis of landing and take-off is sometimes included as an element of performance calculation. The former, being independent of engine power and governed very largely by the design of the landing gear, belongs rather with a treatment of that part of the structure, but the behavior of the airplane in taking off may properly be analyzed along with its speed and climb subsequent to entry into full flight.

The problem of take-off of a landplane is obviously one of simple mechanics, of acceleration from a condition of rest up to a known velocity. The acceleration depends on the mass to be moved and on the force available for the purpose, and the force is

in turn the difference between the propeller thrust and the total resistance to forward motion, which is made up of two parts, the air resistance, or drag, and the frictional resistance to rolling along the ground.

The variation of propeller thrust with speed of flight has already been explained.[1] The Hartman method of presentation of propeller data[2] is particularly valuable in analyzing it, and it was used in developing Fig. 400, the variation of full-throttle thrust with speed along the ground for two fixed-pitch propellers

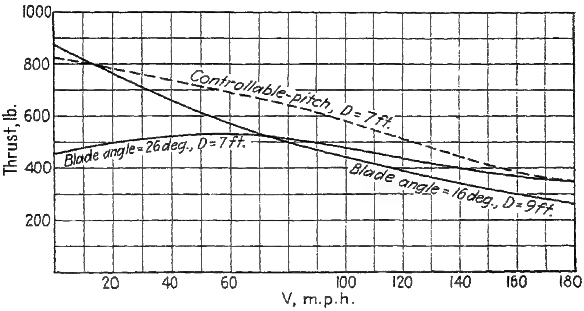


FIG. 400.—Typical thrust curves, over the take-off region, for three propellers.

and for a controllable-pitch design of identical blade form and of diameter equal to the smaller of the two.

Air Resistance in Taking Off.—The determination of air resistance must always be somewhat indefinite, as the attitude assumed by the airplane in taking off is dependent on the pilot's use of the controls. Where the quickest possible take-off is being made, however, there will be little error in assuming that the thrust line is horizontal during the run along the ground until the minimum possible speed of flight is very closely approached. The air resistance is of course least for a small angle of attack; the pilot therefore endeavors to lift the tail off the ground as quickly as possible; the combined influence of the diving moment due to the resistance of the wheels acting far below the center of gravity and of the slip stream on the controls makes it possible

[1] Chap. XIV, *supra*.

[2] "Working Charts for the Determination of Propeller Thrust at Various Air Speeds," by Edwin P. Hartman, *Rept.* 481. For a brief explanation of the method, see p. 501, *supra*, and Fig. 349.

to get into the desired attitude almost instantly when the throttle is opened. There would be little error, then, in assuming a constant attitude with the thrust line at zero angle of attack through the major part of the run and starting from its beginning. Thereafter, toward the end, the plotting of resistance must be largely a matter of guesswork, depending on the methods of the individual pilot. Sometimes, indeed, the tail is forced down to get the machine into flying attitude comparatively early in the run, a tail-low take-off being made, but except that that method insures against holding the machine on the ground too long and taking off at an unnecessarily high speed, fundamental theory and direct experiment[1] are agreed in showing that the best results are obtained by keeping to the angle of minimum resistance until the last instant before take-off speed is obtained and then pulling the elevator up to depress the tail and increase the angle of attack and so the lift coefficient. It is of course true, however, that there are other considerations than that of minimum run to get off the ground, and it may be advisable to hold the tail up and let the machine acquire a higher speed before pulling it off the ground in order that the control may be better and the danger of stalling when starting to climb less. The more quickly a speed at least 25 per cent above the minimum can be attained, too, the smaller are the chances of disaster in case of an engine failure on a multi-engined machine. In the special case of a soft field and a very large coefficient of rolling friction, too, it proves advantageous to hold the tail fairly low throughout the run in order that the wings may gain lift more quickly and decrease the pressure of the wheels on the ground, and therefore the total rolling friction.

Having decided on the attitude which is to be maintained through the major part of the take-off, the total drag coefficient

[1] "A Study of Taking-off and Landing an Airplane," by T. Carroll, *Rept.* 154; "Take-off Characteristics of the DH-4," *Information Circ.* 480, Airplane Eng. Div., U. S. Air Service (a very detailed treatment of the mechanics of taking off); "Landing Speed and Run and Take-off of a Training Airplane," by Harold R. Wells and Paul M. Boyd (an unpublished thesis in aeronautical engineering at the M. I. T.), 1926; "The Distance Required to Take Off an Airplane," by R. McK. Wood and F. B. Bradfield, *R. and M.* 680. "Considerations on the Take-Off Problem," by Edwin P. Hartman, *Tech. Note* 557; "Take-Off and Landing of Aircraft," by D. Rolinson, *R. and M.* 1406; "The Effect of Wind, Weight, and Atmospheric Conditions on the Distance to Take-Off and Land an Aircraft," by B. H. Rolles and H. L. Stevens, *R. and M.* 1172.

for the airplane can be read off from Fig. 374. Of course there is a large variation in the slip-stream effect during the run, as the speed of the airplane along the ground changes much more rapidly than the slip-stream velocity over the control surfaces and fuselage. It can be roughly allowed for by breaking the coefficient of total drag into two parts, the parasite drag in the slip stream and the remainder, and assuming the slip-stream portion subjected to an air stream having a velocity equal to half the airplane's maximum speed of flight at sea level. For very fast airplanes, that makes the total drag at low rolling speeds somewhat higher than it should be; for slow machines with propellers turning at high r. p. m. it comes out somewhat below the truth—but it is near enough to the mark to serve in lieu of a complete recalculation of the slip-stream effect in most practical cases. Fig. 401 has been plotted for the standard illustrative example on the assumption that the angle of attack of the wings is 0 deg. until the speed of 60 m. p. h. is reached and that it gradually increases thereafter to 9 deg. at the instant of take-off. That is a compromise figure (in good accord with usual good piloting practice), for it would be necessary to increase the angle to 15 deg. for take-off at the absolute minimum speed, and so for the absolute minimum distance of run.

There is one additional factor to be taken into account—ground effect. Its nature and the manner of its variation with distance from the ground have already been treated.[1] Substantially without influence in the early part of the run, where the angle of attack and the induced drag are small, it becomes important during the final period in which the angle is being increased. It reduces the induced drag, often by as much as 40 per cent, and at the same time increases the lift at a given angle of attack. In the present case ground effect is of more than average significance, for the low-wing arrangement puts the wing in the closest proximity to the ground during take-off and the unusually low aspect ratio increases the relative importance of changes in the induced component of the drag. The wing lies only 16 per cent of its own span above the ground (in a low-wing transport or bomber that ratio may fall as low as 9 per cent), and the induced drag at the take-off attitude would therefore be reduced by 35 per cent. Induced drag being 63 per cent of the

[1] See pp. 101, 335, *supra*.

total drag at 9 deg., the reduction of total drag would measure 22 per cent. Figure 401 has been adjusted accordingly.

It has been adjusted, too, for the effect of the ground upon lift. Though the effect on maximum lift may be small, the increase of lift at intermediate angles of attack is very definite.[1] At an angle of 9 deg. it is likely to be about 7 per cent, corresponding to a reduction of take-off speed by about 3½ per cent. That in turn would increase the induced drag by 7 per cent and reduce the parasite and profile components (which are directly proportional to V^2) by a like amount. With all these effects added together, the final result is to change the equilibrium speed at 9 deg. from 78 m. p. h. to 73, and the drag at that angle from 262 lb. to 204. In this calculation the airplane leaves the ground at a speed about 9 m. p. h. higher than the minimum for steady flight. The speed of take-off for any particular angle of attack, and in particular the minimum possible speed, are substantially affected by the slip-stream effect on the wings.[2] Take-off may actually be accomplished at speeds as much as 10 per cent below the calculated minimum flight speed.[3]

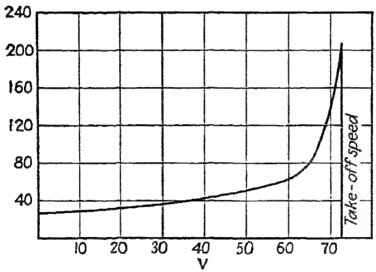


FIG. 401.—Air drag during take-off.

Rolling Resistance.—The evaluation of ground resistance is equally important, and that quantity has been discussed by a number of writers.[4] The coefficient of friction of the wheels appears in the light of their experiments to vary rather widely, ranging from a minimum of about 0.015 on a smooth concrete runway up to a maximum of about 0.2 on the worst surface from

[1] "Effect of the Ground on an Airplane Flying Close to It," by E. Tönnies, *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Mar. 29, 1932, translated as *Tech. Memo.* 674.

[2] See p. 585, *infra*.

[3] *R.* and *M.* 1406, *cit. supra.*

[4] "Landing Speed and Run and Take-off of a Training Airplane," *loc. cit.;* "Landing Run and Get-away for Standard Airplanes," by Alexander Klemin, *Jour.*, Soc. Automotive Eng., March, 1920; "Take-off Characteristics of the DH-4," *loc. cit.; R. and M.* 1406, *cit. supra.* More modern data, now in process of accumulation, are not yet published.

which a take-off would be likely to be attempted; 0.03 is probably a good average value for a good dry grass-grown field, rising to around 0.06 on fields which, although less firm in surface, would still be considered quite satisfactory; 0.03 is used in the illustrative examples given here. Whatever the coefficient of friction may be, the total frictional resistance is of course equal to that coefficient multiplied by the weight borne by the wheels, and that in turn has to be determined by subtracting from the total weight the part carried by the air at any instant. The proportion of the total weight carried by the wings is calculable from a knowledge, obtained in working up the general performance curves, of the speed at which the airplane would have to travel in order that its whole weight might be supported by the wings at the given attitude. If slip-stream effect were neglected the total lift would be simply proportional to the square of the speed, but the angle of attack is usually above that of zero lift, and the slip stream in flowing across the wings is turned downward through a small angle and so provides a lift reaction. In the present case the slip stream affects about 15 per cent of the wing area, and at the assumed constant slip-stream speed of 90 m. p. h. it gives a lift of 5 per cent of the weight of the airplane.

There is, too, a correction to be made for the lift of the tail. It does not vary with speed in the same manner when on the ground as when in the air, since (on a machine with the conventional type of landing gear with main wheels forward of the center of gravity and a tail wheel aft) the stick must be thrust forward during the first part of the take-off to lift the tail, and the elevator derives a considerable lift from the slip-stream action. It is advisable to subtract from the load on the wheels a correction for tail lift varying approximately lineally with speed from $0.08W$ at the beginning of the take-off to 0 when flying speed is reached. Since the machine must be in equilibrium during its run, the exact load on the tail can be determined by taking the moments of all forces (including the airplane's weight) around the axle of the wheels as an axis. The portions of the total weight borne by the wings, tail, and wheels, respectively, are plotted against speed for the illustrative example in Fig. 402.

It is now possible to solve for the length of run, and in Fig. 403 the propeller thrust, the total resisting force, and the difference between them have been plotted. The unbalanced force of

course represents acceleration, after dividing the scale of ordinates by W/g.

It has been pointed out by Diehl[1] that a single step can be taken from the acceleration curve to the graphical evaluation of take-off distance if the reciprocal of acceleration is multiplied by

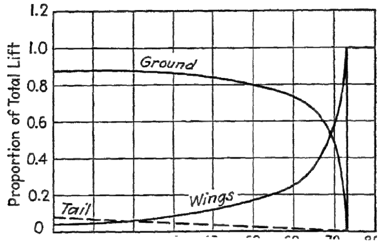


FIG. 402.—Distribution of weight between wings, tail, and wheels during take-off.

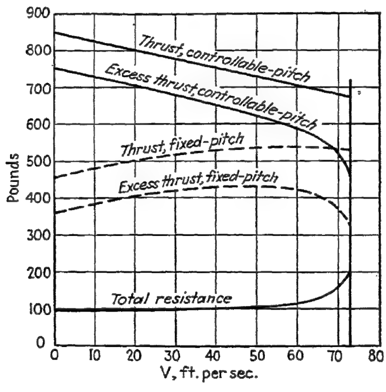


FIG. 403.—Thrust and total drag during take-off.

speed and plotted against V. V for this purpose must of course be in ft. per sec. Since

$$v = \frac{dx}{dt}$$

$$v\frac{dt}{dv} = \frac{dx}{dv} \tag{127}$$

[1] "The Calculation of Take-off Run," by Walter S. Diehl, *Rept.* 450.

and

$$\int_0^{V_0} v\frac{dt}{dv}dv = X_0$$

The method is illustrated in Fig. 404, where the take-off distance proves to be 1,021 ft. with a fixed-pitch and 686 ft. with a con-

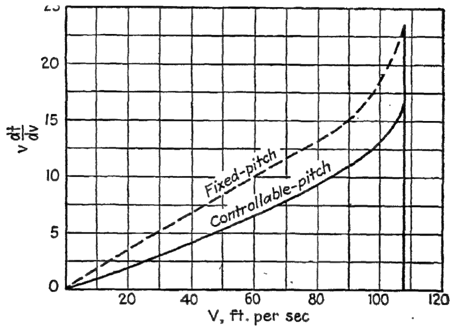


FIG. 404.—Graphical construction for computing take-off.

trollable-pitch propeller. The notable difference in favor of controllable pitch is typical, especially of airplanes of high maximum speed and those with propellers limited to small diameter.

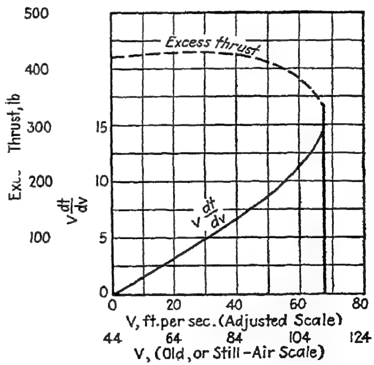


FIG. 405.—Effect of wind on take-off distance.

All of the work so far has been predicated on the assumption of calm air, a condition happily seldom realized, as it is the most unfavorable one possible. The modification of the work to compute the take-off distance against a wind is simple and straightforward. The V which serves as a base for plotting the curves and enters into the integration is ground speed. It is then only necessary to slide the scale of abscissas to the right by the amount of the wind velocity, to throw away the portion of the acceleration curve that lies to the left of the new zero and still

carry the integration up to the original point on the curve, the abscissa of which is now equal to the take-off speed minus the wind speed. In Fig. 405 the work is done for the illustrative airplane with the fixed-pitch propeller and a 30-mile wind. The take-off is reduced to 401 ft., 61 per cent below the calm-air figure.

All of the illustrative calculations have been made for sea level and for an unsupercharged engine. The same process can, of course, be applied for any other altitude. It can be applied with equal facility for the supercharged case, where the manifold pressure is commonly allowed to rise for purposes of take-off to 10 per cent above its maximum-continuous-operation value.

Take-off of Seaplanes.—Exactly the same sort of work can be done for a seaplane, except that the method becomes less general, there being nothing like a constant coefficient of frictional resistance for floats and hulls. The water, like the air, is a fluid medium, and the resistance to motion in it depends on speed.

Not, however, by any simple V^2 law. The drag coefficient depends on the depth of submergence of the hull and on the wave formation around it, and changes abruptly with changes of speed. The characteristic form of the curve of hydrodynamic resistance includes a rapid rise, approximately in proportion to V^2, while the seaplane is plowing through the water as a displacement boat; passage through a peak, commonly known as the hump; and a gradual and somewhat unsteady decline, often interspersed with brief periods in which the resistance resumes its rise. During this final phase, the seaplane is acting as a hydroplane and running over the surface of the water rather than through it. Most of the support is derived from the dynamic reactions of the air on the wings and of the water on the hull. Buoyancy has become almost negligible.

The angle of attack is subject to no such arbitrary adjustment as in the case of the landplane. Hydrodynamic reaction first becomes apparent well forward on the hull, forward of the step, and imposes a stalling moment too large to be readily overcome by the controls. To hold the angle of attack down to 6 deg. may require a force of as much as 8 lb. per sq. ft. on the horizontal tail surfaces when running through the water at 30 m. p. h., and even with full allowance for slip-stream effect that is likely to demand that the elevators be pulled down to the full limit of their travel. To hold an angle of 3 deg. would call for

about twice as much force, and that certainly would be beyond the capacity of the controls. Furthermore, the resistance of a hull or float at intermediate speeds in the neighborhood of the hump is much increased by holding the nose down—much more than the air resistance can be reduced by keeping the wings to a small angle of attack during the early stages of the take-off run.[1]

The best general rule, then, is that the angle should be allowed to increase steadily with speed, with only a comparatively gentle check applied by the elevator control on the natural free trimming angle that the hydrodynamic moments alone would produce, up to about 7 or 8 deg. at the hump speed; and that it should thereafter decrease to something between 2 and 5 deg. at just below the take-off speed. From there the machine can be pulled off the water by pulling the tail down and increasing the lift coefficient of the wings, precisely as in the last stages of the take-off of a landplane.

Assuming that the angle of trim is varied in accordance with the ideal curve, good hulls prove to have a hump resistance of about one-fifth of the total weight that they are supporting. The ratio of resistance to weight is generally lowest on lightly loaded, shallow-draft hulls. The intensity of loading on the hull is best expressed in terms of beam at the step rather than of total area of the projection of the submerged section on a horizontal plane, as most of the weight is carried dynamically on a narrow band of area running across the hull just forward of the step.[2] The com-

[1] The effect of trim angle on resistance and on general take-off behavior is studied in detail in "The Effect of Trim Angle on the Take-off Performance of a Flying Boat," by James M. Shoemaker and John R. Dawson, *Tech. Note* 486. The same report analyzes the effect of changing the angle at which the wings are set upon the hull. The "angle of attack" to which hull performance directly responds is, of course, the angle of the keel to the water, but the distribution of load between the water and the air load on the wings is fixed by the angle of the wings to the relative wind. The relation between the two angles can be varied over a wide range at the will of the designer, but only at the cost of diminished air performance if the angle of the hull to the wings is made such as to increase its air-drag coefficient at the attitude of maximum speed to substantially above the minimum possible at the optimum angle.

[2] The derivation of hull performance and loading coefficients in a non-dimensional form and some illustrations of their application will be found in "Towing Tests of Models as an Aid in the Design of Seaplanes," by P. Schröder, translated as *Tech. Memo.* 676; and in numerous reports from the

mon expression is $W = k_b b^3$, where b is the beam of the hull (or total beam of the floats in a twin-float machine) in feet, W the gross weight in pounds, and k_b a design coefficient which generally runs between 20 and 35 for modern flying boats, between 30 and 60 in float seaplanes. The ratio of water resistance to displacement is plotted against k_b, for several typical hulls of good design, in Fig. 406. The decrease of relative resistance with diminishing bottom loading is steady and apparently without limit. What actually happens is that the hump, as a distinct maximum, disappears below values of k_b of 10 or 15. Below that point it

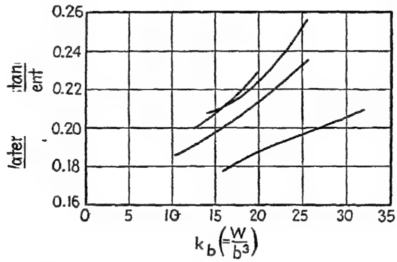


FIG. 406.—Variation of water resistance with depth of submergence for typical hulls.

survives only as a small irregularity in the resistance curve, and the resistance of the hull increases steadily from the at-rest condition up to the take-off speed. If the beam were made large enough to give the resistance curve that form (it would have to be about 25 per cent more than is now customary), the total resistance at low speeds would be decreased, but it would have to be paid for with a much increased resistance just below the take-off speed. Hence the rule that with a heavily loaded hull ($W > 30b^3$ or thereabouts) it is the hump resistance that provides the critical factor in take-off, while with a light loading the critical point comes immediately prior to take-off. The propeller pitch is also a factor, as the critical point is of course the point at which the thrust shows the smallest margin over total resistance. Other things being equal, therefore, the hump condition

N. A. C. A. towing basin, particularly "The N. A. C. A. Tank—A High-speed Towing Basin for Testing Models of Seaplane Floats," by Starr Truscott, *Rept.* 470; and "Tank Tests of a Family of Flying-boat Hulls," by James M. Shoemaker and John B. Parkinson, *Tech. Note* 491.

is most likely to be critical on a machine with a high speed range, a small propeller diameter, or a geared engine—conditions favoring a large propeller blade angle and a falling off of propeller thrust at very low speeds.[1] With a controllable-pitch propeller, showing its most pronounced effect on thrust at speeds well below that of take-off, the hump loses most of its terrors.

The location of the hump is only a little less important than its height. Remembering that it is the point at which the major support of the hull becomes dynamic, the analogy with the minimum speed of flight of an airplane is obvious. The minimum speed is that at which the dynamic reaction on the wings is able to support the machine's weight; the hump speed is substantially, or just below, that at which the dynamic reaction on the bottom

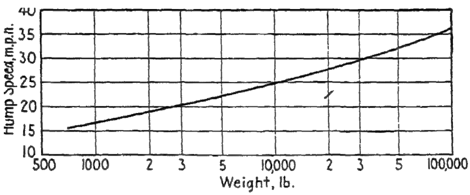


FIG. 407.—Characteristic variation of hump resistance speed, for flying-boat hulls, with gross weight.

of the hull is able to do the same. The minimum speed is proportional to the square root of the wing loading; the hump speed should be proportional to the square root of the loading on the hull bottom or, since the pressure on each element of the bottom when it is being statically supported by buoyancy is in direct proportion to the depth of its submergence, to the square root of the mean draft when at rest. For a series of geometrically similar hulls, the draft would vary as $\sqrt[3]{W}$ and the hump speed ought to vary as $\sqrt[6]{W}$.

Experiment shows that it actually does so, and that even when there is no approach to geometrical similarity the coefficients still run remarkably uniform. With an error never likely to exceed 10 per cent for any hull and less than 5 per cent in most cases, the hump speed in m. p. h. equals $5.3\sqrt[6]{W}$. A curve is drawn against weight in Fig. 407. Although the exact ratio depends of course upon the wing loading and the airfoil section

[1] See p. 494, *supra*.

used, it can be set down as a general rule that the hump speed is from 35 to 45 per cent of the minimum flight speed, or from 30 to 40 per cent of the minimum practical take-off speed.

In Fig. 408 is reproduced a sheaf of resistance curves for varying displacements for a particular hull, that of the Sikorsky Clipper, a 34,000-lb. transport with 2,400 hp.[1] The gradual disappearance of the hump as the weight is reduced will be noted.

By computing the lift of the wings at each speed and subtracting it from the weight of the airplane, the weight remaining to be

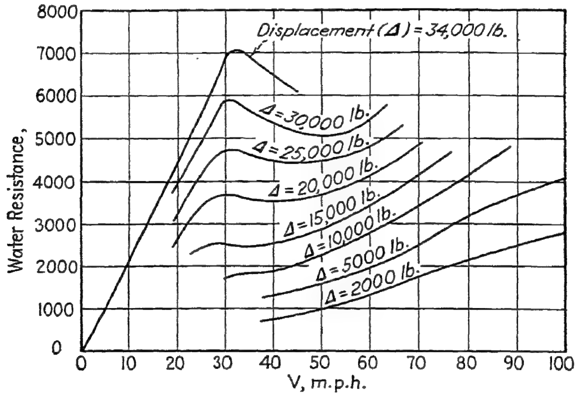


FIG. 408.—Variation of form of resistance curve with displacement, for a typical hull.

carried by the hull can be plotted against speed. It has been done in Fig. 409 both for calm air and for a 30-mile wind—the method of allowing for the effect of wind on the take-off of a seaplane not being quite so simple as for a landplane—and for two different wing loadings. Slip-stream effect on lift is allowed for precisely as in the landplane. By interpolating between the curves of Fig. 408 to get the appropriate resistance for each speed, the curves of total hydrodynamic resistance in Fig. 410 have been drawn. It remains only to draw a total resistance curve and to superpose upon it the curve of thrust. For the purposes of this demonstration, the power has been arbitrarily reduced to 1,700 hp. to emphasize the critical conditions for an under-

[1] Data taken from "A Complete Tank Test of the Hull of the Sikorsky S-40 Flying Boat," by John R. Dawson, *Tech. Note* 512.

powered seaplane. The actual Clipper has of course an ample reserve of power for take-off under all conditions.

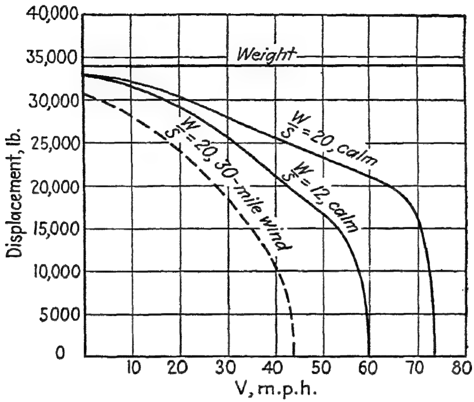


FIG. 409.—Effect of wind on distribution of weight between air and water during take-off of a flying boat.

The thrust curves are superposed on those of total resistance in Fig. 411, with the power arbitrarily reduced as noted and on the assumption of a propeller of 9.2 ft. diameter, absorbing 425 hp. at 150 m. p. h. and 2,000 r. p. m. and giving its maximum

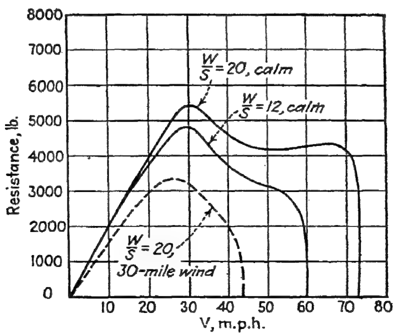


FIG. 410.—Variation of total water resistance with speed.

efficiency under those conditions, for each of the four engines. Take-off, it will be observed, would just fail of being possible in

calm air but would be easy with a 30-mile head wind[1] or with the wing area increased to reduce the take-off speed. These calculations are, of course, for smooth water. The take-off is materially influenced by surface conditions—favorably by a

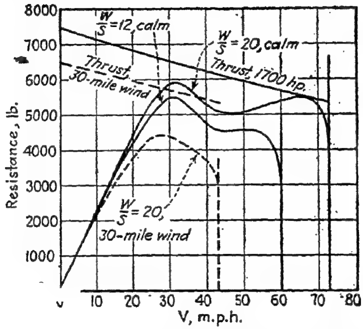


FIG. 411.—Variation of excess thrust of a flying boat with speed, for several illustrative cases (with engine power arbitrarily reduced to emphasize points where take-off difficulty would first appear).

small ripple, unfavorably by a heavy sea—but over the range likely to be encountered in sheltered waters the smooth-surface computations are found to apply with very satisfactory accuracy.

[1] The difference between the no-wind and the 30-mile-wind cases, though it is in the customary sense, is of more than customary magnitude. E. T. Jones ("Effect of Wind on the Take-off of Seaplanes," *R. and M.* 1593) goes so far as to deny that wind has any influence on the maximum gross weight that a seaplane can lift from the water. That may be a frequent rule, especially with hulls of the forms commonly used in Great Britain (to which Jones's numerous illustrative calculations and tests relate), but that it is not by any means a universal one the present illustrative example proves.

CHAPTER XVII

PERFORMANCE BY FORMULA AND CHART

The procedures outlined in the preceding chapters are somewhat elaborate and time consuming. It is useful to devise something simpler, giving approximations to the major elements of performance in terms of a few of the general dimensions and characteristics of the machine. Carefully constructed, and taking a sufficient number of independent variables into account, the performance formula may transcend the bounds of approximation and become an alternative method of final calculation, scarcely less accurate than the step-by-step construction of power curves.

Minimum Speed.—The minimum speed of flight is of course the simplest element to handle, so simple indeed that the equation by which it is predicted is hardly worthy of the name of performance formula. Since $W = C_{L_{\max}}\frac{\rho}{2}SV^2$

$$V_{\min} = \sqrt{\frac{W}{C_{L_{\max}}\frac{\rho}{2}S}} \tag{128}$$

The equation may be left in that form, or still further simplified by the assumption of an average value for $C_{L_{\max}}$. In any case it may be written

$$V_{\min} = K_S\sqrt{\frac{W}{S}} \tag{129}$$

If $V_{\min}$ is to be given in m. p. h.,[1]

[1] Throughout the present chapter, speeds will be expressed in m. p. h. wherever practicable, except as specifically indicated otherwise. Though the departure from dimensional homogeneity complicates the derivation of many of the expressions, the desirability of employing familiar engineering units justifies the complication. Hence the frequent appearance, through the next 40 pages, of the factors 2.15, 0.00255, and $^{60}/_{88}$ or 0.683.

$$K_S = \frac{60}{88}\sqrt{\frac{1}{C_{L_{max}}\frac{\rho}{2}}} \tag{130a}$$

or, at sea level,

$$K_S = \quad 19.8 \tag{130b}$$

the relationship between K and $C_{L_{max}}$ being plotted in Fig. 412. K varies from approximately 15 for a very high-lift section to about 19 for a very thin or almost symmetrical one. Since the

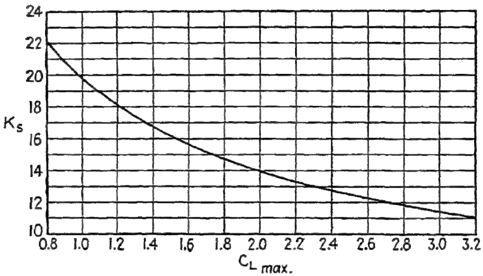


FIG. 412.—Variation of minimum-speed coefficient with maximum-lift coefficient.

airfoil sections which are most popular, at least in the United States, lie well away from either end of this range, the arithmetical mean can be taken and the formula written:

$$V_{min} = 17\sqrt{\frac{W}{S}} \tag{131}$$

but with a mental reservation that the speed will be somewhat underestimated for a thin section of low camber or for a symmetrical one of any thickness, and rather overestimated, the formula being on the conservative side, for an airfoil with a deep mean camber like the Göttingen 387.

With a variable-lift device in operation, $C_{L_{max}}$ is increased and K_S is correspondingly reduced. An ordinary flap, split or plain, cuts K_S down to about 14. The best combinations of leading-edge slot and slotted flap lower it to about 13; a slot alone lowers it to 14.5. A Fowler wing or some other device of that

order, combining an increase of mean camber with an increase of area, may bring K_S as low as 11. Curves of minimum speed are plotted in Fig. 413 with the values 17, 19 (for a symmetrical airfoil without flap), 14 (for a wing with flap), and 11 (for a Fowler wing) successively assumed for K_S.

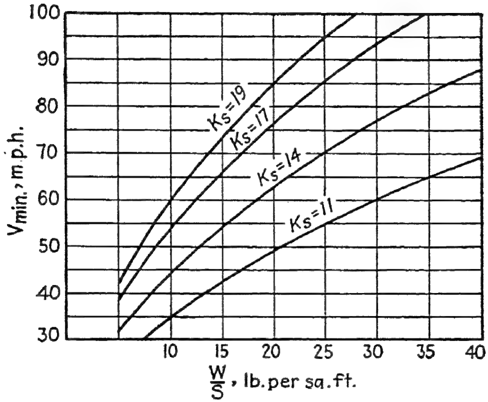


FIG. 413.—Variation of minimum speed with wing loading.

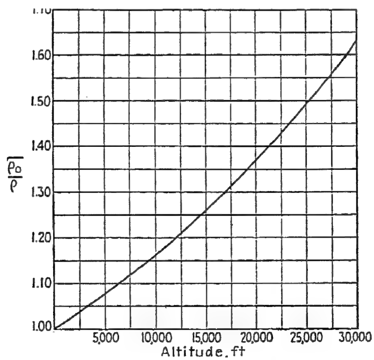


FIG. 414.—Variation of $\sqrt{\rho_0/\rho}$ with altitude.

If minimum speed elsewhere than at sea level is sought, as for determining the landing-speed on a field in high country, K_S has of course to be multiplied by $\sqrt{\rho_0/\rho}$. The curve for that factor, destined to be needed repeatedly in the course of the present chapter, is plotted in Fig. 414. Alternatively, it may be approxi-

mated, with an error of less than 0.25 per cent up to 23,000 ft. and with an error of less than 1 per cent up to 29,000 ft., by the formula

$$\frac{\rho_0}{\rho} = 1 + 0.145 \quad 10{,}000 \quad + 0.02 \frac{h}{10{,}000} \tag{132}$$

Maximum Speed.—The maximum speed involves a greater complexity of elements. Such factors as parasite drag and the form of the drag curve of the wing section must be taken into account for a complete solution, but if averages be assumed for some of these quantities a formula can be developed in terms of power and wing area alone. The simplest of the approximations available include only those two variables.

The condition of maximum speed is that the power required for level flight shall be equal to the power made available through the propeller, or

$$375P\eta = 2.15(C_D S + C_{D_p} S)\frac{\rho}{2}V^3_m \tag{133}$$

where 2.15 is the conversion factor required to put $V_{\max}$ into m. p. h., C_D is the drag coefficient for the wing, and C_{D_p} is the coefficient of parasite drag, reduced to terms of drag per unit of wing area and increased to allow for slip-stream effect (and equal to 1.24 times the equivalent flat-plate area of parasite drag). Induced drag is a small factor at maximum speed, and as a first approximation it can be neglected and C_D and C_{D_p} can be considered as constant. The constant value of C_D for the wing is then the profile drag, henceforth to be denoted by C_{D_0}. If it be further assumed that η is the same in all cases, it follows that

$$\sqrt[3]{\frac{375P\eta}{2.15(C_{D_0}S + C_{D_p}S)\frac{\rho}{2}}} = K\sqrt[3]{\frac{P}{S}} \tag{134}$$

where K is a constant which ought at least to lie in the same region for all well-designed airplanes and which must either be determined by trial or computed from a further examination of the individual quantities entering into (134).

C_{D_0} ranges, for modern airfoils, from 0.0075 to 0.0120; η from 0.72 to 0.86; C_{D_p} from 0.01, for the very cleanest machines, with

landing gears either retractable or very fully streamlined and with the best known forms of cowling on the engines, to 0.05 or even more for the crudest. In the existing state of engineering knowledge, only a very extraordinary set of specifications (such as a requirement that a large amount of equipment be hung on the outside of the fuselage) justifies a value of C_{D_p} above 0.03.

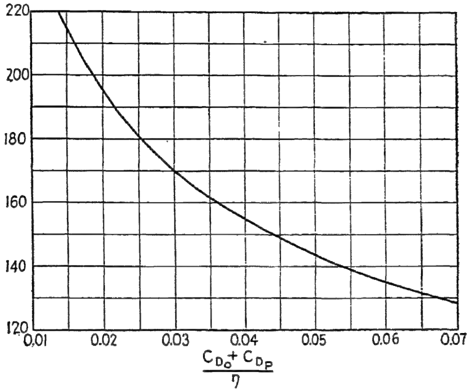


FIG. 415.—Relation of maximum-speed coefficient to profile and parasite drag.

Furthermore, the highest values of C_{D_0} and the highest C_{D_p} do not appear in combination, for the wings of largest profile drag are thick enough to be used only in cantilever structures of which the parasite drag is relatively small. $(C_{D_0} + C_{D_p})/\eta$ can then be taken as ranging between the extreme limits of 0.02 and 0.052. Since

$$K = \frac{375\eta}{\sqrt[3]{2.15(C_{D_0} + C_{D_p})\frac{\rho}{2}}} \tag{135}$$

the corresponding values of K at sea level are 195 and 141. K is plotted against $(C_{D_0} + C_{D_p})/\eta$ in Fig. 415. Application of the formula to existing aircraft shows that the assumption of 0.02 as an attainable minimum was a little too optimistic, for there is no reliably recorded case of a value of K exceeding 183 and there are only a half-dozen instances (mostly twin-engined bombers or transports) of its going above 170. K for a single-engined machine is unlikely, even with the greatest care, to exceed 165.

If every particle of parasite resistance could be eliminated, nothing remaining except a wing with a profile-drag coefficient of 0.0075, K would rise to 253.

K tends to decrease with high wing loading, for the parasite drag per unit of wing area rises as the wings are clipped. The machine that won the Schneider Trophy for Great Britain in 1931—one of the cleanest seaplanes ever built—had a K of but 160. The explanation was not only in the float drag but also in the wing loading, which was over 40 lb. per sq. ft. K tends to decrease with decrease of the operating V/nD, keeping pace with the drop of maximum propeller efficiency. It tends to decrease with decreasing speed-range ratio, for beyond a certain point the induced drag can no longer be neglected. All these factors can be allowed for—some of them empirically, others analytically.

On a machine with a very light wing loading most of the parasite drag is likely to be in the wing bracing, and its amount will be roughly proportional to the wing area. With a heavy unit loading, on the other hand, the fuselage and landing gear are the major factors, and their size is substantially unaffected by the spread of the wings. As a somewhat rough approximation but an adequate one, since it is never likely to lead to errors over 2 per cent in estimating the maximum speed, the parasite drag on a clean airplane ($C_{D_0} + C_{D_p} = 0.020$) with a wing loading of 15 lb. per sq. ft. can be assumed 60 per cent invariant and 40 per cent varying directly with area. Then

$$C_{D_p} = 0.004 + 0.006\left(\frac{W/S}{15}\right)$$

$$K = \sqrt[3]{\frac{375 \times 0.86}{\left(0.014 + 0.0004\frac{W}{S}\right)0.00255}} - \frac{180}{\sqrt[3]{0.60 + 0.020\frac{W}{S}}} \qquad (136)$$

The corresponding values of K are plotted against wing loading in Fig. 416, together with similar curves for airplanes of average cleanness and for those of as poor aerodynamic quality as one ought to expect to encounter in the present stage of the art.

The next correction is for propeller efficiency. Since its magnitude is small, it is better to determine the speed first without the correction and to use this first approximation to the

speed in finding the correction to be applied, rather than to develop a more general and direct expression necessarily somewhat complicated in form. The variation of efficiency with power-plant and airplane characteristics has already been discussed at length,[1] and need not be repeated. Determining the

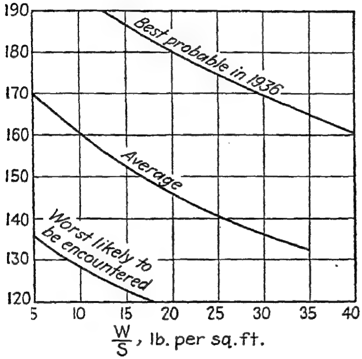


FIG. 416.—Typical variation of maximum-speed coefficient with wing loading.

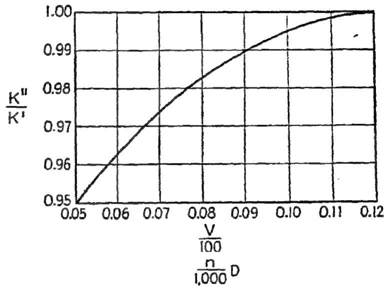


FIG. 417.—Correction factor for variation of propeller efficiency.

first approximation to V from a value of K read off from Fig. 416, and multiplying it by 0.95 to take account of the average values of further corrections, all of which are to be negative, the true maximum efficiency corresponds approximately to the formula

$$\frac{\eta_{max}}{0.86} = 1 - 0.4\left(1 - \frac{\frac{V}{100}}{\frac{n}{1{,}000}D}\right) \qquad (137)$$

where V is in m. p. h. and n in r. p. m. Then

$$\frac{K''}{K'} = \sqrt{1 - 0.4\left(1 - \frac{\frac{V}{100}}{\frac{n}{1{,}000}D}\right)} \qquad (138)$$

where K' is the first approximation to K (read off directly from Fig. 416) and K'' is the second. K''/K' is plotted against $(V/100)/(n/1{,}000)D$ in Fig. 417.

[1] Chap. XIV, *supra*.

The work may stop at this point. It may have stopped immediately after finding K'. It may even have stopped before consulting Fig. 416, a final value of K being arrived at by inspection and simple empirical rule. Everything depends on how much accuracy is sought and upon how near to normal the airplane is in its various characteristics. For a machine of which the wing loading is not above 22 lb. per sq. ft. or the power loading above 15 lb. per hp., and which turns its propeller at an r. p. m. of not more than 15 times the maximum speed in m. p. h., it ought to be sufficient to bear in mind the simple rule that K for the very cleanest airplanes is about 180, while for machines of very inferior aerodynamic arrangement it may drop as low as 135. A little practice in spotting particular airplanes along the scale that connects those extremes ought to make it possible to estimate K, and so the maximum speed at sea level, within 8 per cent in practically every case and within 5 per cent in most cases. However the ultimate value of K may be obtained, if the wing loading is taken into account as a variable and use is made of Fig. 416, the value of K' so obtained should be multiplied by 0.95, the average composite value of the corrections, which will then be omitted. If the correction for maximum propeller efficiency is to be made, by (138), the correction to be applied to K'' should be 0.96. The difference between 0.95 and 0.96 is the average value of the propeller-efficiency factor. What remains, the difference between 0.96 and unity, is the average for the induced-drag correction—yet to come.

In Fig. 418 are plotted the speeds that correspond to various degrees of cleanness of design, as recorded by the value of K, and to various ratios of the power to the wing area—or, alternatively, of the wing loading to the power loading.

As an example, take the airplane used for purposes of illustration throughout the last chapter. Though very clean in appearance, it is a single-engined machine with a fixed landing gear and a fuselage width of almost 10 per cent of the span. Obviously it falls short of the degree of aerodynamic refinement that could be attained with a twin-engined design with the best possible arrangement of cowling, or with a racing plane with liquid-cooled engine and wing-surface radiator. K will therefore be supposed halfway between the "best" and the "average" curves in Fig. 416, and a value of 170 for K' is read off from the chart opposite a wing load-

ing of 15.33. P/S being $^{200}\!/_{150}$, or 1.33, and its cube root 1.10, the preliminary estimate of maximum speed is 170 × 1.10, or 187 m. p. h. $(V/100)/(n/1{,}000)D$ is then $(1.87 \times 0.95)/(2.2 \times 7)$, or 0.115. The correction for propeller efficiency is then substantially zero $\left(\frac{K''}{K'} = 0.999\right)$. The final value of K, on the basis

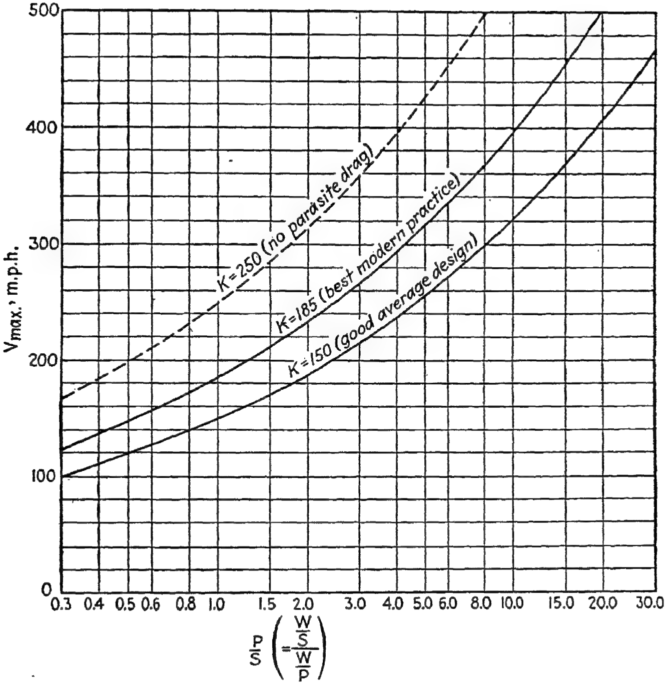


FIG. 418.—Variation of maximum speed at sea level with P/S.

of all the refinement so far exposed, is then 170 × 0.96. Instead of going through the work of computing V all over again with a new value of K, however, steps may obviously be saved by applying the correction factor directly to the speed. V_{max} becomes 187 × 0.96, or 180 m. p. h.—within 2 per cent of the value obtained by construction of the performance curves in the last chapter. The present process should not take over 5 min. to

determine the sea-level speed for an airplane which the estimator has never before seen.

It need take only slightly longer, and the method gains immensely in flexibility, if a correction for induced drag be determined for the specific case. Even in dealing with sea-level performance, the variations of the induced-drag factor may be large enough to make it worth watching. For estimating speed at heights of more than 3,000 or 4,000 ft. it becomes indispensable.

The drag coefficient without induced drag is

$$C_{D_o} + C_{D_p}$$

With induced drag, it becomes

$$C_{D_o} + C_{D_p} + \frac{C_L^2}{\pi R}$$

R being the aspect ratio. Since K is inversely proportional to the cube root of the total drag coefficient,

$$\frac{K}{K''} = \sqrt[3]{\frac{C_{D_o} + C_{D_p}}{C_{D_o} + C_{D_p} + \dfrac{C_L^2}{\pi R}}} \tag{139}$$

Furthermore

$$C_L = \frac{W}{2.15\frac{\rho}{\rho_0} S V^2} = \frac{W\frac{\rho_0}{\rho}}{0.00255 S V^2} \tag{140}$$

and the profile- and parasite-drag coefficients and propeller efficiency are already known to be attached to the value of K obtained prior to any induced-drag correction, either specific or empirical (K''), by the relation

$$C_{D_o} + C_{D_p} = \frac{375 \times 0.86 \times \frac{\eta_{\max}}{0.86}}{0.00255 K''^3} = \frac{125{,}000}{K''^3} \times \frac{\eta_{\max}}{0.86}$$

Then

$$\frac{K}{K''} = \sqrt[3]{\frac{125{,}000}{125{,}000 + \dfrac{49{,}000}{R} \cdot \dfrac{W}{S} \cdot \dfrac{W}{P} \cdot \dfrac{1}{V_{\max}''} \dfrac{0.86}{\eta_{\max}}\left(\dfrac{\rho_0}{\rho}\right)^2 \left(\dfrac{V_{\max}''}{V_{\max}}\right)^4}}$$

$$= \sqrt[3]{\frac{1}{1 + \dfrac{0.392\dfrac{W}{S} \cdot \dfrac{W}{P}\dfrac{0.86}{\eta_{\max}}}{V_{\max}''R} \times \dfrac{\left(\dfrac{\rho_0}{\rho}\right)^2}{(V_{\max}/V_{\max}'')^4}}} \tag{141a}$$

$V_{\max}''$ being the maximum speed as determined from K'', induced drag being ignored. Striking out the density ratio, to limit consideration for the moment to the sea-level case, and letting m represent $(W/S) \times (W/P) \times (0.86/\eta_{\max})/(R \times V_{\max}'')$, a fundamental parameter of the airplane design,

$$\frac{K}{K''} = \sqrt[3]{\frac{1}{1 + \dfrac{0.392m}{(K/K'')^4}}} \tag{141b}$$

Direct solution for K/K'' is impossible, but a series of values of that ratio can be assumed and m solved for and a curve plotted. It has been done in Fig. 419; there results a curve of sea-level induced-drag correction on maximum speed. The term in propeller efficiency is of course much less subject to variation than the others that enter into m, and except for very close work it can be ignored, giving to m the form shown on the scale of abscissas in Fig. 419. The extreme values of m likely to be encountered correspond to the tabulation:

	m	
	Minimum	Maximum
W/S	5.	20.
W/P	3.	25.
R	10.	4.
$V_{\max}''$	215.	120.
m	0.007	1.04

The various elements might of course reach values more extreme than those tabulated, but not in conjunction with the other

figures given in the same column. The combinations shown are as a matter of fact extreme beyond all probability of practical use, and in at least 99 cases out of 100 m will lie between 0.1 and 0.5. In a corresponding proportion of practical airplanes the sea-level value of K/K'' will lie (by Fig. 419) between 0.92 and 0.99. The reasonableness of having assumed an average of 0.96 is apparent.

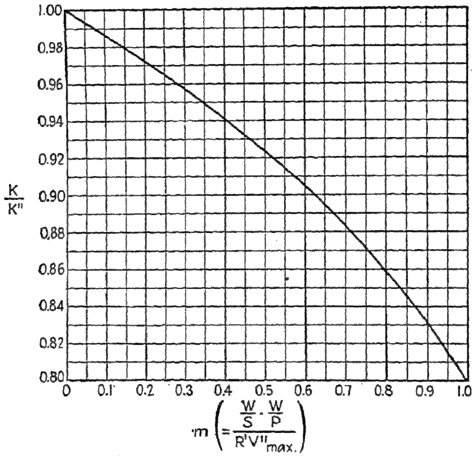


FIG. 419.—Factor for correcting maximum speed at sea level for effect of induced drag.

Since the primary purpose in this chapter is to construct formulas that can be remembered, rather than charts that have to be carried, Fig. 419 should be converted to a formula. With a resulting error of less than 1 per cent in speed for any value of m up to 0.6, the curve can be replaced by a straight line. The equation for the induced-drag correction to maximum speed at sea level then becomes

$$\frac{K}{K''} = 1 \quad 0.16m \qquad (142)$$

One factor not so far taken into account is the variation of profile- and parasite-drag coefficients with angle of attack. It has been suggested by Oswald,[1] in connection with the develop-

[1] "General Formulas and Charts for the Calculation of Airplane Performance," by W. Bailey Oswald, *Rept.* 408.

ment of his performance charts,[1] that if the rise of those quantities above their minimum be assumed proportional to $C_L{}^2$ their effect can be treated as simply increasing the induced drag, or as reducing the aspect ratio of the airplane in the ratio of what he calls the "airplane efficiency factor." Into the same factor can be lumped the change with speed of the slip-stream effect and the effect of departure of the curve of distribution of load along the span from the ideal elliptical form.

The profile drag increases as the cube of C_L rather than the square,[2] but the parasite component is likely to increase with angle approximately as $C_L{}^2$, or even a little less rapidly than that. Adopting Oswald's proposal as a fair approximation to the physical facts, the total of parasite and profile drag (including slip-stream allowance) is found to increase by from 4 to 10 per cent (the exact figure depending principally upon the ratio of parasite to profile, the shape of the fuselage, and the proportion of the total parasite drag that comes from vertical struts) when C_L is increased from 0 to 0.5. The induced-drag correction is then increased in the ratio

$$\frac{\frac{1}{\pi R} + c(C_{D_0} + C_{D_p})}{1/\pi R} \tag{143}$$

where c is somewhere between 0.15 and 0.4. Since c tends to be largest when $(C_{D_0} + C_{D_p})$ is smallest, and vice versa, (143) approximates to the constant $(1 + 0.024R)$. R ought also to be reduced by about 4 per cent on the average for the effect of departure from an elliptical load-grading curve. The average net result is a reduction of the effective aspect ratio, to be used in computing m, by about 15 per cent. The relative reduction is largest (up to 20 per cent) for a machine with high aspect ratio and square fuselage and wing tips; it is smallest (to a minimum of 8 per cent) when the aspect ratio is low, the wings are tapered and rounded at the tips, and all the corners are carefully rounded and filleted. The values of the aspect ratio corrected for all these effects will henceforth be denoted by R'.

In the particular case of the illustrative airplane, the true aspect ratio was 4.47. Reducing that by 12 per cent, and taking

[1] See p. 637, *infra*.

[2] See p. 177, *supra*.

$\eta_{max}/0.86$ from Fig. 417 as 0.995 the fictitious aspect ratio to be used in calculating m becomes 3.93, and m is

$$(15.33 \times 11.5)/(3.93 \times 187),$$

or 0.24. The induced-drag correction factor then is 0.966, and the speed by formula is 187 × 0.966, or 181 m. p. h.

Speed at Altitudes above Sea Level.—The generalization of the work just done, to make it apply at all altitudes, can be undertaken on either of two lines. The most straightforward method is to treat each altitude as a separate case, determining the engine power available at each in turn, figuring a new value of K'', which will be inversely proportional to the cube root of the density, a new value of m, etc. An alternative, and a much less tedious, method is to take the sea-level condition as an established foundation and to develop laws connecting it with the conditions prevailing and the performances attainable at higher levels.

The major factor of variation is the induced drag, and, since the induced-drag coefficient varies inversely as the fourth power of the speed of flight, and inversely as the square of the air density, the speed at altitude can be obtained by solving the equation

$$\frac{V_h}{V_0''} = \sqrt[3]{\frac{P_h/P_0}{\frac{\rho_h}{\rho_0}\left[1 + 0.392m\left(\frac{\rho_0}{\rho_h}\right)^2\left(\frac{V_0''}{V_h}\right)^4\right]}} \qquad (144)$$

V_0'' is still the same V'' as before, taken always at sea level and with a subscript 0 added now as a reminder of the fact. V_h is the true speed at altitude, and the other symbols have their several familiar meanings.

The number of variables can be reduced by one by using known relations between power and density. There are, it will be recalled,[1] four major engine-propeller cases.

	Power Available Proportional to
Unsupercharged fixed-pitch	$\rho^{1.25}$
Unsupercharged controllable-pitch	$\rho^{1.2}$
Supercharged fixed-pitch	$\rho^{-0.5}$
Supercharged controllable-pitch	ρ^0(const.)

Substituting the appropriate power of (ρ_h/ρ_0) for (P_h/P_0) in (144), there result four equations of the form

[1] See pp. 484, 551, *supra*.

$$\frac{V_h}{V_0''} = \left(\frac{\rho_0}{\rho_h}\right)^x \sqrt[3]{\frac{1}{1 + 0.392m(\rho_0/\rho_h)^2(V_0''/V_h)^4}} \tag{145}$$

where x has the values -0.08, -0.07, 0.50, and 0.33, respectively, for the four cases just itemized.

Were the propeller efficiency a constant at all speeds, nothing would remain except to solve the four forms of (145). But the condition is not met. The efficiency is far from constant with a fixed-pitch propeller, and even with the controllable-pitch type the variations are too large to be neglected. If an analytical form is to be retained and an analytical solution found, there must be an analytical approximation for available power, taking into account not only the variations in efficiency but also (in the fixed-pitch case) the variation of r. p. m. and of engine power developed.

No simple expression can be strictly accurate; no single expression can fit all cases; nevertheless a study of propeller data reveals the possibility of a very satisfactory approximation, never likely to be out by more than 3 or 4 per cent over the range of speeds and air densities having to be covered. For the fixed-pitch propeller the starting-point is Hartman's method[1] of plotting thrust produced at constant torque. Experimental curves indicate a probable thrust proportional to

$$\left[1 + \frac{3}{4}\left(1 - \frac{V}{V_o}\sqrt{\frac{\rho/\rho_0}{Q/Q_0}}\right)\right]Q \tag{146}$$

where Q is the engine torque, upon the assumption that the full-throttle power is absorbed at rated r. p. m. at V_o and that the propeller delivers its maximum efficiency at a V/nD corresponding to rated r. p. m. and to $0.9V_o$. If the maximum efficiency were to correspond exactly to maximum speed (which is not, in general, the best practice[2]) the coefficient of the second term would be somewhat less than $\frac{3}{4}$.

Multiplying the thrust by V to get the available power, the ratio of power available under any stated conditions to that available under the design conditions is

$$\frac{P_a}{P_{a_0}} = \frac{V_h}{V_o}\cdot\frac{Q}{Q_0}\left[1 + \frac{3}{4}\left(1 - \frac{V_h}{V_o}\sqrt{\frac{\rho}{\rho_0}\cdot\frac{Q_0}{Q}}\right)\right] \tag{147}$$

[1] *Rept.* 481, *cit. supra.*

[2] See pp. 476, 557, *supra.*

where P_{a_0} is the product of rated engine power and maximum propeller efficiency. Substituting $(\rho/\rho_0)^{1.2}$ for (Q/Q_0) for the unsupercharged engine, and treating the torque as a constant in the supercharged case, there result expressions involving only the speed ratio and the density ratio, which can be combined with (147) to give two new equations.

The controllable-pitch propeller can be similarly treated, except that in that case n can be considered constant and the approximation process can better be applied directly to the power-output charts devised by Weick (like the sample shown in Fig. 335). The experimental facts are best fitted by the expression

$$\frac{\eta}{\eta_{\max}} = 1 - \frac{1}{4}\left(1 - \frac{V}{V_o}\sqrt[5]{\frac{\rho}{\rho_0}}\right) \tag{148a}$$

when the engine is supercharged, and

$$\frac{\eta}{\eta_{\max}} = 1 \quad \frac{1}{4}\left(1 - \frac{V}{V}\right)^{12} \tag{148b}$$

when it is not. The two different forms are required in recognition of the very different mode of variation of the blade angle with altitude in the two cases.

Again there result two composite expressions, somewhat elaborate in form but involving (V_h/V_o) and (ρ_0/ρ_h) as their only variables and m as their only design parameter. Bringing all the results together, the final equations for the four cases are:

Unsupercharged fixed-pitch:

$$\frac{V_h}{V_0''} = \left(\frac{\rho_0}{\rho_h}\right)^{-0.07} \sqrt[3]{\frac{V_h}{V_0}} \sqrt[3]{\frac{1 + \frac{3}{4}\left[1 - \frac{V_h}{V_0}\left(\frac{\rho_0}{\rho_h}\right)\right]}{1 + 0.392m(\rho_0/\rho_h)^2(V_0''/V_h)^4}} \tag{149}$$

Simplifying, and raising both sides of the equation to the $3/2$ power, this becomes:

$$\frac{V_h}{V_0''} = \left(\frac{\rho_0}{\rho_h}\right)^{-0.10} \sqrt{\frac{V_0''}{V_0}} \sqrt{\frac{1 + \frac{3}{4}\left[1 - \frac{V_h}{V_0}\left(\frac{\rho_0}{\rho_h}\right)\right]}{1 + 0.392m(\rho_0/\rho_h)^2(V_0''/V_h)^4}} \tag{150a}$$

Similarly, for the other cases:

Unsupercharged controllable-pitch:

$$\frac{V_h}{V_0''} = \left(\frac{\rho_0}{\rho_h}\right) \sqrt{\frac{1 - \sqrt{1 - \frac{V_h}{V_0}\left(\frac{\rho_0}{\rho_h}\right)}}{1 + 0.392m(\rho_0/\rho_h)^2(V_0''/V_h)^4}} \qquad (150b)$$

Supercharged fixed-pitch:

$$\frac{V_h}{V_0''} \quad \sqrt{\frac{\rho_0''}{\rho_h}}\sqrt{\frac{V_0''}{V_0}}\sqrt{\frac{1 + \sqrt{1 - \frac{V_h}{V_0}\left(\frac{\rho_0}{\rho_h}\right)}}{1 + 0.392m(\rho_0/\rho_h)^2(V_0''/V_h)^4}} \qquad (150c)$$

Supercharged controllable-pitch:

$$\frac{V_h}{V_0''} \quad \sqrt[3]{\frac{\rho_0}{\rho_h}}\sqrt[3]{\frac{1 - \sqrt{1 - \frac{V_h}{V_0}\left(\frac{\rho_0}{\rho_h}\right)}}{1 + 0.392m(\rho_0/\rho_h)^2(V_0''/V_h)^4}} \qquad (150d)$$

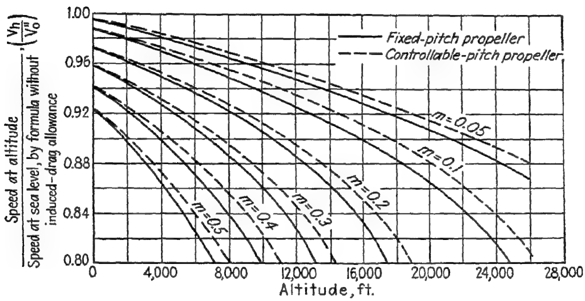


FIG. 420.—Variation of speed with altitude under various conditions, unsupercharged engines.

The case of the supercharged engine driving a fixed-pitch propeller and maintaining a constant torque may be passed over. It does not arise in practice, for it would involve running the engine at dangerously high r. p. m. and power output at high altitudes. But the other three cases have been solved for various values of m, and the results are plotted in Figs. 420 and 421. Superposing the information that these figures provide on that previously drawn from Figs. 416 and 417, it becomes possible to compute the speed-altitude curve for any airplane within a couple of minutes.

Figure 421 is of course usable, in connection with full-throttle operation, only up to the critical altitude. But in analyzing

cruising performance, it serves the purpose up to the greatest height at which the cruising power can be maintained. Suppose, for example, that the performance at a 50 per cent power factor were to be determined for the standard illustrative case. First, m must be recalculated. W/P is doubled by having the power, going from 11.5 to 23. W/S is unchanged; so is the aspect ratio. K'' remains the same; V_0'' therefore varies as the cube root of P; its value becomes 187×0.79, or 148; the new m is 0.61.[1]

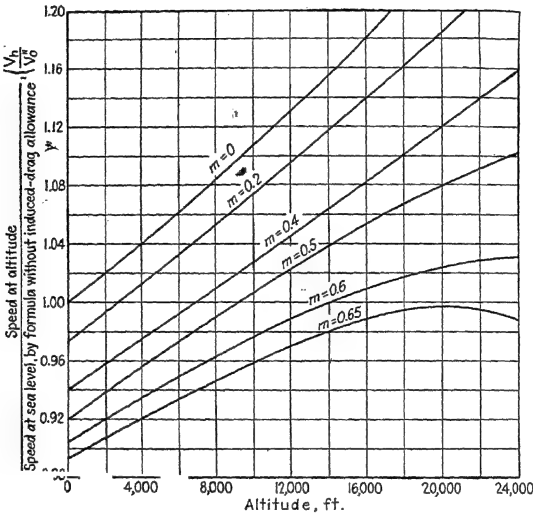


FIG. 421.—Variation of speed with altitude, supercharged engines.

The speed at various altitudes, up as high as 50 per cent of rated power can be maintained, or about 20,000 ft., is then to be found by multiplying 148 by the ordinates of the curve for $m = 0.61$ in Fig. 421. At sea level the machine would cruise at 133 m. p. h.; at 10,000 ft., at 144 m. p. h.; at 20,000 ft., at 151 m. p. h.

[1] Combining the effects of P upon the two factors, m varies as $(P/P_c)^{4/3}$. To a very good approximation (always within 1 per cent)

$$\frac{m_c}{m_0} = 1 + 1.40\left(\frac{1}{P_c/P} - 1\right) + 0.12\left(\frac{1}{P_c/P} - 1\right)^2$$

where m_c is the value for cruising, and P_c/P is the cruising power factor.

For every airplane and every power factor, there comes a point of transition from the supercharged to the unsupercharged case. The most accurate method of passage from one chart to the other at the critical altitude requires a projection backward from that altitude to sea level and a recalculation of the problem on the basis of values of P and V'' equalling those that would have had to exist if the airplane had had no supercharging whatever and yet actually attained the performance that it shows at the critical altitude in the actual supercharged case. The engine power must be assumed increased, in other words, in the ratio $(\rho_0/\rho_c)^{1.2}$, and V'' in the ratio $(\rho_0/\rho_c)^{0.4}$, where the subscript c indicates conditions at the critical altitude.

Approximately the same result is obtained, however, if the value of m is assumed increased in the ratio $(\rho_0/\rho_h)^2$ in passing from Fig. 421 to Fig. 420 at the critical altitude, if all altitudes on Fig. 420 are measured from the critical altitude, and if the ordinates on Fig. 420 are not read directly, but as the ratio of the actual ordinate at the desired altitude to the ordinate of the same curve at sea level.

Thus in the illustrative case the full-power speed at 6,000 ft., which was assumed as the critical altitude, would be determined from Fig. 421 as 187×1.044, or 194 m. p. h. To find the speed at 20,000 ft., take m as $0.24 \times (1.195)^2$, or 0.283, since ρ_0/ρ at 6,000 ft. is 1.195. On Fig. 420, read off the ordinate corresponding to $m = 0.283$ and an altitude of 14,000 ft., which proves to be 0.825 for the controllable-pitch-propeller case, and also for the same value of m at zero abscissa, which gives 0.965. The ratio of the two is 0.86. Multiplying that by 194 gives 167 m. p. h., the required speed within 2 per cent of the figure plotted on page 550.

Again it is desirable to put all this into a formula, or a series of them. Again it proves that formulas of reasonable simplicity can be constructed to fit the curves within 1 per cent for a very wide range of conditions. Thus, for the unsupercharged case, the correction factor for maximum speed can be given the general form

$$\frac{V_h}{V_0''} = 1 - 0.16m - \frac{\sqrt{10m}(h/1{,}000)}{170}$$

with a fixed-pitch propeller, and

$$\frac{V_h}{V_0''} = 1 - 0.16m - \frac{\sqrt{10m}(h/1,000)}{190}$$

with controllable pitch. These formulas are accurate enough for most practical service up to about two-thirds of the absolute ceiling.

When there is supercharging, or when cruising at below the critical cruising altitude, the expression becomes (assuming a controllable-pitch propeller in every case)

$$\frac{V_h}{V_0''} = 1 - 0.16m + (0.011 - 0.005m)\frac{h}{1,000}$$

except for very high altitudes and high values of m. When the altitude exceeds 16,000 ft. with $m = 0.5$, or 12,000 ft. with $m = 0.65$, the more elaborate form

$$\begin{aligned} \frac{V_h}{V_0''} = 1 - 0.16m + (0.011 - 0.005m)\frac{h}{1,000} \\ \frac{(m - 0.4)^2\left(\frac{h}{1,000} - 10\right)^2}{100} \end{aligned}$$

must be used.

Ceiling Formulas.—Formulas for climb characteristics and for ceiling are somewhat more complex in their derivation than those for maximum speed, as they necessarily involve weight as well as power and area, and require also some assumption relative to the rate of decrease of engine power with increasing altitude.

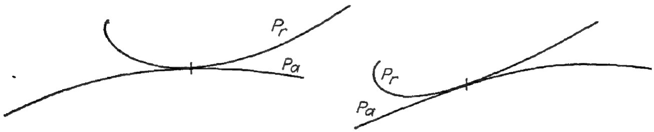
The ceiling of an airplane is the point at which the curves of power available and power required become tangent to each other. Their point of tangency might conceivably lie anywhere along the curve, and the common tangent line might have almost any slope. The relationship of the curves might, in an extreme case, be as in Fig. 422*a*, with the minimum point of one curve just making contact with the maximum of the other. But it is much more likely, especially with a fixed-pitch propeller, to be as in Fig. 422*b*.

If power variation is written in terms of density, for the controllable-pitch case and an unsupercharged engine,

$$P_r = P_{r_0}\sqrt{\frac{\rho_0}{\rho}} \tag{151}$$

$$= P_{a_0}\left(\frac{\rho}{\rho_0}\right)^{1.2} \tag{152}$$

where the subscripts r and a represent required and available power, and the subscript 0 refers to sea-level conditions.



(a) (b)

FIG. 422.—Relation of power required and power available at the absolute ceiling.

The condition at the ceiling is that

$$C_1 P_{a_{\max}} = C_2 P_{r_{\min}}$$

where C_1 and C_2 are the ratios of the ordinates at the point of tangency to the maximum and minimum ordinates, respectively, as in Fig. 422*b*. Substituting from the equation just written

$$\frac{C_1}{C_2} P_{a_0\max}\left(\frac{\rho_H}{\rho_0}\right)^{1.2} = P_{r_0\min}\left(\frac{\rho_0}{\rho_H}\right)^{0.5} \tag{153}$$

at the ceiling, or

$$\left(\frac{\rho_0}{\rho_H}\right)^{1.7} = \frac{P_{a_0\max}}{P_{r_0\min}}\frac{C_1}{C_2} \tag{154}$$

$$1.7 \log_{10} \qquad = \log_{10}\frac{P_{a_0\max}}{P_{r_0\min}} + C \tag{155}$$

the subscript H indicating conditions at the ceiling height. Air density and altitude stand approximately in the relationship[1]

$$\log_{10}\frac{\rho_0}{\rho} = \frac{h}{76{,}000} \tag{156}$$

[1] For a discussion of this and other atmospheric density formulas, see 'Aerostatics," by Edward P. Warner, Chap. I, New York, 1926.

Combining this with (155), there results a formula[1] for ceiling in terms of sea-level power curves. If C_1/C_2 were unity, the formula would be:

$$H = \frac{76{,}000}{1.7} \log_{10} \frac{P}{P_{r_0 \min}} = 45{,}000 \log_{10} \frac{}{P_{r_0}} \tag{157}$$

This is for a controllable-pitch propeller. With a fixed pitch, the value 1.7 would be changed to 1.75, and the coefficient 45,000 on the right-hand side of the equation would become 43,500.

The value of C might be computed by the use of propeller-performance formulas such as (148), but the resultant form would be far too complex to deserve listing among simplified methods. It is better to turn to empiricism and to modify the form of (157) in accordance with experience. So doing, it proves that C_1/C_2 varies with power reserve in such a way that it could better have been treated as an exponent, rather than as a factor, in (153). In (157), then, the allowance for the location of the point of tangency of the curves appears as a coefficient, not as an added term, and the form justified by experience becomes

$$H = 35{,}000 \log_{10} \frac{}{P_{r_0}} \tag{158}$$

With a fixed-pitch propeller, the coefficient is 32,000.

This formula in itself is useful in making it possible to approximate high-altitude performance without going to the trouble of constructing additional sets of power curves for altitudes other than sea level; however, further simplification, obviating the necessity of any curve plotting at all, is of course to be sought.

Before going on to that, however, it must be noted that (158) applies directly only where there is no supercharging. If the engine were completely supercharged to a critical altitude indefinitely high, the available power would be substantially constant. There would be a little change of propeller efficiency with changing blade angle, but it would be small enough to be

[1] For further discussions of the formula and of alternative forms, in some instances containing numerical constants based on out-of-date or incomplete data, consult "Die Steigfähigkeit der Flugzeuge," by Heinrich Kann, *Technische Berichte*, vol. II, p. 231.

neglected in a first approximation. The expression (154) would then be replaced by

$$\left(\frac{\rho_0}{\rho_H}\right)^{0.5} = \frac{P_{a_0\max}}{P_{r_0\min}} \tag{159}$$

and (157) by

$$H = 150{,}000 \log_{10} \frac{P_a}{P_{r_0\min}} \tag{160}$$

and fabulous ceilings would be in prospect.

Since the ratio of the constants in (157) and (160), 45,000/150,000, is 0.3, it follows that 1,000 ft. of climbing under supercharged conditions takes out of the airplane's reserve of power only as much as a climb of 300 ft. by the same machine from the same level without the supercharger. From that in turn it is evident that a ceiling beyond the critical altitude can be approximated by counting the climb up to the critical altitude at only 30 per cent of its true extent. The ceiling in such a case can be determined by using (158), ignoring the supercharging completely, and then adding 70 per cent of the critical altitude to the ceiling figure that (157) gives. This assumes C to have the same value for the supercharged and unsupercharged cases, which is a fair approximation.

In turning to the quest for a more general formula, the obvious requirement is that the horsepower available and horsepower required should be put in terms of the general characteristics of the airplane. For the former that is very easy, the maximum power available being equal to the product of rated engine power and maximum propeller efficiency, which can be taken from (137).

The most direct and obvious approach to the determination of minimum power requirement is through a differentiation of the expression for total power, treating the drag coefficient as the sum of a constant, representing the invariant portions of the parasite and profile drags, and a quantity inversely proportional to the fourth power of the speed, which is the induced drag, suitably modified to allow for the change of profile drag with angle of attack and other such irregularities. The process runs, as first indicated by Klemperer and Bienen,[1]

[1] "Comparing Aerodynamical Properties of Wings," by W. Klemperer and T. Bienen, *Aviation*, p. 368, Sept. 24, 1923.

$$P_r = \frac{C_D \frac{\rho}{2} S V^3}{550} = \frac{\frac{C_D}{C_L^{3/2}} W \sqrt{\frac{W}{S}} \cdot \frac{1}{\sqrt{\rho/2}}}{550} \tag{161}$$

$$\frac{C_D}{C_L^{3/2}} = \frac{\sqrt{C_L}}{\pi R} + \frac{C_{D_o} + C_{D_p}}{C_L^{3/2}} \tag{162}$$

$$\frac{d(C_D/C_L^{3/2})}{dC_L} = \frac{1}{2\pi R\sqrt{C_L}} - \frac{3}{2}\frac{C_{D_o} + C_{D_p}}{C_L^{5/2}} = 0 \tag{163}$$

The power function is then a minimum when

$$C_L^2 = 3\pi R(C_{D_o} + C_{D_p}) \tag{164}$$

Then

$$\frac{C_D}{C_L^{3/2}} = \sqrt{C_L}\left(\frac{1}{\pi R} + \frac{1}{3\pi R}\right) = \frac{4}{3\pi R}\sqrt[4]{3\pi R(C_{D_o} + C_{D_p})}$$

$$= 0.74\sqrt[4]{\frac{C_{D_o} + C_{D_p}}{R^3}} \tag{165}$$

$$P_{r_{o\min}} = 0.039W\sqrt{\frac{W}{S}}\sqrt[4]{\frac{C_{D_o} + C_{D_p}}{R^3}} \tag{166}$$

R in this expression must be the net effective aspect ratio, corrected for rate of change of profile drag and other factors as explained on page 607. It will henceforth be denoted by R'. Aspect ratio thus appears paramount, the power consumed varying inversely as $R^{3/4}$ and directly only as the fourth root of the total parasite and profile drag. Numerical values given directly by (166) must, however, be regarded with great suspicion, for the value of C_L, given by (164) is commonly in excess of the maximum lift of the airfoil, or at least well above the range within which the fundamental assumptions regarding the profile-drag coefficient can be considered as holding true. But the general indication regarding the controlling position of aspect ratio is perfectly valid. For an airplane of given weight, it is the span used, above anything else that the designer can do, that determines the ceiling. The actual optimum value of C_L is likely to be about one-third lower than (164) would give, and the minimum power requirement is raised about 6 per cent thereby.

Increasing $P_{r_{o\min}}$ by 6 per cent, an expression for ceiling can now be written directly from (158). It is

$$H = 35{,}000 \log_{10} \frac{24P\eta}{W\sqrt{\frac{W}{S}}\sqrt[4]{\frac{C_{D_0} + C_{D_p}}{R'^3}}} \tag{167}$$

It will not be apparent at sight, but the variable part of this equation is almost identical with m, just used in studying the effect of induced drag on maximum speed. Thus,

$$m = \frac{W/P \times W/S \times 0.86/\eta_{\max}}{R'V_{\max}''} \tag{168}$$

$$'' = \frac{375P\eta}{\sqrt[3]{2.15(C_{D_0} + C_{D_p})S\frac{\rho}{2}}} \tag{169}$$

$$m^3 = \frac{\frac{W^3}{P^3} \times \frac{W^3}{S^3} \times \frac{S}{P}(C_{D_0} + C_{D_p})\frac{\rho}{2} \times \left(\frac{0.86}{\eta_{\max}}\right)^3}{175\eta R'^3}$$

$$= \frac{W^4\left(\frac{W}{S}\right)^2 \frac{C_{D_0} + C_{D_p}}{R'^3}}{232{,}000\,\eta_{\max}^4} \tag{170}$$

The fourth root of this expression is the exact reciprocal of the expression whose logarithm was taken in (166), except that the constant terms are not quite the same. Substituting, (167) takes the form

$$H = 35{,}000 \log_{10} \left(1.09\frac{1}{m^{3/4}}\right) = 26{,}250 \log_{10} \left(\frac{1}{m}\right) + 1{,}400 \tag{171}$$

Empirical modification again becomes desirable. The adjustment is a small one, for the largest of the factors that do not respond well to analytical treatment have already been taken into account, but checking against actual performance shows better results when the constant term and the coefficient are both changed, to give the form

$$H = 24{,}000 \log_{10} \left(\frac{1}{m}\right) + 2{,}000 \tag{172}$$

With a fixed-pitch propeller, the constant in the form last given would become 21,500. Ceiling is plotted against m for the two cases in Fig. 423. If propellers were chosen with reference to no consideration but ceiling, however, as when an altitude record is to be attacked, the coefficient might run as high as 26,000.

A simpler form may be evolved from (167) if it is assumed that general over-all aerodynamic efficiency can be maintained at a standard level and that $C_{D_o} + C_{D_p}$ can be taken as proportional to R'. This last hypothesis is not so arbitrary as it may appear at first sight, for large increases of aspect ratio require a thickening of the airfoil section, thus increasing the profile drag; they tend also to call for added external members, and so for more parasite, as well.

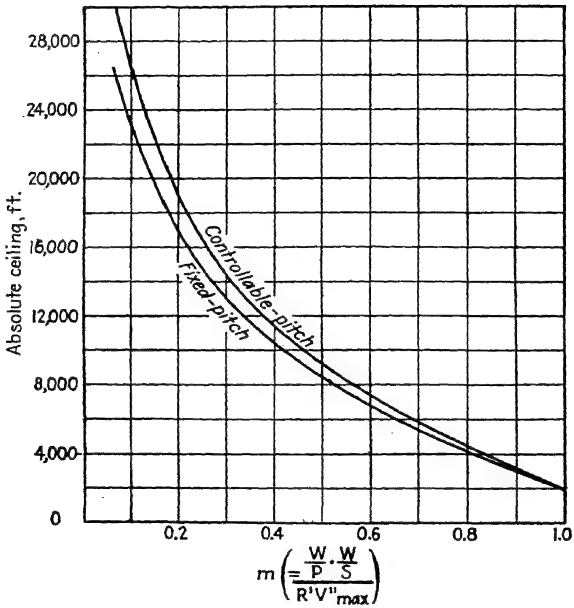


FIG. 423.—Variation of absolute ceiling with m.

On this basis, since

$$SR' = b^2e \tag{173a}$$

(e being the "efficiency" factor that gives the net aspect ratio), $\sqrt{W/S}\sqrt[4]{(C_{D_o} + C_{D_p})/R'^3}$ is proportional to $\sqrt{W/b^2e}$. $\sqrt[4]{(C_{D_o} + C_{D_p})/R'}$ may range from 0.25 up to 0.35, but only very infrequently should it exceed 0.30. The peak propeller efficiency

will not depart very far from 0.83. The ceiling formula then becomes

$$H = 35{,}000 \log_{10} \frac{K_H}{\frac{W}{P}\sqrt{\frac{W}{b^2e}}} \tag{173b}$$

where K_H ranges from 50 to 80. If a single value is required, 60 will be found to approximate the ceiling with fair accuracy in most

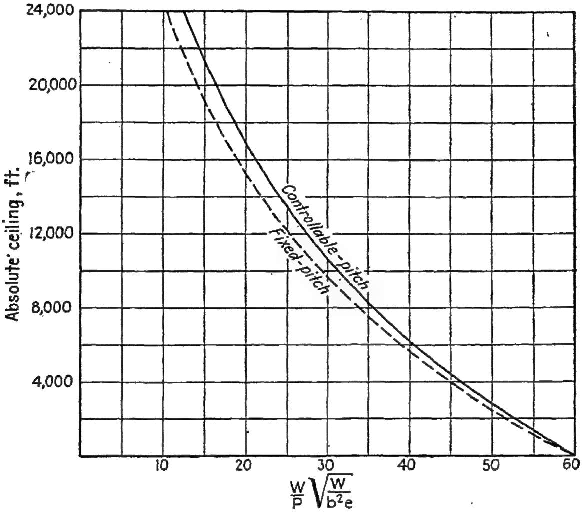


FIG. 424.—Variation of absolute ceiling with power loading and span loading, for average aerodynamic efficiency.

cases. The foregoing derivation is of course valid only for the unsupercharged engine and the controllable-pitch propeller, but to fit the case of a fixed-pitch propeller it is necessary only to change the constant from 35,000 to 32,000. Where there is supercharging, 70 per cent of the critical altitude can be added to the ceiling.

In Fig. 424, ceiling is plotted against $(W/P)\sqrt{W/b^2e}$ for both types of propeller, and for an assumed value of K_H of 60.

Since it is unsafe to operate an aircraft with a ceiling of less than about 3,000 ft., the reserve of power being too much reduced to allow proper control even at sea level, Fig. 424 indicates that

the extreme limit of $(W/P)\sqrt{W/b^2e}$ for continued flight with normal efficiency is about 48. In a very efficient machine the figure might possibly rise to 60. With a "span loading" of 3.3 lb. per sq. ft., ($W/S = 20$ and $R' = 6$), it ought to be possible to fly normally at low altitudes with a power loading of $48/\sqrt{3.3}$, or 26. In an extraordinary case, the allowable power loading with that span loading might be pushed up to 33.

This has an obvious bearing on the possibilities of the ultra-light airplane, which can raise its power loading to very high figures, and so fly with a miniature engine, only by keeping the wing loading exceedingly low and making use of a high aspect ratio. Much more important, it has a bearing on the behavior of the multi-engined airplane after the failure of an engine.

If one engine goes out in a twin-engined machine, the power loading is doubled thereby. Whatever the value of K_H, the computed ceiling is reduced by 35,000 $\log_{10} 2$, or 10,500 ft.

If nothing had happened beyond the loss of half the power, and if C in (155) had remained constant, that would be the total of the loss of ceiling. But it is necessary to reckon also with the drag of the stopped propeller, and with drag on the rudder due to the dissymmetry of thrust. The efficiency of the airplane is decreased, which is equivalent to a decrease of K_H. Also, C does not remain constant. The displacement of the point of tangency from the maximum and minimum points is relatively much more important at reduced power than at full power. There results an actual total loss of ceiling that averages about 16,000 ft. When it is a side propeller on a three-engined machine that stops, the loss of ceiling is likely to be about 10,000 ft. The failure of one engine out of four ought not to cost more than 7,500 ft. of ceiling. The full safety incident to use of multiple power plants is then to be had only by designing, in the case of a twin-engined machine, for a full-power ceiling of at least 19,000 ft. above the highest terrain over which it will be necessary to pass in the course of service. That, in turn, by reference to Figs. 423 and 424, indicates the undesirability of using two engines in any case for which m exceeds 0.2 or for which $(W/P)\sqrt{W/b^2e}$, with average aspect ratio and cleanness of design, exceeds 17. With three engines it is safe to let $(W/P)\sqrt{W/b^2e}$ run up to about 25, and with four engines it may safely go as high as 30, always provided that the anticipated operations are near sea level. For flight among

mountains running up to 7,000 ft. in height, all the figures just given ought to be reduced by one-third, unless of course the engines are supercharged and the mountainous country lies below their critical altitude, in which case the height of the mountains can be considered as reduced to 30 per cent of its true value.

There is a further relationship of some interest between the ceiling and the speed range. Since

$$m = \frac{\frac{W}{P} \times \frac{W}{S} \times \frac{0.86}{\eta_{\max}}}{R' \times V_{\max}''}$$

and $V_{\max}''$ is approximately equal to the sea-level maximum speed, while

$$V_{\min} \quad K_s\sqrt{\frac{W}{S}}$$

$$m = K_m\frac{W}{P} \times \sqrt{\frac{W}{S}} \times \frac{1}{\eta_{\max}} \times \frac{1}{R'} \times \frac{V_{\min}}{V_{\max}''} \tag{174}$$

But

$$\frac{W}{P} \times \sqrt{\frac{W}{S}} = \frac{W/P}{W/S} \times \left(\frac{W}{S}\right)^{3/2} \tag{175}$$

and

$$\frac{W/P}{W/S} = \frac{1}{P/S}$$

$V_{\max}''$ ($V_{\max}$ without allowance for induced drag) is proportional, for a given value of K, to $\sqrt[3]{P/S}$; $V_{\min}$ is proportional to $\sqrt{W/S}$; and, therefore,

$$\frac{W}{P} \times \sqrt{\frac{W}{S}} = K_m'\left(\frac{V_{\min}}{V_{\max}''}\right)^3 \tag{176}$$

If variations in η and R' are neglected,

$$m = K_m''\left(\frac{V_{\min}}{V_{\max}''}\right)^4 \tag{177}$$

Though K_m'' may range widely with R and other variables, it averages about 8 to 9. Then, from (172),

$$H = 96{,}000 \log_{10} \left(\frac{V_{\max}''}{1.7 V_{\min}} \right) + 2{,}000 \tag{178}$$

Since H is a function of m, by (170), and $V_{\max}/V_{\max}''$ is also a function of m, there is a direct relationship between H and $V_{\max}/V_{\min}$. It cannot be expressed through a simple formula, but it could be plotted.

Rate of Climb.—Having separate expressions for power required and power available, the rate of climb can of course be figured at once. The propeller efficiency at best climbing speed, in the first instance, will be left as a variable, with the further assumption that at the point of maximum separation between the two power curves the power required will be 20 per cent in excess of its minimum value. $\sqrt[4]{(C_{D_p} + C_{D_0})/R'}$ will be assumed to be 0.30. The expression for excess power available then becomes, by (166), and (173a),

$$P_a - P_r = \eta P - \frac{1.2W\sqrt{W/b^2e}}{85} = \eta P - \frac{}{70} \tag{179}$$

The expression for propeller efficiency can be made progressively more accurate by making it more complicated, until finally a point is reached at which further complication would become intolerable. The primary objective being to secure simple formulas for instant approximation, no such lengths as would be implied in a return to the expressions (146) and (147) will be considered.

The simplest of all approximations is a single flat figure. Fortunately the two factors tending to vary such a figure work against each other. It is on a machine with a large power reserve that the conditions of best climb will be found farthest removed from the peak of efficiency; but the machine with a large power reserve is also, in practically every instance, the one with a high maximum speed, a propeller working at a high V/nD, and therefore a high maximum propeller efficiency; it accordingly has more efficiency to spare. Though it cannot be pretended that the approximation is a very close one, a flat 62 per cent for a fixed-pitch propeller and 74 per cent for one of controllable pitch will

lie within 5 per cent of the true values at maximum climb in 7 cases out of 10, within 10 per cent in practically all cases. The figure for the fixed-pitch propeller is a net output ratio, taking into account the loss of r. p. m. and full-throttle power output at low speeds of flight.

The simplest possible formula for rate of climb at sea level then becomes

$$\frac{dh}{dt} \quad \frac{24{,}500}{W/P} - 470 \tag{180}$$

with a controllable-pitch propeller, while with fixed pitch the constant 24,500 must be changed to 20,500. It is clear how very intimately the rate of climb of a high-performance airplane depends on power loading. For such a machine, all other factors are negligible in comparison with that one.

A more elaborate formula will take account of the variations of η and of the ratio of the power required at the speed of best climb to the minimum required power. The former is empirically determined with satisfactory accuracy by the formula

$$\eta_c \text{ (in per cent)} = 80 - \frac{0.25H}{1{,}000} \tag{181a}$$

in the controllable-pitch, and

$$\eta_c \text{ (in per cent)} = 75 - \frac{0.75H}{1{,}000} \tag{181b}$$

in the fixed-pitch case (H being the absolute ceiling). The latter, heretofore assumed at a flat 1.2, can be better approximated by

$$1 + 0.008\frac{H}{1{,}000} \tag{182}$$

with controllable pitch. With fixed pitch, 0.008 should be changed to 0.015.

Including all this, the expression for rate of climb becomes

$$\frac{dh}{dt} = \frac{330(80 - 0.25H/1{,}000)}{W/P} - 390\left(1 + 0.008\frac{H}{1{,}000}\right)\sqrt{\frac{W}{b^2e}} \tag{183a}$$

with controllable-pitch propellers, or

$$\frac{dh}{dt} = \frac{330(75 - 0.75H/1{,}000)}{W/P} - 390\left(1 + 0.015\frac{H}{1{,}000}\right)\sqrt{\frac{W}{b^2e}} \quad (183b)$$

where the pitch is fixed.

Again, as several times in the past, it happens that charts can be plotted by the more elaborate and exact formula quite as readily as by the simple one. Since H is itself (for an assumed average of aspect ratio and general aerodynamic efficiency) a function of

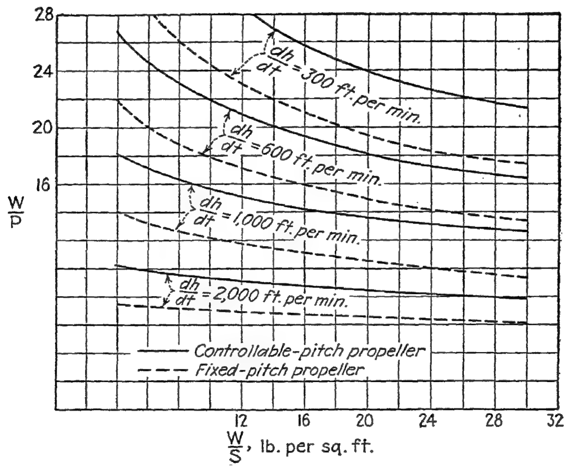


FIG. 425.—Variation of rate of climb with power loading and wing loading, for average aerodynamic efficiency.

W/P and W/b^2e, the value of H corresponding to any desired combination of W/P and W/b^2e can be inserted in (183). Solution for dh/dt is then direct. In that fashion, Fig. 425 was prepared. But if the desire is for a formula that can be easily carried in mind, to supply a rough but useful check on performance claims and a basis for quick estimate of what can be done with a given weight and power, without pause to spread out charts, (180) will serve the purpose. To hold to more accustomed units, and since W/b^2e plays a secondary part in determining the rate of climb in most cases (as compared with W/P), the curves in Fig. 425 have

been drawn in terms of W/P and W/S. The wing loading equivalent to given values of W/b^2e was computed upon the assumption that $R' = 6$.

It would of course be a simple matter to construct similar formulas and curves for the rate of climb at any other altitude. It would be so simple that the work will be left to be done by those who may be interested, merely multiplying the first term on the right-hand side of the equation by ρ/ρ_0 to the appropriate exponent to indicate the rate of change of power output with altitude for the particular engine-propeller combination being used, and multiplying the second term on the right-hand side by $\sqrt{\rho_0/\rho}$. If the form (183) is to be used, H should be placed wherever it appears by $H - h_0$, h_0 being the altitude at which the rate of climb is being determined.

It is equally easy to get the time required to climb to any altitude. If the factors affecting the time are woven into a single mathematical expression, however, this becomes somewhat involved, and it is better to proceed through three successive steps. First find the absolute ceiling, by (172) or (173*b*); then the rate of climb at sea level, by (180) or (183); then the time of climb, by combining the two factors first determined, through the medium of (119) or 120.[1]

Range of Flight.—Maximum radius of action and maximum duration as well, if that is of interest, are also susceptible of simplified treatment. Indeed it becomes particularly desirable in those cases because of the complexity which the calculations assume when the attempt is made to work them through directly for a particular case, with allowance for the variation of the weight during the course of the flight.

Radius of action has been the subject of many careful studies,[2] some of which have involved an exhaustive analysis of variation of engine and propeller characteristics with speed, but for the present purpose it seems hardly worth while to use a very elaborate formula. If anything more than a simple approxima-

[1] See p. 562.

[2] "Analytical Theory of Airplanes in Rectilinear Flight and Calculation of the Maximum Cruising Radius," by A. Rateau, *Trans.* Amer. Soc. Mech. Eng., vol. 42, p. 55, 1920; "A Study of Airplane Ranges and Useful Loads," by J. G. Coffin, *Rept.* 69; "Range of Aircraft," by C. R. Fairey, *J. Roy. Aero. Soc.*, March, 1930.

tion is required, a complete calculation for the specific case can be undertaken.

Writing the derivative of weight with respect to distance flown

$$\frac{dW}{dx} = -\frac{P'}{V}w_f = -\frac{T}{\eta}\cdot\frac{w_f}{375} = -\frac{W}{375}\cdot\frac{w_f}{\eta}\cdot\frac{D}{L} \tag{184}$$

where P' is the power actually being developed, T is the propeller thrust, and w_f is the unit consumption of fuel and oil (combined) per hp.-hr. A process of differentiation of the drag equation, analogous to that which produced (165), gives

$$\frac{D}{L} = 1.14\sqrt{\frac{C_{D_0} +}{R'}} \tag{185}$$

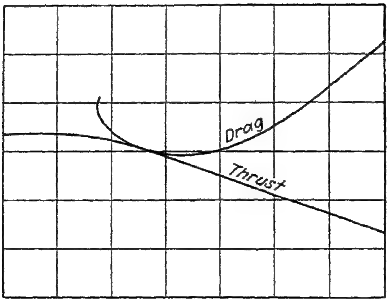


FIG. 426.—Typical relation of curves of drag and thrust under flight conditions corresponding to maximum range.

To be strictly accurate, it is not the minimum value of D/L that is required but the value at the point where the curve of total resistance and the curve of propeller thrust with the engine throttled become tangent to each other, as illustrated in Fig. 426. Though it is difficult to generalize, as much depends on the characteristics of the propeller used, one will not be far off the mark in the average case to assume that the drag at the point of tangency exceeds the minimum drag by 5 per cent. D/L as drawn from (185) may then be multiplied by 1.05.

In determining the ratio η/w_f, variations of power and of V/nD are the controlling factors. As a general rule, the power requirement at the beginning of a long-distance flight is above the

figure for maximum economy, and w_f is likely to be at a fairly high level in the initial stages, decrease to a minimum, and then increase again in the latter half of the flight. The tendency toward increase with weight reduction after passing a minimum is much more marked with fixed-pitch than with controllable-pitch propellers, as the power can be reduced in the fixed-pitch case only by closing the throttle to reduce both the manifold pressure and the r. p. m.; with pitch control the drop of manifold pressure can be kept small, the primary dependence being placed on overloading the propeller by increasing the blade angle and so cutting down the r. p. m.—a much more efficient method of control, from the point of view of fuel economy, than that dependent only on the throttle.

With fixed pitch, V/nD remains surprisingly near to a constant value throughout the flight (not, of course, including the portions of take-off, climb, and descent). The most economical speed decreases slowly as the flight proceeds and the weight is diminished by the consumption of fuel, but at the same time the cruising r. p. m. are both falling off, and at the same rate.[1] The curves of drag and thrust are raised or lowered, moved to the right or left, and changed in curvature by changes of weight, but they always move in conjunction, and the relative location of the point of tangency remains unchanged. Tangency still occurs at the same angle of attack, at the same V/nD, and even corresponds to the same slip-stream velocity ratio (since V_S/V, for a given propeller, is a function of V/nD alone), however the weight may vary.

On the whole, it seems impossible to do better in general terms than to take η/w_f as a constant. With a modern engine, properly adjusted for minimum consumption, η/w_f may be expected to run about 1.45 with a fixed-pitch propeller and 1.60 with controllable pitch. Those are good, though not record-breaking, figures. With a propeller chosen especially for the work to be done, and with a normal V/nD at top speed of at least 1.2, they might be increased by about 5 per cent, or even a bit more than that, especially in the fixed-pitch case. With a propeller running at high r. p. m., and subject to severe limitations on allowable diameter, and with an engine so installed as not to cool readily, the ratios indicated might be reduced by 10 per cent, or even 15.

[1] See also p. 578, *supra*.

D/L, by (185), ranges from 0.057 to 0.13, with 0.09 as a probable value for good normal design. Substituting the values so obtained in (184), and integrating, the range formula becomes

$$X = C \log_{10} \frac{W_1}{W_2} \tag{186}$$

where

$$C = \frac{375}{1.05} \cdot \frac{L}{D} \cdot \frac{\eta}{w_f} \cdot \log_e 10 = 820 \frac{L}{D} \cdot \frac{\eta}{w_f}$$

C averages about 14,500 with a controllable-pitch propeller and 13,200 with a fixed pitch. The limits of possibility, as they now

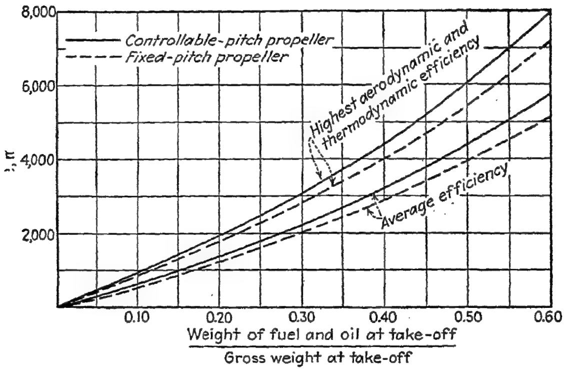


FIG. 427.—Maximum attainable range of flight.

appear, are about 20,000 and 18,200 in the two cases, assuming the cleanest design possible in the present state of the art and the best possible engine-propeller performance. In Fig. 427, curves of range are plotted, in terms of the ratio of total weight of fuel and oil to total gross weight at take-off, for all four values of the constant.

Take-off.—The take-off run is to be computed, as previously explained,[1] by integrating $\frac{v}{dv/dt}dv$. Diehl, to whom that device was due, has also shown[2] that if the excess thrust available for

[1] See p. 586, *supra*.

[2] "The Calculation of Take-off Run," by Walter S. Diehl, *Rept.* 450.

acceleration is a linear function of V the take-off function takes the form

$$\frac{vdv}{dv/dt}dv = \frac{vdv}{\left[1 - \left(\frac{T_1 - T_2}{T_1}\right)\left(\frac{v}{V_t}\right)\right]g\frac{T_1}{W}} \qquad (187)$$

where v_t is the take-off speed, T_1 is the excess thrust at zero speed (the difference between the static thrust and the sum of the rolling resistance of the airplane and the drag produced by the action of the slip stream), and T_2 is the excess at take-off speed. Integrating, the take-off distance becomes

$$X_t = \frac{V_t^2}{gT_1/W}\left[\frac{T_1}{T_1 - T_2}\left(\frac{T_1}{T_1 - T_2}\log_e\frac{T_1}{T_2} - 1\right)\right] \qquad (188)$$

The assumption of linear variation of excess thrust is not quite accurate, for the thrust is likely to fall off quite rapidly with increasing speed in the latter part of the run, and at the same time the drag is increasing sharply as the pilot increases the angle of attack to secure the necessary lift for take-off. There is, however, no other practicable assumption that would be closer to the truth, and the effect of the actual form of the excess-thrust curve can be empirically allowed for by neglecting the last-instant increase in drag in figuring the final excess thrust. If a take-off is being made in the shortest possible distance, the tail will be pulled down very abruptly, and the resultant extra drag will act through so short a time prior to getting off the ground as to have very little effect on the run. What loss is due to the drag may be considered as offset by the gain due to the form of the curve of propeller thrust against speed, a curve customarily concave downward instead of slightly concave upward, as it would have to be (paralleling the curve of drag against speed) to keep the variation of net thrust strictly linear.

By no stretch of the imagination can (188) be called a simple formula. But it can be considerably simplified, with but little error, by making the empirical substitutions, based on experience and on propeller tests in the wind tunnel:

$$\frac{T_1}{W} = (27 - 0.75\beta)\frac{5.25}{n/1{,}000 \cdot D \cdot W/P} - 0.04 \qquad (189)$$

where β is the propeller blade angle at three-fourths maximum radius, D is the diameter, and n is the propeller r. p. m., and

$$\frac{T_1 - T_2}{T_1} = 0.025\frac{W}{P} - 0.01\ (\beta - 24) \tag{190a}$$

It is discovered to be empirically true that

$$\frac{T_1}{T_1 - T_2}\left(\frac{T_1}{T_1 - T_2}\log_e \frac{T_1}{T_2} - 1\right) \approx 0.45 + 0.8\left(1.2 - \frac{T_2}{T_1}\right)^2 \tag{190b}$$

The run then becomes

$$X_t = \frac{V_t^2\left[0.45 + 0.8\left\{0.2 + 0.025\frac{W}{P} - 0.01(\beta - 24)\right\}^2\right]}{\dfrac{80(27 - 0.75\beta)}{n/1{,}000 \cdot D \cdot \dfrac{W}{P}} - 0.60} \tag{191}$$

where V_t is in m. p. h.

It would be quite possible, after a propeller and a gear ratio had been chosen, to use this form directly. Still further simplification can be obtained, however, by assuming a mean value for the propeller tip speed. β will then be approximately expressible as a function of the maximum speed of flight. If 800 ft. per sec. is selected as the most probable peripheral speed, and if it is assumed that the propeller will be chosen to give maximum efficiency at $0.9V_{max}$, β will be approximately equal to $V_{max}/9$, V_{max} being given in m. p. h. Since the take-off should in every case first be computed without allowing for the use of high-lift devices, bringing them in at a later stage if desired, V_t^2 may be replaced by $275W/S$. Furthermore, β at take-off with a controllable-pitch propeller may be taken as 75 per cent of the blade angle for the same installation at maximum speed. The expression (191) then takes the final forms

$$X_t = \frac{52\frac{W}{S}\left[0.45 + 0.8\left(0.2 + 0.025\frac{W}{P} + \frac{220 - V_{max}}{900}\right)^2\right]}{\dfrac{27 - \dfrac{V_{max}}{12}}{W/P} - 0.115} \tag{192a}$$

for a fixed-pitch, and

$$X_t = \frac{52\frac{W}{S}\left[0.45 + 0.8\left(0.2 + 0.025\frac{W}{P} + \frac{275 - V_{\max}}{1,200}\right)^2\right]}{27 - \frac{16}{W/P} - 0.115} \tag{192b}$$

for a controllable-pitch propeller.

In Fig. 428, take-off run is plotted (in terms of W/S, since it is directly proportional to that ratio) against W/P for two different values of $V_{\max}$.

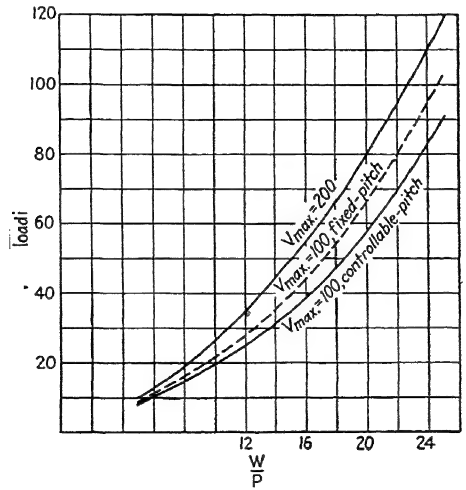


FIG. 428.—Variation of take-off with wing loading and power loading.

If a simpler formula, of a more approximate order, is sought, it can be given the empirical form

$$X_t = C\frac{W}{S}\frac{W}{P} \tag{193}$$

where C averages 2.7 with controllable pitch and 3.3 with fixed pitch. The ratio of those values would indicate a constant saving of 18 per cent in take-off run by the use of controllable pitch, but reference back to (192), or an inspection of the thrust curves

reproduced in Chap. XIV, or a few trial calculations, will show the untrustworthiness of any such generalization. Everything depends on the design of the fixed-pitch propeller. On the average, 18 per cent may be a fair margin of improvement, but there may be as little as 8 per cent in some cases and as much as 30 per cent, or even more, in others. In recent years, the relative advantage of controllable pitch in taking off has been growing in importance, as flight speeds have risen and as geared engines have become more common.

One take-off problem of very serious practical import can be dealt with very simply after the basic calculation has been made for the take-off run with full power. Operators of multi-engined planes always face the danger of a failure of one engine before getting clear of the ground, yet after the take-off has proceeded too far to apply the brakes and come to a stop within the confines of the field. Experience with many experimental calculations shows that the total distance required for a take-off after the failure of an engine, whether the distance is being computed up to the point of leaving the ground or up to that of clearing a barrier of assigned height (not more than 100 ft.), is satisfactorily given by

$$X_t' = x_1 + C_t(X_t - x_1) \tag{194}$$

where X_t is the normal take-off distance with full power (off the ground or over the barrier, as the case may be), X_t' is the corresponding distance in the case where an engine has failed, x_1 the distance covered from the standing start up to the point of losing the dead engine, and C_t a constant depending primarily on the number and arrangement of power plants.

On a twin-engined machine, C_t can be taken as 5.5; on a three-engined one, as 2.5 where the center engine fails and 4.0 where it is a wing engine that goes out; and in the four-engined case, with the four engines arranged in a line along the leading edge of the wing, as 2.0 where it is an outboard engine that fails, and 1.75 for an inboard failure. So that if an airplane normally capable of getting over a 50-ft. obstacle in 1,800 ft. were to suffer the loss of an engine after 1,000 ft. of run, the distance needed to rise clear would be increased to 5,400 ft. for a twin-engined plane, and to from 2,400 to 2,600 ft. for a four-engined one (the

choice between the two figures in the latter case depending upon where the errant engine was located along the span).

Performance Calculation for Small Design Changes.—Closely related to the use of formulas is the practice utilized when the effect of a particular change in form, or in some single design characteristic, has to be determined with great accuracy. If power is to be increased by 2 per cent, for example, it would be foolish to start again at the beginning and to carry through a completely new computation. Not only can a great amount of time be saved, but the effect of the change can actually be ascertained with greater accuracy than by the straightforward repetition of the initial work. Where the effect of a small difference is to be determined, it is always better to approach it directly than indirectly.

The direct approach, in the case of a power change, would require: (*a*) the writing of the power equation for the various flight conditions, and in particular for maximum speed, for such cruising conditions as may be of interest, and for minimum power; (*b*) the differentiation of the equation; (*c*) the insertion of the appropriate numerical values for the various factors of the derivative, for they can be regarded as constants if the change to be made is a small one; (*d*) the determination, on the assumption of a constant value of the derivative, of the change in one of the two main variables which would correspond to a specified change in the other.

So for example in the case just suggested (that of a small increase of power), the general power equation takes the form

$$P = \frac{(D_{C_o} + D_{C_p})\frac{\rho}{2}SV^3 + \dfrac{W^2}{\frac{\rho}{2}\pi b^2 eV}}{550} \tag{195}$$

Differentiating with respect to speed

$$\frac{dP}{dV} = \frac{3(D_{C_p} + D_{C_o})\frac{\rho}{2}SV^2 - \dfrac{W^2}{\frac{\rho}{2}\pi b^2 eV^2}}{550} \tag{196}$$

D_{C_p}, D_{C_o}, ρ, S, W, and b are all constants in this particular problem.

V is very nearly constant for small changes in P, and can be so considered. For the greatest possible accuracy, a mean value of V should be approximated. It is of course obvious that the speed will vary a little less rapidly than the cube root of the power. If the power is to be increased by 2 per cent, then it may be assumed as a first approximation that the speed will be increased by 0.5 per cent. Since it is a mean of the initial and final values that is wanted, the value of V to be substituted in (196) should be 0.25 per cent higher than the initial value. If the derivative then indicates that this particular airplane, under the particular conditions of the problem, requires an addition of 6 hp. to gain 1 mile an hour, the means of finding the gain in speed that will result from the particular power change contemplated are too clear to need explanation.

Essentially the same method, though with a different equation and with the derivative taken with respect to a different quantity, can be applied when the weight, or the span, or any other quantity is to be changed. The method is a particularly useful one for studying the effect on performance of changes in gross weight.

Performance Charts.—Most of the formulas to which the last 40 pages have been devoted are frankly approximations, with more concern for speed than for accuracy. But in the course of their derivation there have been presented equations which could have been made the basis of a very accurate prediction indeed. It would only have been necessary, instead of approximating to representative values of the various design constants, to give them the specific values appropriate to a particular airplane. If R and $C_{D_0} + C_{D_p}$ had been left as variables, together with W/S and W/P in their various combinations, every element relating to performance could have been expressed in analytical form with substantial rigor. Even the propeller characteristics could be given analytical representation, by such empirical fitting of formulas to the experimental curves as has been practiced several times in the earlier sections of this chapter.

The analytical forms so obtained would generally be too complex for direct solution, but they can be handled as the problem of variation of speed with altitude has already been treated here. They can be developed into charts.

The making of such charts and nomograms in various forms has been a common pastime both in America and in Europe for

more than a dozen years.[1] All depending on a common principle, essentially the same that governs the derivation of the quick-reference formulas, they have been gradually refined and extended to a culmination, up to the present time, in the charts of Oswald.[2]

The Oswald charts are too numerous to be reproduced in a general text. Reference to the original source, where more than a dozen will be found, is strongly recommended. Given some experience in their use, and limiting their application to designs not differing too radically in general conformation from those upon which the experience has been gained, they may even replace direct calculation entirely. But it is not advised that they be resorted to so exclusively that capacity to use the direct-calculation method is allowed to atrophy. Both the direct method and the book of charts, and the approximation by formula as well, ought to be in the designer's arsenal.

The charts are built around various combinations of four basic parameters. Keeping the symbols for familiar quantities consistent with those heretofore used, they are

$$l_p = \frac{W}{(C_{D_o} + C_{D_p})S} \tag{196a}$$

$$l_s = \frac{W}{k^2b^2e} - \frac{W}{R'S} \tag{196b}$$

$$l_t = \frac{W}{\eta_{\max}P} \tag{196c}$$

$$\Lambda = \frac{l_s l_t^{4/3}}{l_p^{1/3}} = \frac{W^2\sqrt[3]{(C_{D_o} + C_{D_p})S}}{P^{4/3}\eta_{\max}^{4/3}R'S} \tag{196d}$$

where e is the "span efficiency"[3] and k is the biplane span factor

[1] "An Empirical-theoretical Method of Comparative Prediction of Airplane Performance," by L. V. Kerber, *Information Circ.* 68, Airplane Eng. Div., U. S. Air Service; "Graphic Method for Calculating the Speed and Climbing Ability of Airplanes," by Adolf Rohrbach and Edwin Luberger, *Technische Berichte*, vol. III, No. 6, translated in *Tech. Note* 163; "Airplane Performance and Design Charts," *Information Circ.* 183, Airplane Eng. Div., U. S. Air Service; "Airplane Design Graphs," by W. F. Gerhardt and F. W. Herman, Dayton, 1924.

[2] "General Formulas and Charts for the Calculation of Airplane Performance," by W. Bailey Oswald, *Rept.* 408; "Methods of Performance Calculation for Airplanes with Supercharged Engines Developed by W. Bailey Oswald," by R. B. Ashley, *Air Corps Information Circ.* 679.

[3] See p. 607, *supra*.

or any other such factor modifying the effective span for the purpose of determining the induced drag.

All these expressions have a certain familiarity of appearance, and that for Λ, which is the most important of all the parameters, is interchangeable with the expression for m.[1] The two differ only by a constant coefficient. As between the analytical expressions used, that for Λ is the more closely attached to the fundamentals, but the formula for m has the advantage of being the easier to remember and the easier to apply mentally, at least to a close approximation, when once the maximum speed has been determined.

Logarithmic Charts.—There is another device, of quite a different order, for solving certain problems both of airplane and of propeller performance. The logarithmic diagram of Eiffel,[2] though it has fallen into disuse in recent years because of its requirement of a neat graphical construction whenever it is used, still deserves to be understood, to be remembered, and occasionally (especially in connection with propeller computations) to be resorted to.

The basic principle is identical with that of slide-rule construction. It is the taking of the logarithm of each side of an equation, to the end that a product of terms raised to various exponents may resolve itself merely into a laying end to end of the logarithms of the several components, multiplied by numbers equalling their several exponents in the equation. But it is easier to show it than to explain it.

The familiar power-requirement equation is written

$$C_D S = \frac{550 P\eta}{\frac{\rho}{2} V^3}$$

The equation describing the relation between weight and speed is

$$C_L S = \frac{W}{\frac{\rho}{2} V^2}$$

[1] See p. 605, *supra*.

[2] "Nouvelles recherches sur la résistance de l'air et l'aviation," by G. Eiffel, Paris, 1914.

Taking the logarithm of each expression,

$$\log C_D S = \log (P\eta) - \log \frac{\rho}{2} - 3 \log V + \log 550$$

and

$$\log C_L S = \log W - \log \frac{\rho}{2} - 2 \log V$$

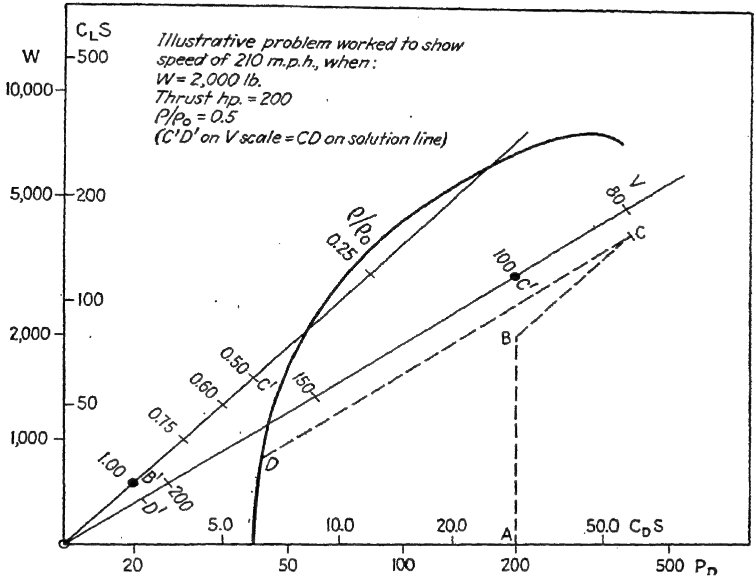


FIG. 429.—Logarithmic diagram for an airplane.

A chart is then constructed as in Fig. 429. The polar of the airplane, obtained either by wind-tunnel test or (more accurately) by computation, is plotted on a logarithmic scale. An inverted scale of densities is plotted (logarithmically) along a 45-deg. diagonal, with the scale elongated in the ratio of 1.41 to 1 as compared with that used in plotting the lift and drag, so that the projections of the density line onto the vertical and horizontal axes may be equal to each other, and on a scale (1.41 × 0.71) equal to that used for the coefficients. Speed is similarly plotted (logarithmically) along a line inclined to the horizontal at an angle

of which the tangent is $\frac{2}{3}$ and with a scale equal to $\sqrt{13}$ times the basic scale, so that its projections onto the vertical and horizontal axes, respectively, are $-2 \log V$ and $-3 \log V$. Weight is plotted along the vertical axis, power along the horizontal one. The logarithm of 550 is introduced as a horizontal displacement of the power scale.

In using the chart, it is necessary only to draw a broken line of four segments, starting from the origin and terminating at a point on the polar curve, the four segments being parallel to, and of lengths determined from, the scales of W, P, ρ/ρ_0, and V. Any three of the quantities can be fixed in advance, the corresponding segments drawn, and the solution made for the fourth quantity by simply drawing the closing segment. If the problem is an impossible one, the three figures specified being inconsistent among themselves, the fourth segment will fail to intersect the curve. The origins for the various curves must of course be so chosen as to be mutually consistent. The origin for V having been arbitrarily set at 100 m. p. h., for example, and that for ρ/ρ_0 at unity, the scale of weight must be so placed that the weight opposite each value of C_LS on the vertical scale will be that which would be supported by that lift coefficient at standard sea-level density and 100 m. p. h. The scale of power and that of C_DS must be tied together in the same fashion. An illustrative problem, self-explanatory, is worked in Fig. 429.

There is little that can be done with the logarithmic chart for airplanes that cannot be more simply and more accurately done by more direct means, but application to the propeller brings the logarithmic device into its own. It provides the means of doing with a single chart what might otherwise require three or four, and some special constructions as well. Such problems as that of determining the variation of r. p. m. with flight speed, for example, are solved as rapidly as lines can be drawn across a chart.

The technique is the same as for the airplane; the basic plot is of log C_P against log V/nD. The scales are of P, V, n, D, and ρ, inclined and proportioned to give horizontal and vertical projections adapted to fit into their proper places in the logarithmic equations

$$\log \frac{V}{nD} = \log V - \log n - \log D$$

and

$$\log C_p = \log P - 3 \log N - 5 \log D - \log \rho$$

Figure 430 shows the resultant chart and the method of using it. The figures spotted along the curve are propeller efficiencies, and it will be noted that the efficiency in the example worked (in dotted lines) is almost exactly the peak value for the propeller.

The horizontal scale in Fig. 430 was arbitrarily set at double the vertical one, which required that the angles of the n and D scales

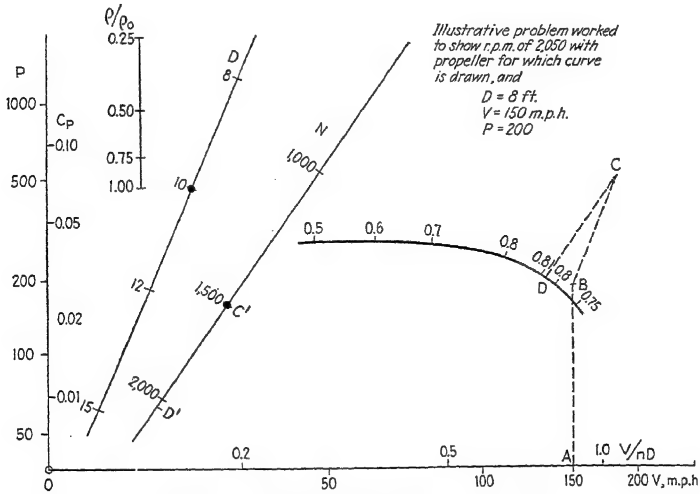


FIG. 430.—Logarithmic diagram for a propeller.

to the vertical axis be doubled. That is common practice in using the logarithmic diagram, as all the segments except that of speed would otherwise be very nearly parallel and an accurate construction would be difficult.

The same sort of diagram can be constructed with C_Q instead of C_P as the ordinate, and with the scale of r. p. m. set at 45 deg. to the vertical (where the doubled horizontal scale is used) instead of the 33.7 deg. used in this case. Although it is the less frequently seen, the torque diagram is somewhat more convenient, as torque is substantially independent of r. p. m. and problems

can be solved on the torque diagram without requiring any such simultaneous manipulation of two of the variables as may be necessary where power is the basic factor. The difference between the power and the torque diagrams is purely in the scales. The characteristic curve of a given propeller plots in exactly the same form on the two, and the same diagram may in fact be used, with merely the addition of a scale of torques along the vertical axis and a supplementary scale of n to be used in conjunction with it.